# 16.- Topics in Industrial Organization

- As the Stackelberg vs. Cournot example showed, going from a static setting to a situation where players move sequentially can immediately increase the range of possible equilibrium outcomes we can observe.
- Here we will quickly study a few more examples of sequential games involving simple economic models. In each case we will look for SPE equilibrium behavior.

- **Example: Advertising and Competition.-** In the oligopoly models we have studied so far (Cournot, Bertrand and Stackelberg competition) we have assumed that market demand is given and exogenous.
- However, in the real world firms can directly affect demand for their products. One way to do this is through **advertising**.
- Thus, we can think about a more complex model in which firms decide how much to advertise and then compete on quantity produced (in Cournot or Stackelberg) or prices.

- Consider the following model:

1. There are two firms in an industry. In the first stage, firm 1 decides how much to spend on advertising. Denote this advertising activity by $a$, and assume it is nonnegative.
2. Suppose that advertising increases the overall market demand for the (homogenous) good. Therefore, firm 2 also benefits from $a$. Suppose that market demand is given by:

$$p = a - q_1 - q_2$$

Note that advertising shifts market demand to the right, benefitting both firms.

3. Suppose that the advertising cost to firm 1 is given by:

$$\frac{2 \cdot a^3}{81}$$

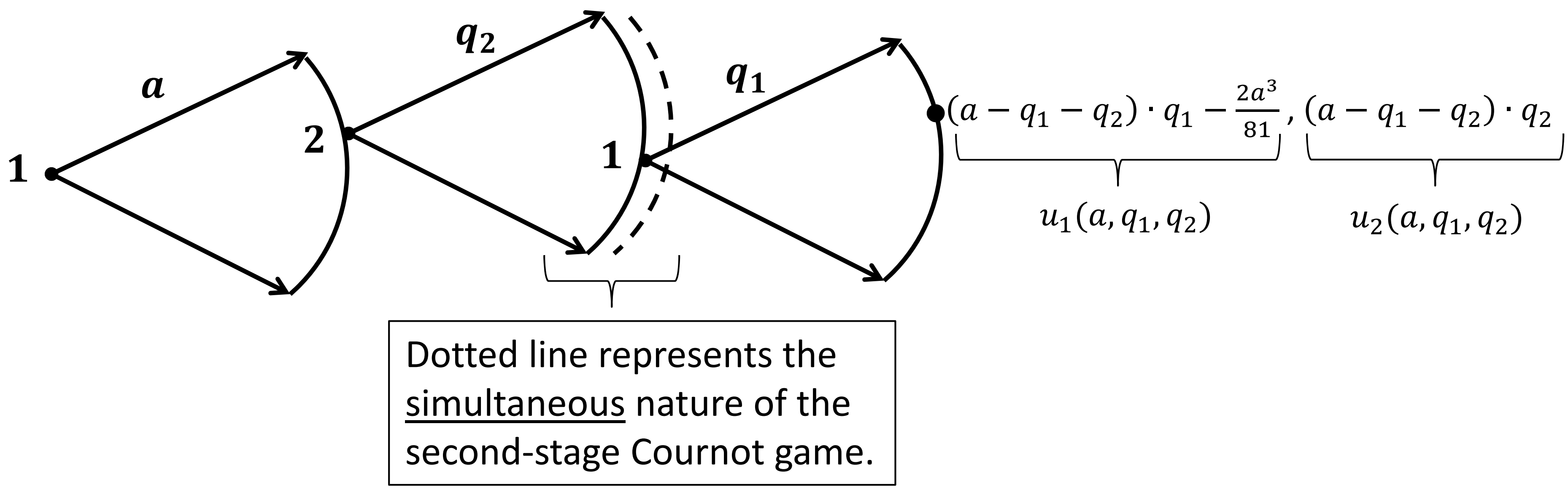
4. Once firm 1 decides how much to advertise, then both firms play a Cournot game where they **simultaneously choose $q_1$ and $q_2$**. Suppose that quantities can be produced at a **cost of zero** to both firms.

- Find the SPE (subgame perfect equilibrium) strategies in this game.

- Firm's payoffs (profits) look like this:

$$u_1(a, q_1, q_2) = (a - q_1 - q_2) \cdot q_1 - \frac{2 \cdot a^3}{81}$$
$$u_2(a, q_1, q_2) = (a - q_1 - q_2) \cdot q_2$$

- The extensive form of this game looks like this:



- How do strategies look like in this game?
  - Firm 1 first chooses the advertising level $a$ before observing quantities produced.
  - Once $a$ is revealed to both firms, they engage in a simultaneous Cournot game. The strategies in this game can be represented as: $q_1(a)$ and $q_2(a)$.
  - Therefore, a strategy for firm 1 is of the form:
$$s_1 = (a, q_1(a))$$
  - And a strategy for firm 2 is simply $q_2(a)$.

- To find the SPE, we first look for the equilibrium in the second stage Cournot game, given the choice of $a$.

- In the second stage Cournot game, both players know the value of $a$, and they simultaneously choose how much to produce. The best response by firm 2 is the solution in $q_2$ to the first order conditions:

$$\frac{\partial u_2(a, q_1, q_2)}{\partial q_2} = 0$$

- And the best response by firm 1 is the solution in $q_1$ to the first order conditions:

$$\frac{\partial u_1(a, q_1, q_2)}{\partial q_1} = 0$$

- We have:

$$\frac{\partial u_2(a, q_1, q_2)}{\partial q_2} = a - 2q_1 - q_2$$

$$\frac{\partial u_2(a, q_1, q_2)}{\partial q_2} = a - q_1 - 2q_2$$

- Therefore the best responses in the Cournot second stage game are:

$$BR_1(a, q_2) = \frac{a - q_2}{2}$$

$$BR_2(a, q_1) = \frac{a - q_1}{2}$$

- The equilibrium in the second stage Cournot game is the profile $\left(q^*{}_1(a), q^*{}_2(a)\right)$ that solves:

$$q^*{}_1 = \frac{a - q^*{}_2}{2}$$
$$q^*{}_2 = \frac{a - q^*{}_1}{2}$$

- This yields:

$$q^*{}_1(a) = \frac{a}{3} \quad and \quad q^*{}_2(a) = \frac{a}{3}$$

- Once we have found the equilibrium in the second stage we need to figure out the **continuation payoffs to firm 1** in the first stage. That is, we need to **express the payoffs to firm 1 in the second stage as a function of the advertising expenditure "$a$".**
- The continuation payoffs for firm 1 are:

$$u_1\left(a, {q^*}_1(a), {q^*}_2(a)\right)$$

$$= \left(a - {q^*}_1(a) - {q^*}_2(a)\right) \cdot {q^*}_1(a) - \frac{2 \cdot a^3}{81}$$

$$= \left(a - \frac{a}{3} - \frac{a}{3}\right) \cdot \frac{a}{3} - \frac{2 \cdot a^3}{81} = \frac{a^2}{9} - \frac{2 \cdot a^3}{81}$$

- Therefore, the continuation payoffs to firm 1 in the first stage are:

$$u_1\left(a, q^*{}_1(a), q^*{}_2(a)\right) = \frac{a^2}{9} - \frac{2 \cdot a^3}{81}$$

- The SPE profile is therefore found by looking at the advertising expenditure level "$a$“ that maximizes the continuation payoffs to firm 1. This is found by solving the first order conditions:

$$\frac{\partial u_1\left(a, q^*{}_1(a), q^*{}_2(a)\right)}{\partial a} = 0$$

- We have:

$$\frac{\partial u_1\left(a, {q^*}_1(a), {q^*}_2(a)\right)}{\partial a} = \frac{2 \cdot a}{9} - \frac{6 \cdot a^2}{81}$$

- Therefore the optimal advertising level given the continuation payoffs is:

$$\boldsymbol{a = 3}$$

- **From here, the SPE profile in this game is:**

$$s_1 = (a, q_1(a)) = \left(3\ , \frac{a}{3}\right) \text{ (for firm 1)}$$

$$q_2(a) = \frac{a}{3} \text{ (for firm 2)}$$

- The outcome in the SPE equilibrium is:

$$a = 3, q_1 = 1, q_2 = 1.$$

- Firms' payoffs (profits) in the SPE equilibrium are:

$$u_1(3,1,1) = \frac{1}{3} \text{ and } u_2(3,1,1) = 1$$

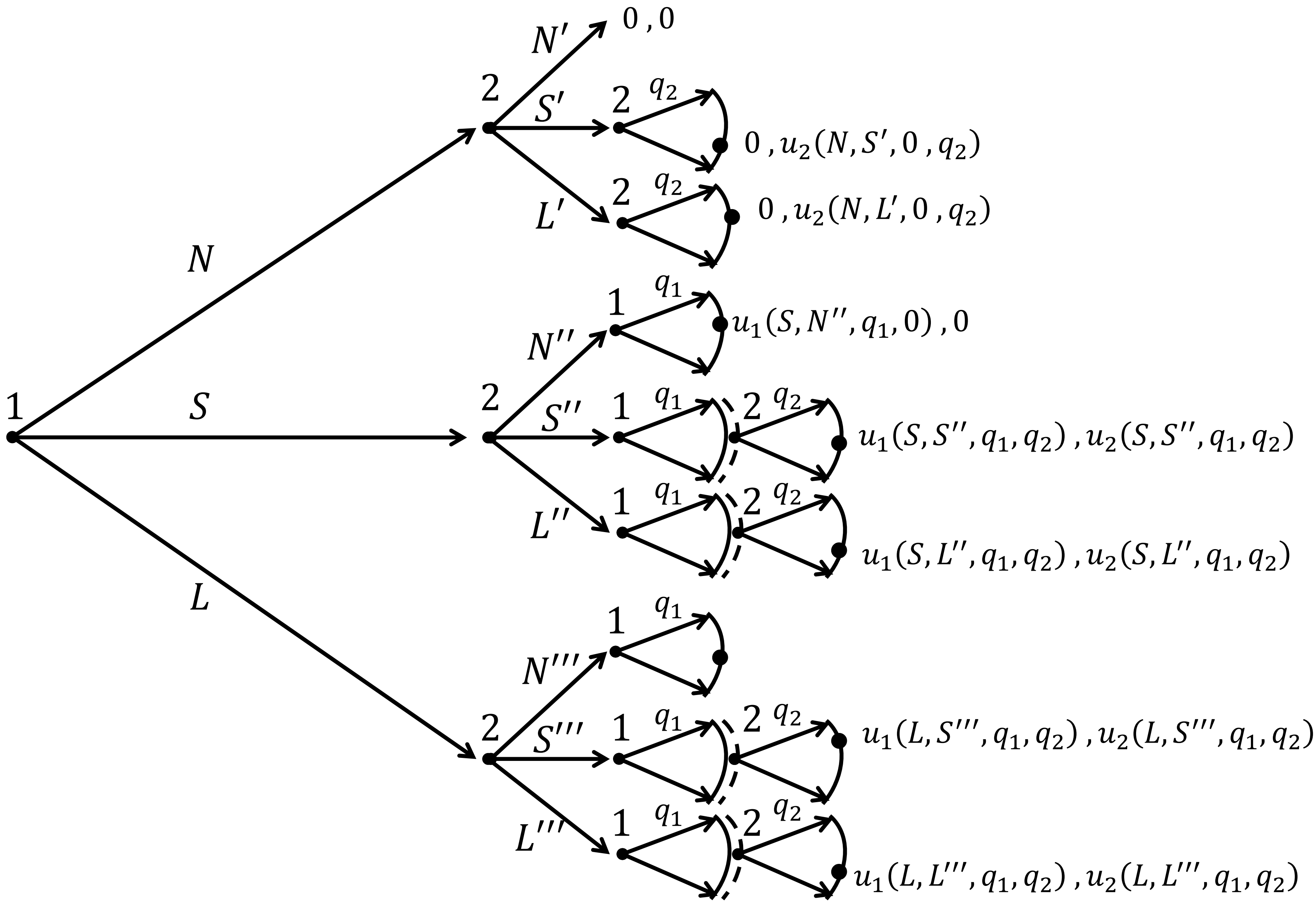
- **Example: A Capacity Model.-** In some industries, a better approximation to real-world behavior could be to model a sequential decision where firms do the following:
  - First, firms decide sequentially whether to enter an industry and if they decide to enter, they choose their capacity (how big of a factory to build).
  - Once entry/capacity decisions are made, players then engage in a simultaneous Cournot model where quantities produced are selected. If there is only one entrant, then this entrant behaves like a monopolist.

- For instance, consider a game described as follows:

1. In stage 1, firm 1 decides between staying out of the industry, building a small facility or building a large facility. Suppose:
   a) Small facility.- A small facility costs $50K to build and it allows the firm to produce up to 100 units of the good at zero marginal cost.
   b) Large facility.- A large facility costs $175K and it allows the firm to produce *any* number of units of the good at zero marginal cost.
   – If a firm decides not to enter, its payoff is zero regardless of what the other firm does.

2. In stage 2, having observed the choice made by firm 1, firm 2 chooses among the same three alternatives: Stay out, build a small facility, or build a large facility.
3. Stage 3: Once both firms have made their capacity decisions, these become observed by both and the following happens:
   a) If both firms enter, they engage in simultaneous, Cournot competition subject to their capacity constraints.
   b) If only one firm enters, it behaves like a monopolist, selecting the quantity that maximizes its profits subject to its capacity constraint.

- Extensive form:



- Relevant cases for the last-stage subgame are:

  1. Both firms enter with large capacity.
  2. Both firms enter, one with large capacity and the other one with small capacity.
  3. Both firms enter with small capacity.
  4. Only one firm enters with a large capacity.
  5. Only one firm enters with a small capacity.
  6. Neither firm enters.

- We will study the equilibrium of the last-stage subgame for each case. From here we will obtain the continuation payoffs in the last stage.

- Let us denote the payoffs of the **last stage subgame** as

$$v_i(q_i, q_j)$$

- The payoffs of the overall game depend on both players' capacity/entry decisions and on the outcome of the last stage subgame.
- For example:

$u_1(L, L''', q_1, q_2)$ =payoff to 1 if both enter with a large facility and then produce $q_1, q_2$ in last stage.

$u_1(S, L'', q_1, q_2)$ =payoff to 1 if 1 enters with a small facility, 2 enters with large facility and then produce $q_1, q_2$ in last stage.

$u_1(S, N'', q_1, 0)$ =payoff to 1 if it is the sole entrant, with a small facility and then produces $q_1$ in last stage.

- **Case 1.- Both firms enter with large capacity.-** In this case, in the Cournot game of the last stage firms 1 and 2 maximize the profits:

$$v_1(q_1, q_2) = (900 - q_1 - q_2) \cdot q_1$$

and

$$v_2(q_1, q_2) = (900 - q_1 - q_2) \cdot q_2$$

- Using our expertise in solving Cournot games, we can easily find that the equilibrium in this stage would be:

$$q^*{}_1 = 300, \; q^*{}_2 = 300$$

- Since both firms invested in a large facility, the overall profits of the game for both firms would be:

$$u_1(L, L''', 300{,}300) = (900 - 300 - 300) \cdot 300 - 175{,}000$$
$$= -85{,}000$$

- Since both firms invested in a large facility, the overall profits of the game for both firms would be:

$$u_1(L, L''', 300{,}300) = (900 - 300 - 300) \cdot 300 - 175{,}000$$
$$= -85{,}000$$
$$u_2(L, L''', 300{,}300) = (900 - 300 - 300) \cdot 300 - 175{,}000$$
$$= -85{,}000$$

- **Case 2.- Both firms enter. One with large facility and the other one with small facility.-** We know that in the game of the final stage, both firms would like to produce 300 units. However, if a firm is constrained it can only produce 100 units.
- For simplicity assume firm 1 has the large capacity (the exact same result will follow if we reverse the players).

- Therefore, the firm who entered with a small capacity (firm 2) will produce 100 units in the final stage game. That is, we will have $q_2 = 100$

- What is the optimal choice for $q_1$? If $q_2 = 100$ , then the best response for firm 1 is to choose the $q_1$ that maximizes:

$$v_1(q_1, 100) = (900 - q_1 - 100) \cdot q_1$$

- The best response is given by the $q_1$ that solves the first order conditions:

$$\frac{\partial v_1(q_1, 100)}{\partial q_1} = 0$$

- It is easy to see that this yields:

$$q_1 = 400$$

- Therefore, if firm 1 enters with a large facility and firm 2 enters with a small facility, in the last stage of the game they will choose:

$$q_1 = 400 \quad \text{and} \quad q_2 = 100$$

- The reverse will happen if firm 2 enters with a large facility and firm 1 enters with a small one.

- The overall payoff to the large firm (taking into account the capacity costs) would be:

$$u_1(L, S''', 400{,}100) = (900 - 400 - 100) \cdot 400 - 175{,}000$$
$$= -15{,}000$$

- And the overall payoff to the small firm would be:

$$u_2(L, S''', 400{,}100) = (900 - 400 - 100) \cdot 100 - 50{,}000$$
$$= -10{,}000$$

- These payoffs would be reversed if it is firm 2 who enters with a large facility and firm 1 with a small one.

- **Case 3.- Both firms enter with small facility.-** We already know that, if they were both unconstrained, they would like to produce 300 units. On the other hand, if only one firm is unconstrained, that firm would like to produce 400 units.
- Therefore, when both firms enter they would each like to produce at least 300 units. Therefore, with capacity constraints they will want to produce as much as they can. That is, they will produce

$$q_1 = q_2 = 100$$

- The overall payoff to both firms (taking into account the capacity costs) would then be:

$$u_1(S, S'', 100{,}100) = (900 - 100 - 100) \cdot 100 - 50{,}000$$
$$= 20{,}000$$

$$u_2(S, S'', 100{,}100) = (900 - 100 - 100) \cdot 100 - 50{,}000$$
$$= 20{,}000$$

- **Case 4.- Only one firm enters with a large facility.-** Suppose that firm 1. In the last stage of the game, recognizing that it is the only firm in the industry, it will want to choose $q_1$ to maximize:

$$v_1(q_1, 0) = (900 - q_1) \cdot q_1$$

- Solving the first order conditions of this problem, it is easy to see that the optimal production level would be:

$$q_1 = 450$$

- And the overall payoff (taking into account capacity costs) would be:

$$u_1(L, N''', 450,0) = (900 - 450) \cdot 450 - 175{,}000$$
$$= 27{,}500$$

- The payoff to the non-entrant is zero:

$$u_2(L, N''', 450,0) = 0$$

- These payoffs would be exactly reversed if it is firm 2 the one who enters.

- **Case 5.- Only one firm enters with a small facility.-** Again, suppose it is firm 1. We know from our previous result that this firm would like to produce $q_1 = 450$. Being capacity-constrained, it will produce as much as it can. That is, it will produce:

$$q_1 = 100$$

- And firm 1's overall payoff (taking into account capacity costs) would be:

$$u_1(S, N'', 100{,}0) = (900 - 100) \cdot 100 - 50{,}000 = 30{,}000$$

- Again, the payoff to the non-entrant is zero:

$$u_2(L, N'', 100{,}0) = 0$$

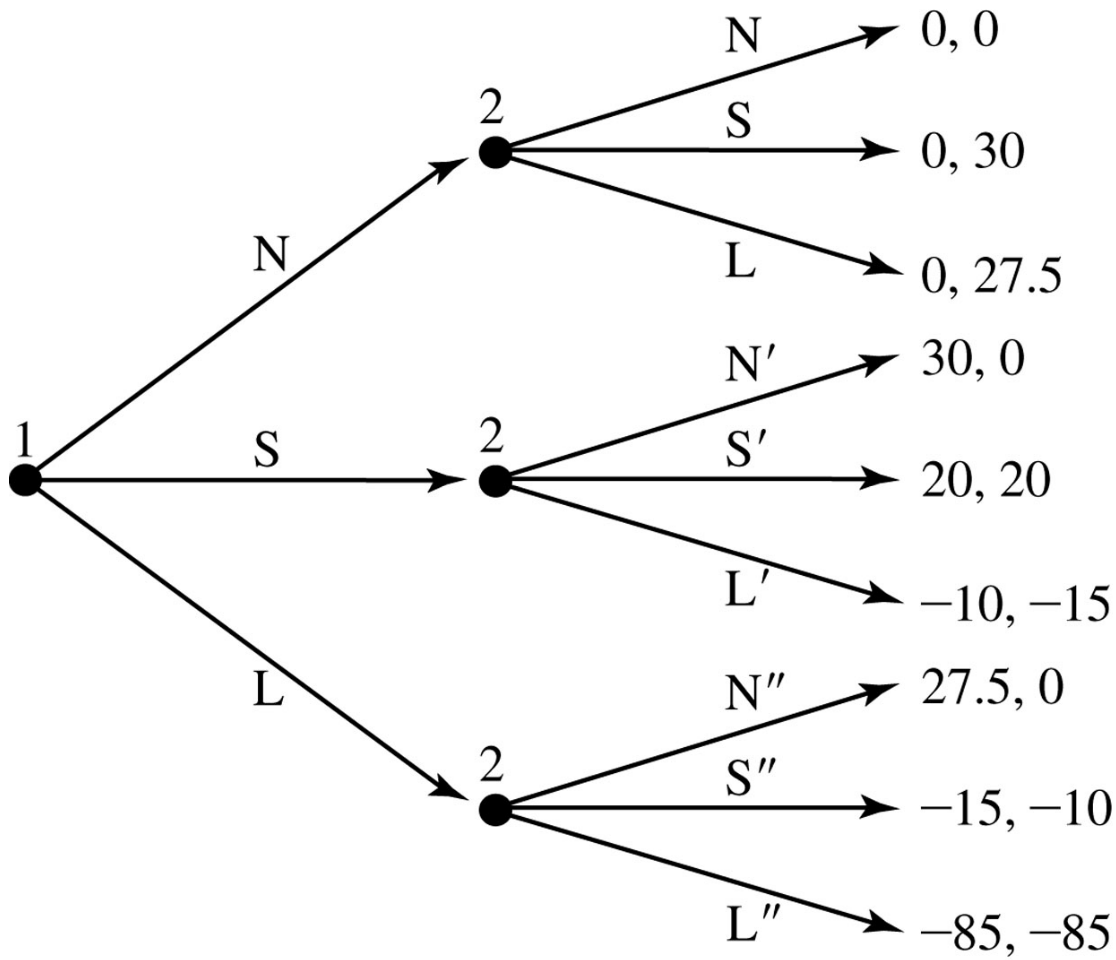
- These payoff figures would be exactly reversed if it is firm 2 the one who enters.

- **Case 6.- Neither firm enters.-** This case is trivial, both firms earn a payoff of zero.

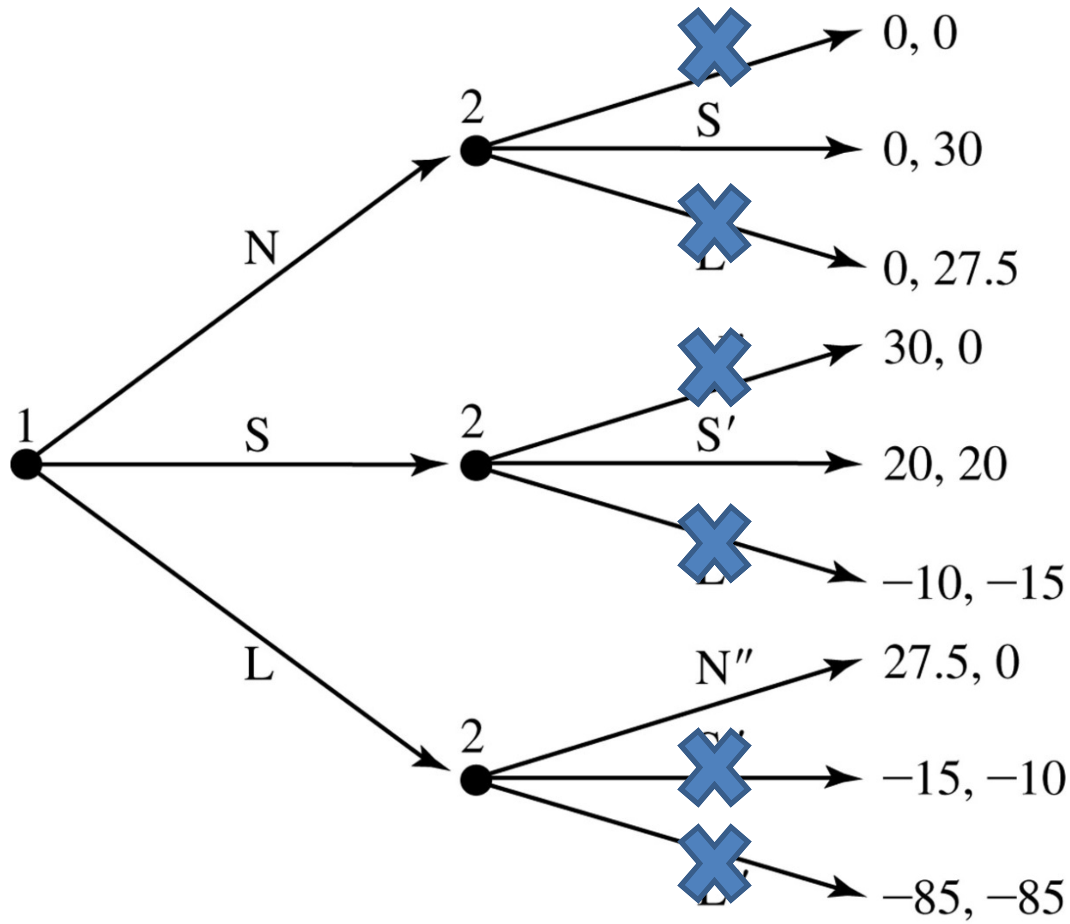
- OK, we have analyzed all six relevant cases and we have described the equilibrium behavior in the last-stage Cournot game. We have also described the payoffs that would result to each player in all cases.

- Using these results we can write down the continuation payoffs of the game in the last stage.

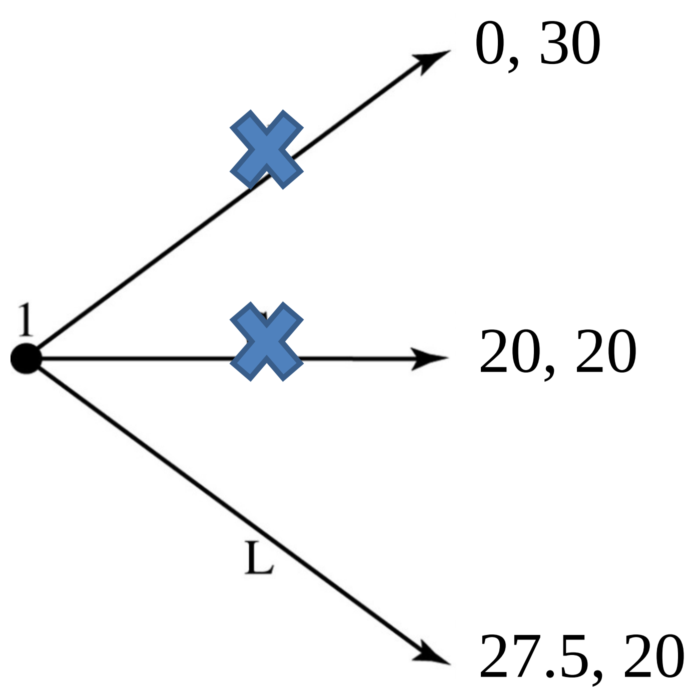
- Extensive form with the continuation payoffs of the last-stage Cournot game (in thousands of dollars):



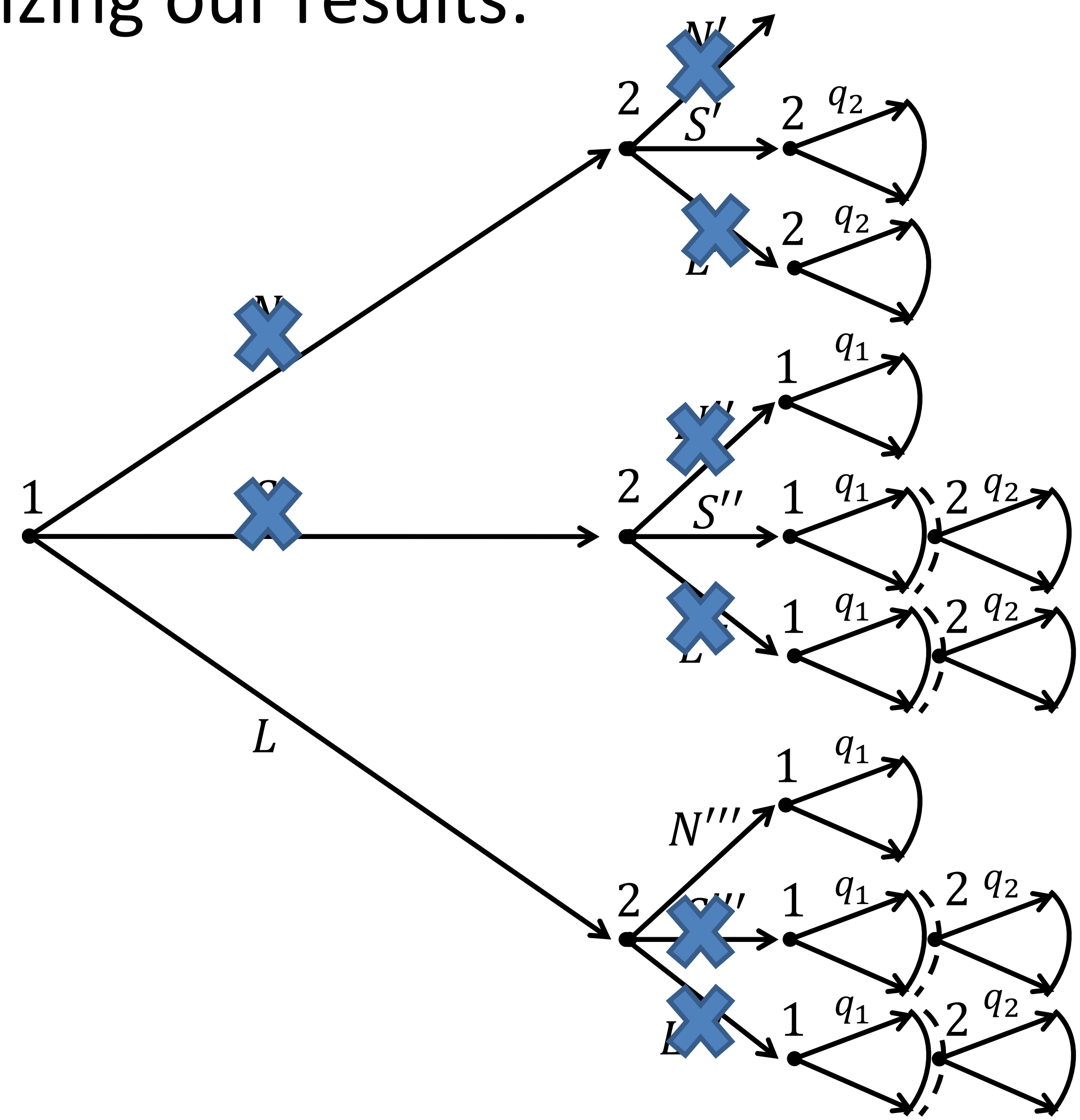
- Using these continuation payoffs, the next step to find the SPE profile is to do **backward induction**. The first step looks like this:



- From here, the last step of backward induction looks like this:



- Summarizing our results:



- In the SPE of this game:
  - Firm 1 enters with a large facility.
  - Firm 2 observes this and decides not to enter.
  - Firm 1 chooses quantity produced that maximizes its monopolist profits.

- The SPE profile is:

$$s_1 = (L, q_1(L, N''') = 450)$$
$$s_2 = (N''', q_2(L, N''') = 0)$$

- In this game, firm 1 **builds up excess capacity as a way to deter entry by firm 2.** In the real world, this type of behavior (building up capacity) has been used as evidence of anticompetitive strategies by firms in antitrust cases.

- **Example: Price discrimination by a monopolist.-** We have learned in our previous examples that sequential (dynamic) games can have many more equilibrium outcomes compared to static, simultaneous games.

- This example will show that dynamics can help a monopolist implement price discrimination that would be impossible in a static, one-shot interaction.

- **Description of the game:**
- Consider a seller of a particular good (for example, flat-screen TVs). Let's treat this seller as a monopolist, meaning that the seller is not competing with anyone else for the costumers.
- Customers can purchase the good in one of two periods: $t_1$ or $t_2$. Think of $t_1$ as "now" and $t_2$ as "next month", for example.
- There are four total customers. These can be classified into two types:
  - High value customers: Two customers of this type, label them as $H_1$ and $H_2$.
  - Low value customers: Two customers of this type, label them as $L_1$ and $L_2$.

- High value customers:
  - Their enjoyment of the good (their valuation of the good) is valued at $1,200 in period $t_1$, and $500 in period $t_2$.

- Low value customers:
  - Their enjoyment of the good (their valuation of the good) is valued at $500 in period $t_1$, and $200 in period $t_2$.

- Therefore:
- High value customers are willing to pay up to \$1,700 for the good if they purchase it in period $t_1$, and they are willing to pay up to \$500 if they buy it in period $t_2$.

- Low value customers are willing to pay up to \$700 for the good if they purchase it in period $t_1$, and they are willing to pay up to \$200 if they buy it in period $t_2$.

- Summarizing, the willingness-to-pay (i.e, the maximum amount they are willing to pay for the good) for each type of customer is:

| | Willingness to pay | |
|---|---|---|
| | Period $t_1$ | Period $t_2$ |
| High-value customers | \$1,700 | \$500 |
| Low-value customers | \$700 | \$200 |

- Ideally, the seller would like to sell the good to each type of customer priced at their willingness to pay, but the law requires to offer the same price to every customer at each point in time.

- However, price discrimination can take place over time by offering a different price in periods $t_1$ and $t_2$.

- **Find the subgame perfect Nash equilibrium (SPE) of the game:**
- We start, as usual, at the last-stage of the game, which is period $t_2$.
- Note first that, if the low-value customers purchased the good in period $t_1$, then the high-value customers must also have bought the good in period $t_1$, since their willingness to pay is higher, so any price that is attractive for the low-value customers is also attractive for the high-value customers.

- Therefore, in period $t_1$ the following must have happened:
    a) All four customers purchased the good.
    b) Only the high-value customers purchased the good.
    c) Nobody purchased the good.
- With this in mind, we can do the backward induction in period $t_2$ for each of the three cases described above:

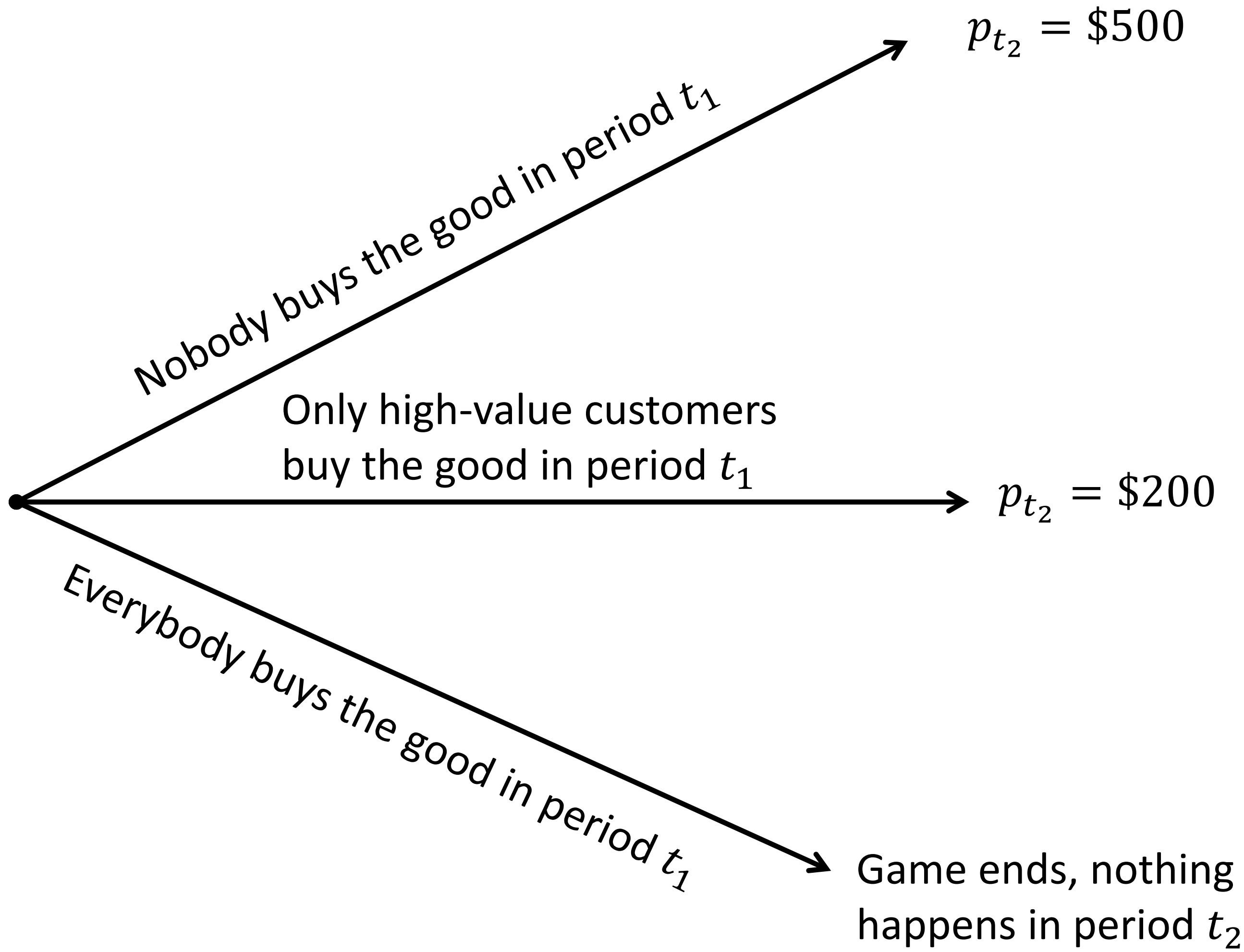
a) If all customers purchased the good in period $t_1$, then nothing happens in $t_2$.

b) If only high-valued customers bought the good in period $t_1$: Then in period $t_2$ the optimal strategy for the seller is to set the price at $\boldsymbol{p_{t_2}} = \mathbf{\$200}$ , which is the willingness to pay of the low-valued customers in period $t_2$. The revenue for the seller in period $t_2$ would be $2 \times \$200 = \$400$.

c) If nobody bought the good in period $t_1$: Then the only two strategies that could make sense would be to either price the good at \$500 (the willingness to pay of the high-value customers) or at \$200 (the willingness to pay of the low-valued customers). <u>Any price in between \$200 and \$500 would be suboptimal</u>.

- If the price is \$500, then the revenue in $t_2$ would be: $2 \times \$500 = \$1{,}000$. If the price is \$200, then the revenue in $t_2$ would be: $4 \times \$200 = \$800$. Therefore the optimal price in period $t_2$ is $\boldsymbol{p_{t_2}} = \mathbf{\$500}$.

- OK, so we have figured out how the seller will behave optimally in period $t_2$ in each of the three possible scenarios.



- Since sequential rationality is common knowledge, all customers know how the seller will behave in period $t_2$ in each possible scenario.
- **The fact that customers can anticipate the behavior of the seller in period $t_2$ will <u>constraint the price that the seller can set in period $t_1$</u>.**

a) Given sequential rationality, what is the optimal price that the monopolist can set that would induce all customers to buy the good in period $t_1$? There are only two relevant prices that the seller could consider for $p_{t_1}$: Either \$1,700 or $\$700$.

- If $p_{t_1} = \$1{,}700$, then only the high value would buy in period $t_1$, so this would not work. Therefore, **the highest possible price that would induce all customers to buy the good in period $\boldsymbol{t_1}$ is $\boldsymbol{p_{t_1} = \$700}$**. The total revenue of the seller would then be: $\mathbf{4 \times \$700 = \$2,800}$.

b) Given sequential rationality, what is the optimal price that the monopolist can set that would induce only high-value customers to buy the good in period $t_1$?

- **Note that in this case, everybody knows that $\boldsymbol{p_{t_2} = \$200}$.**
- Therefore, high-value customers know that if they wait until period $t_2$ to buy the good, their payoff will be: $\$500 - \$200 = \$300$ (the difference between their valuation of the good in period $t_2$ and the price they would pay).
- Therefore, the highest price that the seller can set in period $t_1$ must lead to a payoff of at least \$300 to high-value types. Therefore, the highest price that the seller can set in period $t_1$ is $\boldsymbol{p_{t_1} = \$1,400}$. In this case, the payoff to high-value types of buying the good in period $t_1$ would be: $\boldsymbol{2 \times \$1,400 = \$2,800}$.

c) What price would induce nobody to buy the good in period $t_1$? Any price above \$1,700. Therefore, nobody will purchase the good in $t_1$ if $p_{t_1} > \$1{,}700$. The revenue of the seller would be zero in period $t_1$.

- OK, we have now looked at all possible relevant scenarios in periods $t_1$ and $t_2$:

a) If all customers buy the good in period $t_1$: The total revenue in both periods to the seller is:

$$\mathbf{4 \times \$700 + 0 = \$2,800}$$

b) If only high-value customers buy the good in period $t_1$: The total revenue in both periods to the seller is:

$$\mathbf{2 \times \$1,400 + 2 \times \$200 = \$3,200}$$

c) If nobody buys the good in period $t_1$: The total revenue in both periods to the seller is:

$$\mathbf{0 + 2 \times \$500 = \$1,000}$$

- Scenarios (a), (b) and (c) are the only relevant possibilities that can occur in this game. Clearly, (b) yields the highest payoff to the seller.
- Therefore, the SPE in this game is described as follows:

$$p_{t_1} = \$1{,}400, p_{t_2} = \$200$$

$H_1$ and $H_2$ buy the good in period $t_1$

$L_1$ and $L_2$ buy the good in period $t_2$

- The total profit to the seller in this SPE is **$3,200**

- Note that the seller would prefer to be able to set the price $p_{t_1} = \$1{,}700$ and have the two high-value customers purchase the good in period $t_1$. This would yield a total profit of $\$3{,}400$ to the seller.

- However, the high-value customers would never buy the good in period $t_1$ if $p_{t_1} = \$1{,}700$ because backward induction allows them to deduce that the seller would set $p_{t_2} = \$200$. Therefore it would be best to wait until period $t_2$.

- Thus, the seller would like to have a **credible commitment device** (a binding “contract”) that would publicly guarantee that the seller will match $p_{t_1}$ and $p_{t_2}$.

- We see these **price-matching guarantees** in the real world everyday. For instance, the clothing store Eddie Bauer offers a “price adjustment” whereby, if a good goes on sale within 14 days of a purchase, they will match the sale price and return the customer the difference.

- With a price matching guarantee, if $p_{t_1} = \$1{,}700$, then once period $t_2$ arrives, the seller will not want to sell to the low-value customers, since this would necessitate setting $p_{t_2} = \$200$, and therefore having to reimburse the high-value customers a total amount of $2 \times \$1{,}500 = \$3{,}000$.

- Therefore, a price guarantee constitutes a credible commitment not to set $p_{t_2} = \$200$. This would therefore induce the high-value customers to buy the good in period $t_1$ at price $p_{t_1} = \$1{,}700$