12.- Strictly Competitive Games and Security Strategies

- So far we have assumed that players choose strategies that maximize their expected payoff.
- This chapter briefly discusses an alternative model of behavior where agents choose "security strategies".
- The chapter then describes the relationship between this type of behavior and Nash equilibrium in a class of games called "strictly competitive games".

- **Security strategy:** A security strategy focuses on "worst case scenarios".
- In any given game, let

$$w_i(s_i) = \min_{s_j \in S_j} u_i(s_i, s_j)$$

- Note that $w_i(s_i)$ is the worst payoff that player i can obtain if he chooses the strategy s_i.
- In other words, if player i chooses strategy s_i, then he can assure himself a payoff at least equal to $w_i(s_i)$.

- A **security strategy** $\underline{s}_i$ is defined as the strategy $s_i \in S_i$ that yields the largest value of $w_i(s_i)$.
- That is, a security strategy $\underline{s}_i$ satisfies:

$$w_i(\underline{s}_i) \geq w_i(s_i) \quad \text{for all} \quad s_i \in S_i$$

- **Example:** Consider the following game:

1 \ 2	X	Y
A	3, 2	0, 4
B	6, 1	1, 3

- We have:

$$w_1(A) = 0, \text{ and } w_1(B) = 1$$
$$w_2(X) = 1, \text{ and } w_2(Y) = 3$$

- Therefore,

$$\underline{s}_1 = B \quad \text{and} \quad \underline{s}_2 = Y$$

- The concept of security strategies can also be extended to mixed strategies. We refer to these as **maxmin strategies**. For a given mixed strategy σ_i let

$$w_i(\sigma_i) = \min_{s_j \in S_j} u_i(\sigma_i, s_j)$$

- A **maxmin strategy** is given by the mixed strategy $\underline{\sigma_i}$ that maximizes $w_i(\sigma_i)$ over the set of all mixed strategies $\sigma_i \in \Delta_i$.

- What is the relationship between security strategies and the notions of rational behavior we have studied so far?
- There is no general relationship. For instance, it is easy to construct examples where security strategies are not rationalizable. For example, take the game:

1 \ 2	X	Y
A	3, 5	-1, 1
B	2, 6	1, 2

- Note that (A,X) is the only rationalizable profile. However, B is the security strategy for player 1.

- How about the relationship between Nash equilibrium strategies and security strategies?
- There is no general relationship results between Nash equilibrium strategies and security strategies. However, there are results for a class of **two-player** games called **strictly competitive games**.
- A **two player strictly competitive game** is a two player game with the property that, for every two strategy profiles s and s':

$$u_1(s) > u_1(s') \text{ if and only if } u_2(s) < u_2(s')$$

- In a two-player strictly competitive game, if one player strictly prefers the outcome s to the outcome s', then the other player must strictly prefer s' to s.
- Matching pennies is a special case of a strictly competitive game. It is in fact a zero-sum game (not all strictly competitive games are zero-sum games).
- Intuitively, think about games in which all outcomes can be classified as either:
  - Player 1 “wins” and player 2 “loses”.
  - Player 2 “wins” and player 1 “loses”.
  - Both players “tie”.
- If players prefer winning to tying and tying to losing, then any game like the one described above will be a strictly competitive game.
- For example, games of sports (tennis, football, baseball, etc.) and leisure (checkers, chess, etc.) are strictly competitive games.

- The following result relates Nash equilibrium strategies with security strategies:
- **Result:** If a two-player game is strictly competitive and has a Nash equilibrium $s^* = (s^*{}_1, s^*{}_2)$, then $s^*{}_1$ is a security strategy for player 1 and $s^*{}_2$ is a security strategy for player 2. Furthermore, $s^*{}_1$ and $s^*{}_2$ are also maxmin strategies for players 1 and 2.
- Therefore, if s_i is a Nash equilibrium strategy for player i in a two-player strictly competitive game, then s_i is a security strategy and player i cannot improve the security payoff produced by s_i by using a mixed strategy instead.

- Let us go back to the game:

1 \ 2	X	Y
A	3, 2	0, 4
B	6, 1	1, 3

- We have:

$$u_1(A,X) = 3, u_1(A,Y) = 0, u_1(B,X) = 6, u_1(B,Y) = 1$$
$$u_2(A,X) = 2, u_2(A,Y) = 4, u_2(B,X) = 1, u_2(B,Y) = 3$$

- Note that:

$$u_1(B,X) > u_1(A,X) > u_1(B,Y) > u_1(A,Y)$$
$$u_2(A,Y) > u_2(B,Y) > u_2(A,X) > u_2(B,X)$$

- The rankings are exactly reversed between the two players. Therefore **this is a strictly competitive game.**
- This game has a unique pure-strategy Nash equilibrium: (B,Y). We showed previously that this is also the unique profile of security strategies.

- On the other hand, consider again the game:

1 \ 2	X	Y
A	3, 5	-1, 1
B	2, 6	1, 2

- We showed that B is a security strategy for player 1 even though it is not rationalizable.
- This is **not a strictly competitive game**. To see this, note that $u_1(B, Y) = 1$, $u_1(A, Y) = -1$**,** $u_2(B, Y) = 2$ and $u_2(A, Y) = 1$. Therefore:

  $u_1(B, Y) > u_1(A, Y)$ and $u_2(B, Y) > u_2(A, Y)$
- This violates the description of a strictly competitive game.

- **Recap:** In strictly competitive games (with two players), the behavior produced by the Nash equilibrium concept is consistent with the one produced by security strategies.
- Outside this class of games, both models can produce vastly different predictions even in very simple games.
- These differences could allow us to test in the real world which of these two behavioral models seems to approximate real-world behavior best (in games that are not strictly competitive).