10.- Oligopoly, Tariffs, Crime and Voting

- This chapter includes examples of interesting economic models (games) and their Nash equilibrium properties.

➢ Oligopoly: Competition in quantities (Cournot) and prices (Bertrand).

➢ A simple game of tariff setting between countries.

➢ A game of crime and police.

➢ Using the location game from previous chapters to explain the “median voter theorem”.

- **Oligopoly:** The chapter considers an industry with two dominant firms and two different ways to model how these firms compete with each other:

1. **Assuming firms choose the quantity produced and let the market demand decide the corresponding price:** This gives rise to the Cournot model that we have studied extensively.
2. **Assuming firms choose the price at which they will sell and let the market demand decide the quantity they must produce at those prices:** This gives rise to what is called the Bertrand model of competition.

- We have studied previously the Nash equilibrium properties in the Cournot model. **We will focus on the Bertrand model**.

- **Setup:** Two dominant firms in an industry. Suppose the market demand function in this industry is given by the equation:

$$p = 1000 - q_1 - q_2$$

- If we let $Q = q_1 + q_2$ denote the total quantity produced by both firms, then the previous equation implies that, if consumers face price p, then they will demand a total quantity of:

$$Q = 1000 - p$$

- Suppose the following holds:

a) Consumers purchase from the firm that charges the lower price.

b) If both firms set the same price, then the demand is split evenly between the two.

c) Suppose the cost of producing q_i is given by $100 \cdot q_i$ for each firm $i = 1,2$.

d) Suppose both firms select a price simultaneously. Denote these prices by p_1 and p_2.

- What is the Nash Equilibrium in this game?
- Again, since this is a game with a continuous strategy, to find the Nash equilibrium we first need to characterize the best response functions of each firm.
- Let p_j denote the price set by firm j. **What is $\boldsymbol{BR_i(p_j)}$? (the best response of player $\boldsymbol{i}$)**
- If $p_i > p_j$, then firm j captures the entire demand and firm i sells nothing. Therefore, the profits to i would be zero. That is, $u_i(p_i, p_j) = 0$.
- If $p_i = p_j$, then both firms split the demand evenly, each producing $q_i = \frac{1000-p_i}{2}$ and earning profits:

$$u_i(p_i, p_j) = p_i \cdot \left(\frac{1000 - p_i}{2}\right) - 100 \cdot \left(\frac{1000 - p_i}{2}\right)$$

$$= \frac{1}{2} \cdot (p_i - 100) \cdot (1000 - p_i)$$

- Finally, if $p_i < p_j$, then firm i captures the entire market, producing $q_i = 1000 - p_i$ and earning profits:

$$u_i(p_i, p_j) = p_i \cdot (1000 - p_i) - 100 \cdot (1000 - p_i)$$
$$= (p_i - 100) \cdot (1000 - p_i).$$

- Therefore, given a price p_j chosen by firm j, the profit for firm i is given by:

$$u_i(p_i, p_j) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2} \cdot (p_i - 100) \cdot (1000 - p_i) & \text{if } p_i = p_j \\ (p_i - 100) \cdot (1000 - p_i) & \text{if } p_i < p_j \end{cases}$$

- Therefore, the **best response to $\boldsymbol{p_j}$ is a price $\boldsymbol{p_i}$ "infinitesimally" smaller than $\boldsymbol{p_j}$ as long as $\boldsymbol{u_i(p_i, p_j) \geq 0}$.**

- Note that if $p_i < 100$, then
$$(p_i - 100) \cdot (1000 - p_i) < 0$$

- Therefore, if $p_j = 100$, the best response by player i is to also choose $p_i = 100$.

- And if $p_j > 100$, the best response by player i is to choose a price p_i infinitesimally smaller than p_j.

- Summarizing, the best response function by player i is given by:

$$BR_i(p_j) = \begin{cases} 100 \text{ if } p_j = 100 \\ \text{A price infinitesimally smaller} \\ \text{than } p_j \text{ if } p_j > 100 \end{cases}$$

- Therefore, $(\boldsymbol{p_1}, \boldsymbol{p_2}) = (\mathbf{100}, \mathbf{100})$ **is a Nash equilibrium of this game and it is the ONLY Nash equilibrium of this game** (if any firm chooses a price above 100, the competitor will undercut it and this will continue until they both reach 100).

- Note that, **unlike the Cournot competition, in the Bertrand model both firms earn zero profits in equilibrium**.

- **Cournot vs. Bertrand competition:** Since both models producevery different predictions in equilibrium, the question is which one should we use to study an industry in the real world?

- **It depends on whether it is more realistic to assume that firms commit to quantities or if they commit to prices.**

- In industries where production processes are costly and complex (e.g, automobiles, pharmaceuticals) or where the production cycle is longer and rigid (agricultural products), it may be more realistic to assume that firms have to commit ex-ante to a production plan, and therefore Cournot competition may be a better approximation.

- In industries where firms can adjust their production levels on short notice (e.g, software distributed over the internet), price competition (Bertrand model) may be a more accurate approximation.

- Introducing capacity constraints (where individual firms are not able to satisfy the demand of the entire market by themselves) into the Bertrand model would change the predictions of this model in an important way. We will study this in Chapter 11.

- **Tariff Setting by Two Countries:** Taxes on the importation of foreign commodities are called ***tariffs***.
- When a large country (a country able to influence world prices) imposes a tariff it experiences two effects:

1. A terms of trade improvement.
2. An efficiency loss.

- As the tariff restricts domestic demand, terms of trade improvements are derived from a decrease in the world price of the imported good. Efficiency losses arise from distortions to producer and consumer prices.
- Terms of trade: Relative price (in international markets) of exports compared to imports

- If the country is large enough, the gain in the terms of trade can compensate the efficiency loss from the tariff.

- **However**, if other countries retaliate with tariffs of their own, the gains from terms of trade could be nullified and all that remains would be the efficiency loss from the tariff.

- We can model the decision of how to set tariffs optimally as a game.

- Suppose there are two countries: $i = 1,2$. Let x_i denote the tariff set by country i (with x_i between 0 and 100).

- If country i chooses tariff x_i and if country j chooses tariff x_j, the payoff to i is:

$$\boldsymbol{u_i(x_i, x_j) = 2000 + 60 \cdot x_i + x_i \cdot x_j - {x_i}^2 - 90 \cdot x_j}$$

- Suppose both countries choose their tariff levels simultaneously and independently. **Find the Nash equilibrium of this game**.

- Again, since strategies are continuous, we need to begin by characterizing both players' **best response functions**. This can be done by looking at the **first order conditions:**
- For a given x_j, the best response by player i is given the solution (in x_i) to the first order condition:

$$\frac{\partial u_i(x_i,x_j)}{\partial x_i} = 0$$

- That is,

$$60 + x_j - 2 \cdot x_i = 0$$

- This yields:

$$\boldsymbol{BR_i(x_j) = 30 + \frac{1}{2} \cdot x_j}$$

- The Nash equilibrium of this game is given by the profile of strategies (x^*_1, x^*_2) such that:

$$x^*_1 = BR_1(x^*_2) \quad \text{and} \quad x^*_2 = BR_2(x^*_1)$$

- That is,

$$x^*_1 = 30 + \frac{1}{2} \cdot x^*_2$$
$$x^*_2 = 30 + \frac{1}{2} \cdot x^*_1$$

- The best response functions of each player is increasing in the import tariffs set by the other country, implying that **tariffs are strategic complements in this game**.
- Plugging the first expression into the second equation:

$$x^*_2 = 30 + \frac{1}{2} \cdot \left(30 + \frac{1}{2} \cdot x^*_2\right) = 45 + \frac{1}{4} \cdot x^*_2$$

- This yields:

$$\frac{3}{4} \cdot x^{*}{}_{2} = 45$$

- Therefore:

$$x^{*}{}_{2} = 60$$

- Plugging this back into the first equation, we have:

$$x^{*}{}_{1} = 30 + \frac{1}{2} \cdot 60 = 60$$

- The equilibrium tariffs are (60,60) (sixty percent tariffs in each country).

- **A Game of Crime and Police:** There is an obvious *strategic interaction* relationship between the police and criminals:
- ***Government (police):*** They try to select a level of law enforcement in order to reduce the impact of crime on society, but law enforcement is a costly effort
- ***Criminals:*** They try to select a level of criminal activity, subject to a certain probability of getting caught. The latter is affected by the level of law enforcement.
- This strategic interaction can be approximated through a simple game where the police selects a level of law enforcement and criminals select a level of criminal activity.

- Let $\boldsymbol{x} \geq \mathbf{0}$ denote the level of **law enforcement** by the government.
- Let $\boldsymbol{y} \geq \mathbf{0}$ denote the level of **criminal activity** by the criminals.
- Suppose crime is costly on society, but that this cost can be reduced through law enforcement. Suppose also that law enforcement itself is a costly activity.
- Suppose that the probability of evading capture decreases with the level of law enforcement, and also with the level of criminal activity.
- We will assume payoff functions that capture the features described above.

- Take a profile of strategies (x, y). Suppose payoff functions are given by:
- For the government:

$$u_G(x, y) = -x \cdot c^4 - \frac{y^2}{x}$$

the term $-x \cdot c^4$ represents the cost of law enforcement (the magnitude of this cost depends on the parameter c^4), and the term $-\frac{y^2}{x}$ represents the social cost of crime on society (which can be reduced by higher levels of law enforcement).

- For criminals:

$$u_C(x,y) = \frac{y^{1/2}}{(1 + x \cdot y)}$$

The term $y^{1/2}$ represents the fact that criminals enjoy their activities, while the term $\frac{1}{(1+x\cdot y)}$ represents the probability of evading capture. This probability is assumed to be decreasing in x (more law enforcement makes it more likely to get caught) and also in y (more criminal activity increases the chances criminals will get caught).

- **Characterize the best response functions in this game:** Again, because strategies are continuous, this is done by looking at the first order conditions in the model.
- **For the government:** The best response $BR_G(y)$ is given by the solution (in x) to the first order conditions:

$$\frac{\partial u_G(x, y)}{\partial x} = 0$$

- **For the criminals:** The best response $BR_C(x)$ is given by the solution (in y) to the first order conditions:

$$\frac{\partial u_C(x, y)}{\partial y} = 0$$

- We have

$$\frac{\partial u_G(x,y)}{\partial x} = \frac{y^2}{x^2} - c^4$$

- Therefore, $BR_G(y)$ is given by the solution (in x) to

$$\frac{y^2}{x^2} - c^4 = 0, \quad \text{that is:} \quad \boldsymbol{BR_G(y) = \frac{y}{c^2}}$$

- And we have

$$\frac{\partial u_C(x,y)}{\partial y} = \frac{1 - x \cdot y}{2 \cdot y^{1/2} \cdot (1 + x \cdot y)^2}$$

- Therefore, $BR_C(x)$ is given by the solution (in y) to

$$\frac{1 - x \cdot y}{2 \cdot y^{1/2} \cdot (1 + x \cdot y)^2} = 0, \quad \text{that is:} \quad \boldsymbol{BR_C(x) = \frac{1}{x}}$$

- Note that the best response level of law enforcement is increasing in the level of criminal activity, and the best response level of criminal activity is decreasing in the level of law enforcement.
- **Find the Nash equilibrium of this game:** This is the profile (x^*, y^*) such that:

$$x^* = BR_G(y^*) \quad \text{and} \quad y^* = BR_C(x^*)$$

- That is:

$$x^* = \frac{y^*}{c^2} \quad and \quad y^* = \frac{1}{x^*}$$

- The solution to this system yields:

$$(x^*, y^*) = \left(\frac{1}{c}, c\right)$$

- **The Median Voter Theorem:** The location game (Hotelling model) studied in the previous chapter has been used to study how candidates decide to locate across the *political spectrum*.
- A general result in these types of location games is that the Nash equilibrium consists of players locating "in the middle" of the spectrum.
- The general notion of "in the middle" in these models refers to locating in the point of the spectrum where both players would "split the market" exactly in half. This corresponds to the **median point of the spectrum**.

- This explains what political scientists refer to as the **median voter theorem**, which predicts that candidates will tailor their platforms to the tastes of the median voter.

- **Remark:** The location game we studied is only a simple approximation to a more complicated real-world problem. Its predictions could break down, for example, if we have three or more firms (candidates) or –in the case of the median voter- if there is **strategic voting** on behalf of voters, which could induce them to vote for candidates that are not necessarily "closest" to their political tastes.