18.- Bargaining Problems

- We have studied contracts before, focusing on the <u>implementation</u> (enforcing) phase of contracts. Bargaining models can help us model the <u>contracting phase</u>, where the terms of the transaction (contract) are set.
- **<u>Setup:</u>** Our setup is very general, aimed at studying a **potential economic transaction.** In its basic form, we will assume that this transaction involving two parties (**two players**).

- This potential transaction could encompass many different real-world examples, such as:
  - Hiring a worker.
  - Selling a good.
  - Providing a service.
  - Etc…
- Whatever the specific transaction is, we will fit it into a general, simplified model. This general model includes the following components…

- <u>**Components of the general bargaining model:**</u>
- **Monetary transfer "t".-** The terms of the transaction (contract) are assumed to include a monetary transfer t**.** This is a transfer from player 2 to player 1. If $t > 0$, player 2 gives money to player 1. If $t < 0$, then it is player 2 who receives money from player 1.

- **Other items (features) of the transaction "z".-** These describe other characteristics of the transaction (contract) other than the monetary transfers. For example, they could describe: working conditions, features of the service provided, features of the good to be exchanged, etc.,

- **Players' payoffs:** We assume that payoffs can be represented in monetary terms (that is, we assume that players' utility can be measured in monetary terms). Specifically, we assume that the utility that player $i = 1,2$ derives from "z" can be measured in monetary terms as: $\boldsymbol{v_i(z)}$
- And we assume that **monetary transactions enter additively** into each player's utility function.
- Therefore, if the transaction (contract) involves monetary transfers t and terms (features) z, players' payoffs would be given by:

$$\boldsymbol{u_1 = v_1(z) + t} \quad \textbf{(for player 1)}$$

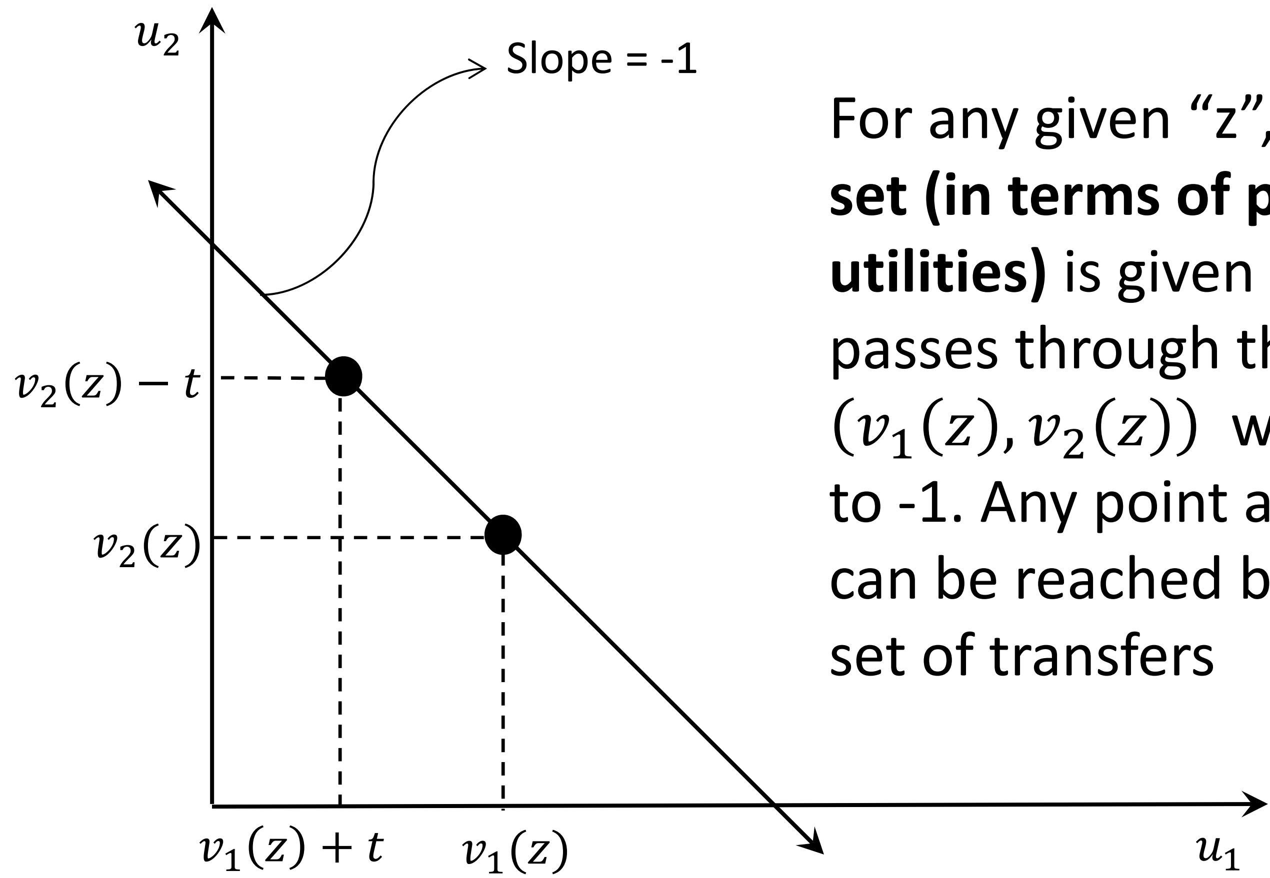
$$\boldsymbol{u_2 = v_2(z) - t} \quad \textbf{(for player 2)}$$

- Assuming that players' payoffs are of these form implies that utility can be transferred between players precisely by making monetary transfers between them. This feature is called "**transferable utility**".
- **Joint value of the transaction.-** Transferrable utility allows us to measure the overall value of the transaction (the joint value of the transaction) simply as **the sum of both players' payoffs**. That is:

$$
\begin{aligned}
Value = u_1 + u_2 &= (v_1(z) + t) + (v_2(z) - t) \\
&= v_1(z) + v_2(z)
\end{aligned}
$$

- Note that since t is just a monetary transfer from player 2 to player 1, it does not affect the overall value of the transaction.
- **Bargaining Set.-** We assume that the potential transaction can involve different values for "z" as well as monetary transfers "t". Let Z denote the set of different values that "z" can take and let T denote the range of possible values for transactions.
- The **bargaining set** "$\boldsymbol{V}$" is the **collection of all combinations** $(\boldsymbol{z}, \boldsymbol{t})$ **such that** $\boldsymbol{z} \in \boldsymbol{Z}$ **and** $\boldsymbol{t} \in \boldsymbol{T}$. This is the collection of all possible terms of the transaction (contract).

- Graphical depictions of bargaining sets (in terms of utility) in two-player bargaining games:

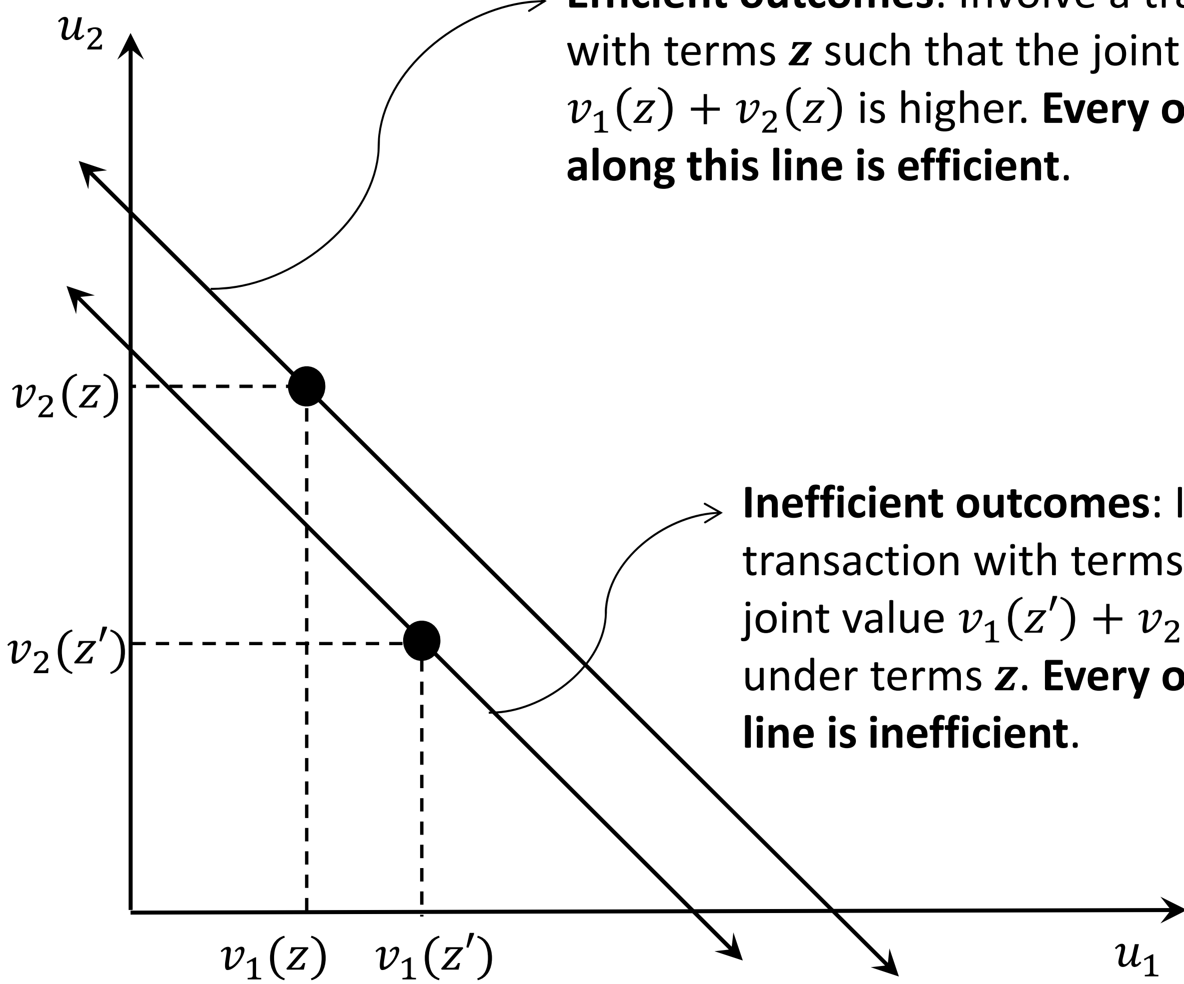


For any given "z", the **bargaining set (in terms of players' utilities)** is given by the line that passes through the point $(v_1(z), v_2(z))$ with slope equal to -1. Any point along this line can be reached by a particular set of transfers

- **Efficient outcome.-** Given that utilities are transferrable, the outcome of the negotiation (contract) is efficient if it **maximizes the joint value within the bargaining set V**.

- Note that since joint value does not depend on the monetary transfers, efficiency depends only on the terms "z" . Therefore **an outcome is efficient if and only if its terms "z" maximize $v_1(z) + v_2(z)$ within the set Z.**

- Graphical depictions of efficient vs. inefficient outcomes:



Efficient outcomes: Involve a transaction with terms $\boldsymbol{z}$ such that the joint value $v_1(z) + v_2(z)$ is higher. **Every outcome along this line is efficient**.

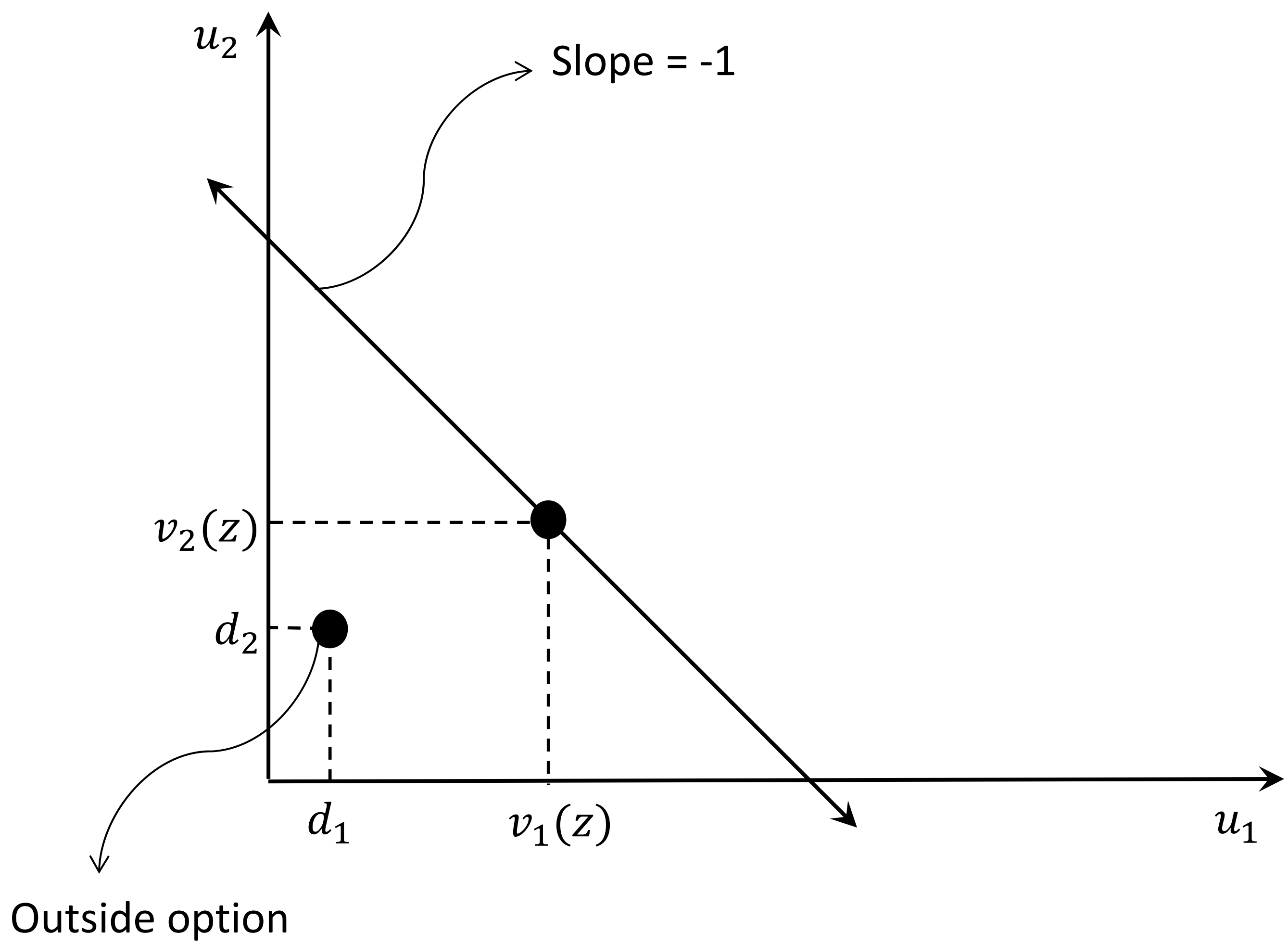
Inefficient outcomes: Involve a transaction with terms $\boldsymbol{z}'$ such that the joint value $v_1(z') + v_2(z')$ is lower than under terms $\boldsymbol{z}$. **Every outcome along this line is inefficient**.

- **Default outcome (or “disagreement outcome”).-** This refers to the payoff that players would obtain if they fail to reach an agreement. This represents the payoff they could obtain if they walk away from the negotiation of the contract. It is denoted as

$$\boldsymbol{d = (d_1, d_2)}$$

- Each player is allowed to unilaterally walk away from the negotiation and secure himself the default outcome.
- Therefore, **in any successful negotiation we must have $\boldsymbol{u_i \geq d_i}$ for $\boldsymbol{i = 1, 2}$.**

- Outside option (graphical illustration):



- **Surplus.-** Given the outcome of the negotiation, the surplus is defined as the difference between the joint value of the transaction and players' joint default outcome. That is:

$$Surplus = v_1(z) + v_2(z) - (d_1 + d_2)$$

Since any successful negotiation must provide both players at least their default outcome payoffs, **negotiation and bargaining boils down to deciding how they will split the surplus between themselves**.

- **Bottom line:** Bargaining models in Economics (even the most complex ones) are seen as involving two main joint decisions between the parties involved:

1. How much surplus to produce.

2. How to divide the surplus between the parties involved in the negotiation.

- **Bargaining weights.-** These measure and fully summarize the <u>relative bargaining powers</u> of both players. Bargaining weights are represented by

$$(\boldsymbol{\pi_1}, \boldsymbol{\pi_2})$$

- Where

$$\pi_i \geq 0 \quad \text{and} \quad \pi_1 + \pi_2 = 1$$

- The bargaining weight $\boldsymbol{\pi_i}$ **represents the proportion of the surplus that will go to player $\boldsymbol{i}$ at the end of the negotiation**.
- Where do these bargaining weights come from? We will study this later on. For now, let us take them as given.

- **Standard bargaining solution.-** This refers to a generic characterization of the outcome of a bargaining problem that results from Nash equilibrium behavior. The proof that this is a Nash equilibrium can be found in Appendix D of the textbook.
- The standard bargaining solution predicts a bargaining outcome with the following features:
  - **Efficiency:** That is, the outcome of bargaining will be the "z" that maximizes the joint value of the transaction.
  - **Transfers:** Monetary transfers will be such that the proportion of the surplus that will go to player i is exactly his bargaining weight π_i

- That is, the standard bargaining solution predicts an outcome with the following features:

a) The joint value of the transaction will be efficient. That is:

$$v^* = \max_{z \in Z}(v_1(z) + v_2(z))$$

b) Players' payoffs will be:

$$u_1 = d_1 + \pi_1 \times (v^* - [d_1 + d_2])$$
$$u_2 = d_2 + \pi_2 \times (v^* - [d_1 + d_2])$$

- Note that $v^* = {v^*}_1 + {v^*}_2$, where ${v^*}_1$ and ${v^*}_2$ correspond to $v_1(z^*)$ and $v_2(z^*)$, where z^* is the efficient level of "z".
- Therefore, in the standard bargaining solution the transfer t has to be such that:

$${v^*}_1 + t = d_1 + \pi_1 \times (v^* - [d_1 + d_2])$$

And

$${v^*}_2 - t = d_2 + \pi_2 \times (v^* - [d_1 + d_2])$$

- Quick algebraic manipulation shows that the **transfer from player 2 to player 1** predicted in the **standard bargaining solution** is given by:

$$t = \pi_1 \times \left[{v^*}_2 - d_2\right] - \pi_2 \times \left[{v^*}_1 - d_1\right]$$

- Intuitive interpretation of the formula for transfers t in the standard bargaining solution:
- Recall that t is the transfer from player 2 to player 1.
- From the previous formula, it follows that:
- The larger π_2 is relative to π_1 (i.e, the larger the relative bargaining power of player 2), the smaller the transfer t will be.
- In fact, if $\pi_2 = 1$ (i.e, if player 2 has all the bargaining power) then $t = -[{v^*}_1 - d_1]$ and **player 2 <u>receives</u> player 1's entire surplus.**

- In contrast, if $\pi_1 = 1$ (i.e, if player 1 has all the bargaining power) then $t = [{v^*}_2 - d_2]$ and **player 2 <u>gives</u> his entire surplus to player 1.**

- **Example:** Suppose a High School principal (label her as "R") is considering hiring a prospective employee (label him as "J").
- Suppose there are two alternative sets of job duties:
  - J could be in charge of teaching a class only. Label this as $\mathrm{z}=0$ (the basic job-responsibility setting)
  - Alternatively, J could be in charge of teaching a class AND coaching the softball team. Label this as "$z = 1$" (the "advanced" job-responsibility setting).

- Suppose the subjective, personal utility that J derives from teaching the class has a monetary value of $\$10{,}000$. However, suppose that if he has to also coach the softball team, then this utility decreases by $\$3{,}000$.

- If J agrees to teach the class AND coach the team, suppose the monetary value of the utility for R is $\$45{,}000$. On the other hand, if R only teaches the class, then the utility for R decreases by $\$5{,}000$

- Therefore, we can summarize both players' preferences over the job description "z" as:

$$v_J(z) = 10{,}000 - 3{,}000 \cdot z$$

and

$$v_R(z) = 40{,}000 + 5{,}000 \cdot z$$

- Suppose J has an alternative job opportunity that is worth $\$15{,}000$ to J.
- Suppose R has an alternative job candidate (less qualified) whose services are worth $\$10{,}000$ to R.

- Therefore, the disagreement payoffs are

$$d_J = \$15{,}000 \quad \text{and} \quad d_R = \$10{,}000$$

- Notice that the joint value of this labor relationship between J and R is given by:

$$\boldsymbol{V = v_J(z) + v_R(z) = 50,000 + 2,000 \cdot z}$$

- Therefore, the efficient outcome would be the one where $z = 1$ (that is, where J performs both duties: teacher and coach).

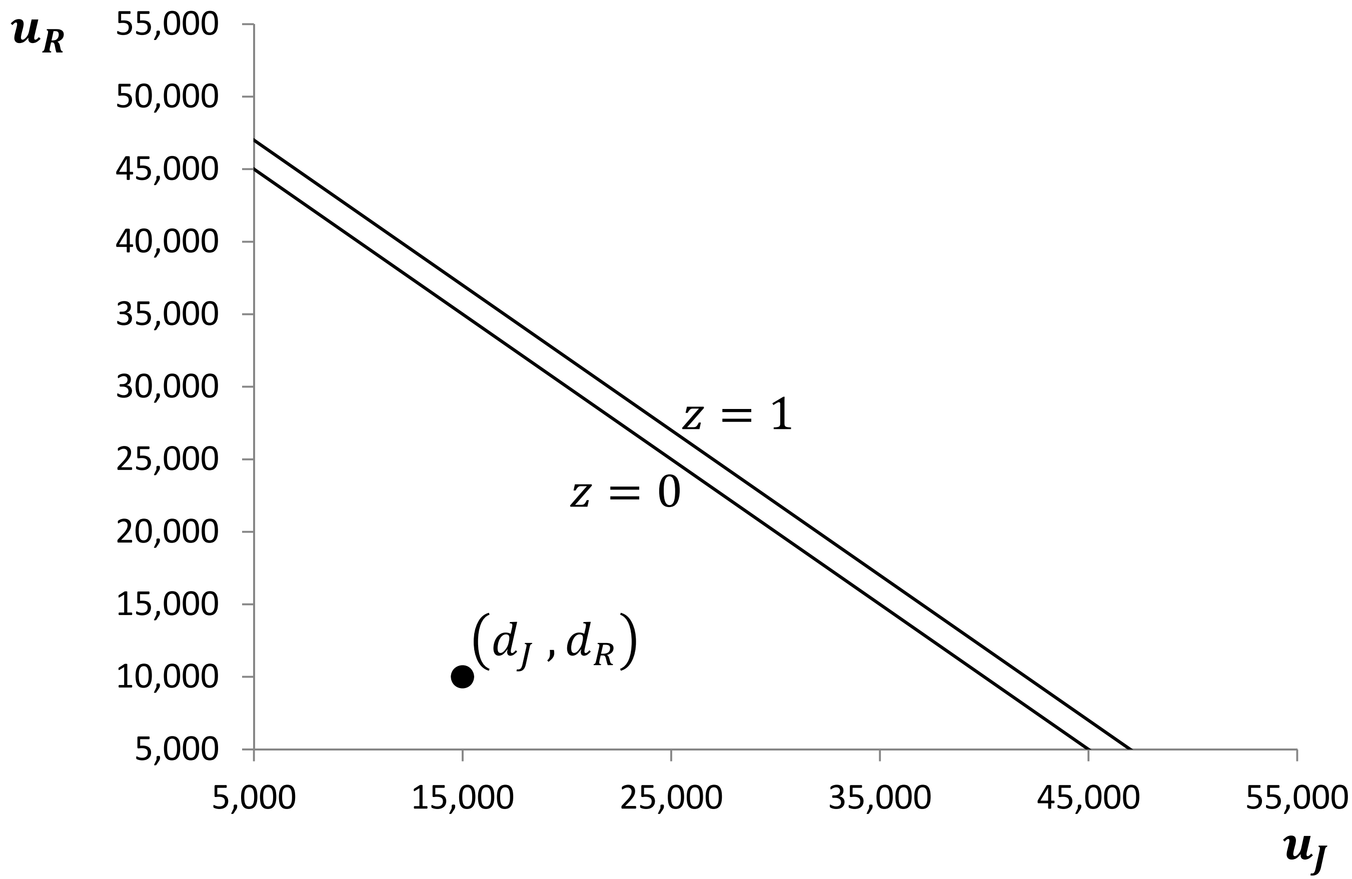
- The bargaining problem in this example involves two parameters of the job:
  - The duties to be performed: $z = 0$ or $z = 1$.
  - The salary to be paid: Measured as the transfer t from player R to player J.

- The total utility to both players is therefore:

$$u_J(z) = v_J(z) + t$$
$$u_R(z) = v_R(z) - t$$

- The bargaining problem can be described graphically as follows…

- Bargaining problem (graphically):



- What does the standard bargaining solution predict for this problem?

- The **standard bargaining solution** predicts:

1. That the outcome will be **efficient**, which in this case means that they will choose $z = 1$ (both teaching AND coaching duties).

2. That the surplus will be divided proportionately according to the bargaining weights π_J and π_R

- More precisely, the transfer (salary) t will be determined according to the formula (which we derived previously):

$$t = \pi_J \times \left[{v^*}_R - d_R \right] - \pi_R \times \left[{v^*}_J - d_J \right]$$

- Both ${v^*}_J$ and ${v^*}_R$ correspond to the efficient outcome $z = 1$. Therefore:

$$ {v^*}_J = v_J(z = 1) = 7{,}000 $$
$$ {v^*}_R = v_R(z = 1) = 45{,}000 $$

- Also we have $d_J = \$15{,}000$ and $d_R = \$10{,}000$.
- Therefore:

$$ t = \pi_J \times [45{,}000 - 10{,}000] - \pi_R \times [7{,}000 - 15{,}000] $$

- That is:

$$ \boldsymbol{t = \pi_J \times 35,000 + \pi_R \times 8,000} $$

- The only thing we have not described are the bargaining weights π_J and π_R…

- Suppose both parties have equal bargaining power. Then $\pi_J = \pi_R = 1/2$ and we would have:

$$t = \frac{1}{2} \times 35{,}000 + \frac{1}{2} \times 8{,}000 = 21{,}500$$

- In this case, J would be hired to both teach AND coach the softball team, and his salary would be $\$21{,}500$.
- Suppose in contrast that R (the employer) has more bargaining power. Specifically, $\pi_R = 2/3$ and $\pi_J = 1/3$. In this case:

$$t = \frac{1}{3} \times 35{,}000 + \frac{2}{3} \times 8{,}000 = 17{,}000$$

- Therefore, in the case $\pi_J = \pi_R = 1/2$, the total utility to each player is:

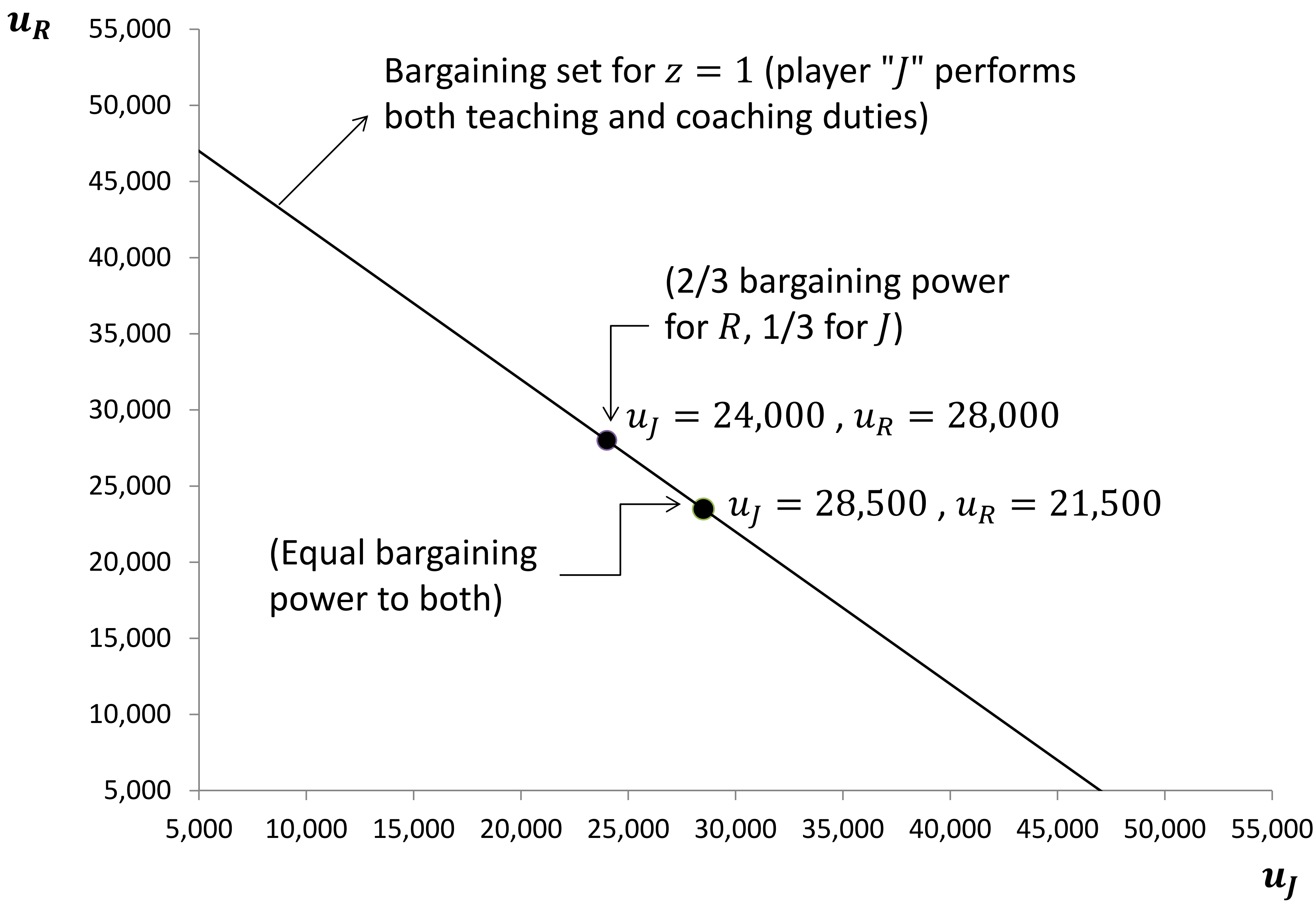
$$u_J(z) = v_J(1) + t = 7{,}000 + 21{,}500 = 28{,}500$$
$$u_R(z) = v_R(1) - t = 45{,}000 - 21{,}500 = 23{,}500$$

- And in the case $\pi_R = 2/3$ and $\pi_J = 1/3$:

$$u_J(z) = v_J(1) + t = 7{,}000 + 17{,}000 = 24{,}000$$
$$u_R(z) = v_R(1) - t = 45{,}000 - 17{,}000 = 28{,}000$$

- These represent two different points in the bargaining set…

- Graphical depiction of the standard bargaining solution for this example:



Key question: Where does bargaining power come from? **Chapter 19** explores this question, and we will study it next…