13.- Contract, Law, and Enforcement in Static Settings

- Under some conditions, the existence of contracts can help alleviate the three "strategic tensions" we have described before:
  - Conflict between individual and group interests.
  - Eliminate strategic uncertainty (by aligning beliefs and behavior).
  - Rule out the possibility of coordinating towards inefficient equilibria.
- This chapter describes such conditions in simple games.

- **Contract (definition):** An agreement about behavior that is intended to be enforced.
- A **contractual relationship** refers to an interaction relationship whose terms have been set ex-ante by the economic agents involved. A contractual relationship has two phases:
  - The **contracting phase** when the terms of the contract are set.
  - The **implementation phase** when the contract is carried out (i.e, when the game is played according to the terms of the contract).

- This chapter focuses on the implementation phase of the contract, without modeling the details of the contracting phase (we will study this later in the course).

- A contract is **self-enforced** if the players have the individual incentives to abide by the terms of the contract without any external parties involved.

- A contract is **externally enforced** if the players have an incentive to abide by its terms only if a **third party** (a judge or arbitrator) intervenes to modify the parameters of the interaction.

- **Example:** Consider two parties (players) who have to undertake a joint project.
- Each player must decide whether to **invest effort in the project (I)** or **not invest effort in the project (N)**.
- Suppose the relationship between these two players and can be described by the following normal-form game:

1 \ 2	I	N
I	z_1, z_2	y_1, x_2
N	x_1, y_2	0, 0

- Suppose the payoffs described above actually correspond to **monetary payoffs.**
- Let us refer to the total monetary payoffs of a given outcome as the joint value of the project. This corresponds to the sum of the payoffs of both players.
- Let us assume that the joint value of the project is at its maximum if both players invest in effort. Specifically, let us assume that:

$$z_1 + z_2 > x_1 + y_2$$
$$z_1 + z_2 > x_2 + y_1$$
$$z_1 + z_2 > 0$$

- Throughout this chapter, suppose that **monetary transfers between players are possible**.
- Specifically, we assume that **players can make monetary transfers between themselves**.
- Given this ability, then the previous assumption about $z_1 + z_2$ implies that $(\boldsymbol{I}, \boldsymbol{I})$ **is the only efficient outcome in this game**.
- The question becomes: Under what conditions can the players write a contract that guarantees that the most efficient outcome $(\boldsymbol{I}, \boldsymbol{I})$ is the outcome actually played in the game?

- If we assume that players always freely and voluntarily agree to play the most efficient Nash equilibrium, then $(\boldsymbol{I}, \boldsymbol{I})$ would be automatically played **as long as it is a Nash equilibrium**. That is, as long as:

$$z_1 \geq x_1 \quad \text{and} \quad z_2 \geq x_2$$

- If either of these conditions is violated, then $(\boldsymbol{I}, \boldsymbol{I})$ cannot be a self-enforced outcome. In this case, an **externally enforced contract** would be necessary.
- Under what conditions can a contract modify the underlying game to ensure that (I, I) is a Nash equilibrium?
- A contract in this setting would consist of **monetary transfers between the players**.

- These monetary transfers would transform the underlying game into an **induced game** (the game induced by the monetary transfers stipulated in the contract).
- The induced game would be of the following form:

1 \ 2	I	N
I	$\boldsymbol{z_1, z_2}$	$\boldsymbol{y_1+\beta, x_2-\beta}$
N	$\boldsymbol{x_1-\alpha, y_2+\alpha}$	$\boldsymbol{\gamma, -\gamma}$

- Here, α, β and γ represent monetary transfers between the players.

- For example:

$\beta =$ Monetary transfer from player 2 to player 1 if player 1 invests effort,

but player 2 does not invest effort.

$\alpha =$ Monetary transfer from player 1 to player 2 if player 2 invests effort,

but player 1 does not invest effort.

$\gamma =$ Monetary transfer from player 2 to player 1 if neither player invests effort.

- A contract could punish players unevenly if they do not invest effort.

- The values of the monetary transfers α, β and γ are specified by the contract. A third party ("the court") is in charge of enforcing these transfers.
- Allowing for the transfers to depend on the specific outcome of the game corresponds to a **complete contract**, defined as a contract that stipulates transfers for each outcome of the game.
- Enforcing a complete contract requires that the court be able to observe exactly the strategies chosen by each player. The court needs to know exactly which was the realized outcome in the game. This is called **full verifiability**.

- Full verifiability would fail if the court cannot distinguish between multiple outcomes.
- Suppose the contract is **complete** (it specifies a monetary transfer for each possible outcome) and **full verifiability** is possible.
- Then, it is easy to see that we can always find transfers α, β and γ such that (I, I) is a Nash equilibrium. All we need is:
  - α should be such that choosing I is a best response for player 1 if player 2 chooses I.
  - β should be such that choosing I is a best response for player 2 if player 1 chooses I.

- That is, α should be such that $\boldsymbol{z_1 \geq x_1 - \alpha}$**.** That is, we must choose α such that:

$$\alpha \geq x_1 - z_1$$

- Note that $x_1 - z_1$ is the monetary gain for player 1 if he deviates from the outcome (I, I). Therefore, the transfer player 1 has to make if he deviates must be at least equal to that gain.
- Similarly, β should be such that $\boldsymbol{z_2 \geq x_2 - \beta}$. That is:

$$\beta \geq x_2 - z_2$$

- The penalty for player 2 once again must offset the gain from deviating from the outcome (I, I).

- As long as the contract stipulates transfers α and β that satisfy these conditions, and if there is full verifiability, then (I, I) will be a Nash equilibrium in the induced game.
- The monetary transfer γ does not affect this result, and can be fixed to zero.
- Therefore, **a complete contract with full verifiability can always enforce the efficient outcome $(\boldsymbol{I}, \boldsymbol{I})$.**

- **What happens without full verifiability?**

- Full verifiability will cease to exist if the court is unable to distinguish between a subset of outcomes. In this case we say that there exists **limited verifiability**.

- Suppose in the previous example, that the court can only distinguish between the following outcomes:
  - Both players invested effort.
  - Some player (maybe both) failed to invest effort, but the court cannot know exactly who.

- In this case, the court could not be able to distinguish between $(I, N), (N, I)$ and (N, N).
- With limited verifiability the court no longer has the ability to enforce a contract with different monetary transfers for $(I, N), (N, I)$ and (N, N).
- Any contract that can be enforced now has to read as follows:

"If both players invest effort, then no transfers take place. Otherwise, if some player fails to invest effort, then there will be a monetary transfer of α from player 2 to player 1"

- Note that α can be positive (player 2 transfers money to player 1) or negative (player 2 receives money from player 1).

- The induced game with limited verifiability looks like this:

1 \ 2	I	N
I	z_1, z_2	$y_1 + \alpha, x_2 - \alpha$
N	$x_1 + \alpha, y_2 - \alpha$	$\alpha, -\alpha$

- Can a contract like this still enforce the outcome (I, I)? Not always…
- (I, I) will be a Nash equilibrium in the induced game if and only if:

$$z_1 \geq x_1 + \alpha \quad \text{and} \quad z_2 \geq x_2 - \alpha$$

- That is, we must have:

$$z_1 - x_1 \geq \alpha \quad \text{and} \quad \alpha \geq x_2 - z_2$$

- In other words, the transfer α must satisfy:

$$x_2 - z_2 \leq \alpha \leq z_1 - x_1$$

- This will be possible if and only if:

$$x_2 - z_2 \leq z_1 - x_1$$

- Rearranging, this condition can be re-expressed as:

$$z_1 + z_2 \geq x_1 + x_2$$

- Thus, with limited verifiability, a contract like the one described above will enforce the outcome (I, I) if and only if:

$$z_1 + z_2 \geq x_1 + x_2$$

- For instance, suppose the underlying game is specifically given by:

1 \ 2	I	N
I	$8, 8$	$-4, 4$
N	$10, -2$	$0, 0$

- In this case, we have:

$$z_1 = z_2 = 8, x_1 = 10, x_2 = 4.$$

- Note that $x_2 - z_2 = -4$ and $z_1 - x_1 = -2$. Also:

$$z_1 + z_2 = 16 \text{ and } x_1 + x_2 = 14.$$

- Therefore $z_1 + z_2 \geq x_1 + x_2$ and we conclude that, with limited verifiability, a contract can enforce (I,I) as long as the monetary transfer α satisfies:

$$-4 \leq \alpha \leq -2$$

- For example, we can have:

$\alpha = -3$ (here, the contract stipulates that player 2 receives \$3 from player 1 if somebody fails to invest effort).

$\alpha = -2$ (here, the contract stipulates that player 2 receives \$2 from player 1 if somebody fails to invest effort).

$\alpha = -3.5$ (here, the contract stipulates that player 2 receives \$3.50 from player 1 if somebody fails to invest effort).

- Any transfer from player 1 to player 2 between \$2 and \$4 would enforce *(I,I)* as the outcome in this game with limited verifiability.

- Recall that the induced game looks like this:

1 \ 2	I	N
I	$\mathbf{8, 8}$	$\mathbf{-4+\alpha, 4-\alpha}$
N	$\mathbf{10+\alpha, -2-\alpha}$	$\mathbf{-\alpha, \alpha}$

- For example, if $\alpha = -3$, the induced game becomes:

1 \ 2	I	N
I	$\mathbf{8, 8}$	$\mathbf{-7, 7}$
N	$\mathbf{7, 1}$	**3, -3**

- Instead, suppose the underlying game is:

1 \ 2	I	N
I	$\mathbf{10, 10}$	$\mathbf{-4, 12}$
N	$\mathbf{12, -4}$	$\mathbf{0, 0}$

- In this case, we have:

$$z_1 = z_2 = 10, x_1 = x_2 = 12.$$

- Note that $x_2 - z_2 = 2$ and $z_1 - x_1 = -2$. Also:

$$z_1 + z_2 = 20 \text{ and } x_1 + x_2 = 24.$$

- Therefore $z_1 + z_2 < x_1 + x_2$ and we conclude that, **with limited verifiability, a contract cannot enforce** $(\boldsymbol{I}, \boldsymbol{I})$**.**

- Often in practice, contracts do not specify monetary transfers for all possible outcomes. That is, **in practice, contracts are often incomplete**.

- In practice, the two players in this game would agree to play (I, I) and would let a court determine damage awards if one of the players breaches the contract.

- That is, in practice players would let a court "complete the contract".

- In the real world, courts determine damage awards according to three legal principles:

  1. The legal principle of **expectation damages**.
  2. The legal principle of **reliance damages.**
  3. The legal principle of **restitution damages**.

- As we will see, which principle is used depends on the information available to the court. That is, whether the court is able to observe precisely x_i, y_i and z_i.

- We will refer to the player who breaches the contract (the who deviated from the agreed outcome (I, I)) as **the defendant**. And we will refer to the player who adhered to the contract as **the plaintiff**.
- **The legal principle of expectation damages**: According to this legal principle, the defendant must give a monetary transfer to the plaintiff **such that the payoff to the plaintiff is equal to the one he expected if the contract had been fulfilled**.

- Therefore, if player 2 is the defendant, then he must give player 1 (the plaintiff) a transfer in the amount of:

$$\beta = z_1 - y_1$$

- Note that z_1 is the payoff player 1 was expecting to receive under the terms of the contract, while y_1 is the payoff received after player 2 breached it.
- Similarly, if player 1 is the defendant, then he must give player 2 a transfer in the amount of:

$$\alpha = z_2 - y_2$$

- If both players breach the contract, we assume that nobody goes to court and their payoffs are zero for each.

- If players know that the court operates under the legal principle of expectation damages, then the induced game effectively becomes:

1 \ 2	I	N
I	z_1, z_2	$z_1, x_2 - (z_1 - y_1)$
N	$x_1 - (z_2 - y_2), z_2$	$0, 0$

- Can a court operating under this legal principle enforce the outcome (I, I)?
- Again, this requires that (I, I) be a Nash equilibrium in the induced game.

- (I, I) will be a Nash equilibrium in the induced game if and only if:

$$z_1 \geq x_1 - (z_2 - y_2)$$

and

$$z_2 \geq x_2 - (z_1 - y_1)$$

- These two inequalities can be re-arranged as:

$$z_1 + z_2 \geq x_1 + y_2$$

and

$$z_1 + z_2 \geq x_2 + y_1$$

- But these conditions are exactly the conditions under which the outcome (I, I) is efficient.
- Therefore we conclude that **with expectation damages, a court can enforce the outcome (I, I) if and only if this outcome is efficient**.

- Thus, expectation damages is a legal rule that can always enforce efficient outcomes.
- This is a very nice result, but notice that expectation damages requires a court with full verifiability and exact knowledge of:
- z_i =payoff the plaintiff would have obtained if the contract had been honored.
- y_i =payoff the plaintiff obtained after the contract was breached.
- In practice, it is reasonable to assume that y_i is easy to determine. But z_i may be hard to estimate.
- In this case we can look at alternative legal principles.

- **The legal principle of reliance damages**: According to this legal principle, the defendant must give a monetary transfer to the plaintiff **such that the payoff to the plaintiff is equal to the one he would have if he had never entered into the contract**.
- Let w_i denote the payoff that player i would have obtained if he hadn't entered into the contract. Then, under reliance damages:
- If player 2 breaches the contract, then the transfer he must give to player 1 is:

$$\beta = w_1 - y_1$$

- If player 1 breaches the contract, then the transfer he must give to player 2 is:

$$\alpha = w_2 - y_2$$

- Therefore, under reliance damages the induced game looks like this:

1 \ 2	I	N
I	$\boldsymbol{z_1, z_2}$	$\boldsymbol{w_1, x_2 - (w_1 - y_1)}$
N	$\boldsymbol{x_1 - (w_2 - y_2), w_2}$	$\mathbf{0, 0}$

- (I, I) will be a Nash equilibrium in the induced game if and only if:

$$z_1 \geq x_1 - (w_2 - y_2)$$

and

$$z_2 \geq x_2 - (w_1 - y_1)$$

- The key question is: How can we determine the payoff w_i that player i would have received if he had not entered into the contract?

- In some cases, w_i may be easy to determine from some outside business opportunity that was forgone by player i when he entered into the contract.

- In other cases, we may think that if there had not been a contract, the game would have been played anyway between the players, and they would have chosen a Nash equilibrium. Then w_i would be the payoff received by i in that Nash equilibrium.

- Recall that the underlying game looks like this:

1 \ 2	I	N
I	z_1, z_2	y_1, x_2
N	x_1, y_2	$0, 0$

- y_i is the payoff to player i if the other player breaches the contract. Suppose we assume that $y_1 \leq 0$ and $y_2 \leq 0$ (contract breach results in losses to the plaintiff).
- Then (N,N) would be the only Nash equilibrium in this game, and each player would obtain a payoff of zero.
- Suppose that without the contract, both players would have simply played the equilibrium (N, N). Then we have $w_1 = 0$ and $w_2 = 0$.

- The game induced by reliance damages then would look like this:

1 \ 2	I	N
I	$\boldsymbol{z_1, z_2}$	$\boldsymbol{0, x_2 + y_1}$
N	$\boldsymbol{x_1 + y_2, 0}$	$\boldsymbol{0, 0}$

- The outcome (I, I) is a Nash equilibrium in this induced game if and only if:

$$z_1 \geq x_1 + y_2$$

and

$$z_2 \geq x_2 + y_1$$

- Intuitively, this means that **the plaintiff's damage from the breach (measured by y_i) must be relatively large in order to enforce (I, I).**

- For instance, suppose the underlying game is:

1 \ 2	I	N
I	$\mathbf{8, 8}$	$\mathbf{-4, 4}$
N	$\mathbf{10, -2}$	$\mathbf{0, 0}$

- We have then:

$$z_1 = 8, y_1 = -4, x_1 = 10$$

$$z_2 = 8, y_2 = -2, x_2 = 4$$

- $x_1 + y_2 = 8$ and $x_2 + y_1 = 0$, and the conditions $z_1 \geq x_1 + y_2$ and $z_2 \geq x_2 + y_1$ are satisfied. Therefore, the outcome (I, I) can be enforced through reliance damages.

- Suppose instead that the underlying game is:

1 \ 2	I	N
I	$8, 8$	$-4, 4$
N	$10, -1$	$0, 0$

- Now the breach damages for player 1 are smaller (-1 instead of -2). We now have $x_1 + y_2 = 9$ and the condition $z_1 \geq x_1 + y_2$ no longer holds. Therefore the outcome (I, I) cannot be enforced through reliance damages.

- **The legal principle of restitution damages**: Seeks to eliminate any gains obtained by the defendant from the breach of contract. According to this legal principle, the defendant must give a monetary transfer to the plaintiff **equal to the payoff obtained by the defendant from the breach of contract**.
- If player 2 breaches the contract, then the transfer he must give to player 1 is:

$$\beta = x_2$$

- If player 1 breaches the contract, then the transfer he must give to player 2 is:

$$\alpha = x_1$$

- The induced game with restitution damages looks like this:

1 \ 2	I	N
I	$\boldsymbol{z_1, z_2}$	$\boldsymbol{y_1 + x_2, 0}$
N	$\boldsymbol{0, y_2 + x_1}$	$\boldsymbol{0, 0}$

- Restitution damages will enforce (I, I) if and only if

$$z_1 \geq 0 \text{ and } z_2 \geq 0$$

- Which of these three legal principles can be followed by a court depends on the information available to it.
- Let i denote the plaintiff and let j denote the defendant. Then:
- **With expected damages, the court needs to know** :
  - z_i: The plaintiff's payoff if the contract had been honored.
  - y_i: The plaintiff's payoff once the contract is breached. That is, the losses incurred by the plaintiff as a result of the breach.

- **With reliance damages, the court needs to know** :
  - w_i: The plaintiff's payoff if he had not entered into the contract.
  - y_i: The plaintiff's payoff once the contract is breached. That is, the losses incurred by the plaintiff as a result of the breach.

- **With restitution damages, the court needs to know:**
  - x_j: The defendant's payoff from breaching the contract. That is, the unjust gains made by the defendant by breaching the contract.

- In addition, in all three cases the court needs to be able to determine whether there was a breach of contract as well as which of the players breached the contract.