17.- Parlor Games

- This is a nice real-world application of two previous results we have studied.
- Two-player parlor games have two main features:
  - They are **strictly competitive** (the outcome can always be classified as either: “player 1 wins”, “player 2 wins” or “both players tie”.
  - Parlor games where players move sequentially (e.g, chess, checkers, tic-tac-toe, etc.,) are also **finite games of perfect information**.

- We have studied two results previously:

1. Every finite game of perfect information has a pure-strategy Nash equilibrium (we stated this result when we studied backward induction).

2. If a two-player game is strictly competitive and has a pure-strategy Nash equilibrium, then the strategies in this Nash equilibrium are security strategies.

- Recall that when players play security strategies, they **ensure themselves a certain outcome regardless of what the other player does**.
- Therefore, in any two-player parlor game where players move sequentially, one of the two (or both) has to be true:

a) There exists a (security) strategy where one player ensures himself a win regardless of what the other player does.

b) There exists a (security) strategy where a tie is ensured.

- This is formally stated in the following result:
- **Result:** Take any two-player, extensive form game that (a) is finite, (b) has finite information, and (c) is strictly competitive.
  - If the possible outcomes of the game are “1 wins”, “2 wins” and “tie”, then either one of the players has a **winning strategy** (a security strategy that ensures a win) or both players have strategies guaranteeing a tie.
  - If the possible outcomes of the game are “1 wins” and “2 wins” then either player 1 has a winning strategy or player 2 has a winning strategy.

- This result obviously extends beyond parlor games, but the latter are a natural class of games where this result applies.

- For example, it predicts that the game of chess has a strategy where either one of the players can ensure themselves a victory, or a tie. We still don't know which one because Nash equilibria for chess have not been characterized yet because even though this game is finite, its extensive form is too rich for today's computers to fully handle (there are too many possible outcomes for today's computer power).

- In contrast, the game of checkers was “solved” in 2007 (it has a total of 10^{20} different board positions). The existence (and characterization) of a strategy that ensures a draw was shown by a team of computer scientists from Canada.

- Other parlor games that have been solved include: Connect Four, Chopsticks, Tic-tac-toe (trivial) among others…