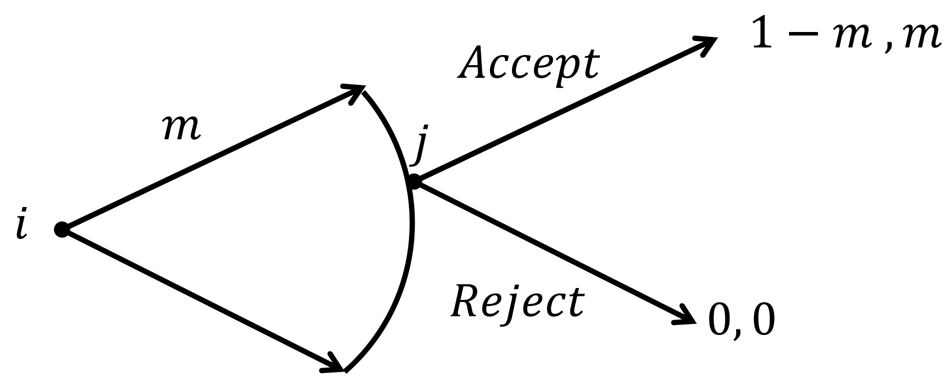
19.- Analysis of Simple Bargaining Games

- Chapter 19 explores two possible sources of bargaining power:

a) Being the proposer in an ultimatum-type bargaining negotiation.
b) Being relatively “patient” in an alternating-offer bargaining negotiation.

- Bargaining power for the **<u>proposer</u>** in a one-shot ultimatum game:
- Consider an ultimatum game where:
- **In stage 1:** The proposer offers the other player a fraction of the surplus, $m \in [0,1]$.
- **In stage 2:** The other player observes the proposed m and decides whether to accept or reject the offer. If the offer is accepted, the payoff to the proposer is $1 - m$ and the payoff to the other player is m. If the offer is rejected, they both get zero.

- Extensive form representation:



- What is the subgame perfect equilibrium (SPE) of this game?
- Notice that **in the second stage, any sequentially rational strategy by the player receiving the offer must accept any offer where $\boldsymbol{m > 0}$. The question is what to do if $\boldsymbol{m = 0}$…**

- If $m = 0$, then the player receiving the offer is indifferent between accepting or rejecting the offer.
- Therefore **there are two sequentially rational strategies for the player receiving the offer**:

$$s^*{}_j = \begin{cases} \text{Accept any offer where } m > 0 \\ \text{Accept the offer } m = 0 \end{cases}$$

$$\hat{s}_j = \begin{cases} \text{Accept any offer where } m > 0 \\ \text{Reject the offer } m = 0 \end{cases}$$

- Both strategies are sequentially rational for player j (the player receiving the offer).

- However, **only one of these strategies can lead to an SPE**.
- Notice that if player j uses strategy $\hat{s}_j$ in stage 2, then **the proposer (player i) does not have a well-defined optimal strategy in stage 1.**
- Why? Because offering $m = 0$ in stage 1 is not optimal (since player j would reject) and if player i offers $m > 0$ in stage 1, then he would be better off offering a smaller amount m’ $< m$.
- Since this is true for any $m > 0$, it means that player i would want to offer an infinitesimally small $m > 0$, but any m’ that is even smaller would be better.
- Therefore, if player j uses strategy $\hat{s}_j$ in stage 2, there does not exist an optimal offer m in stage 1 for player i.

- **Therefore, $\hat{s}_j$ cannot be used in any SPE**. The only SPE must involve strategy ${s^*}_j$, where

$$
{s^*}_j = \begin{cases} \text{Accept any offer where } m > 0 \\ \text{Accept the offer } m = 0 \end{cases}
$$

- If the player receiving the offer uses strategy ${s^*}_j$, then the best response by player i (the proposer) is simply to **offer $\boldsymbol{m = 0}$ in stage 1**.

- The SPE in this game is: $\left(m = 0, {s^*}_j\right)$. In other words, **the proposer offers none of the surplus to the other player and this offer is accepted.**

- Therefore, **in any bargaining game that is a one-shot ultimatum game, SPE behavior predicts that the proposer will have total bargaining power**: $\boldsymbol{\pi_i = 1}$ **and** $\boldsymbol{\pi_j = 0}$, where player i is the proposer and j is the player receiving the offer.

- **Caveat:** One-shot ultimatum games are seldom an accurate description of how negotiation takes place in the real world. A **bargaining process with alternating offers is more realistic**.

- **Alternating offers bargaining models:** These games proceed in stages described as follows:

1. Player i makes an offer in period t. Then player j decides to accept or reject.
2. If the offer is accepted, the game ends. Otherwise both players wait until period $t + 1$, when player j (the one who declined the offer in period t) makes a counteroffer.
3. Player i now decides to accept or reject the counteroffer. If it is accepted, the game ends. Otherwise in period $t + 1$ player i will make a counteroffer.
4. This alternating offer/counteroffer game continues like this until period T, when the game ends.

- **Discount Factor.-** In alternating-offer bargaining games, **players may have to wait some time until an agreement is reached**.

- **Having to wait** until an agreement is reached has an **opportunity cost** (a dollar tomorrow is worth less than a dollar today). This opportunity cost is measured by players' subjective **discount factors**.

- **Definition of a discount factor:** Suppose one dollar received in period $t+1$ is worth δ_i dollars to player i in period t. Then δ_i is called the "discount factor" for player i.

- Discount factors always satisfy $0 \leq \delta_i \leq 1$ (since one dollar tomorrow is worth less than a dollar today).
- In many instances, discount factors are directly related to interest rates, since they both reflect the opportunity cost of waiting to receive money.
- However, discount factors can also be subjective and different from one individual to another.
- **Discount factors are a measure of "patience" for players.** Discount factors closer to one are indicative of patient individuals, while discount factors closer to zero indicate impatience.

- Bargaining power in alternating-offer negotiations will depend on players' discount factors: **Players that are more patient** (players with factors $\boldsymbol{\delta_i \approx 1}$) **will have more bargaining power than impatient players** (players with factors $\boldsymbol{\delta_i \approx 0}$).

- Note: If the discount factor of player i is δ, then:
  - A dollar in period $t+1$ is worth δ dollars in period t.
  - A dollar in period $t+2$ is worth δ^2 dollars in period t.
  - A dollar in period $t+3$ is worth δ^3 dollars in period t.
  - A dollar in period $t+n$ is worth δ^n dollars in period t.

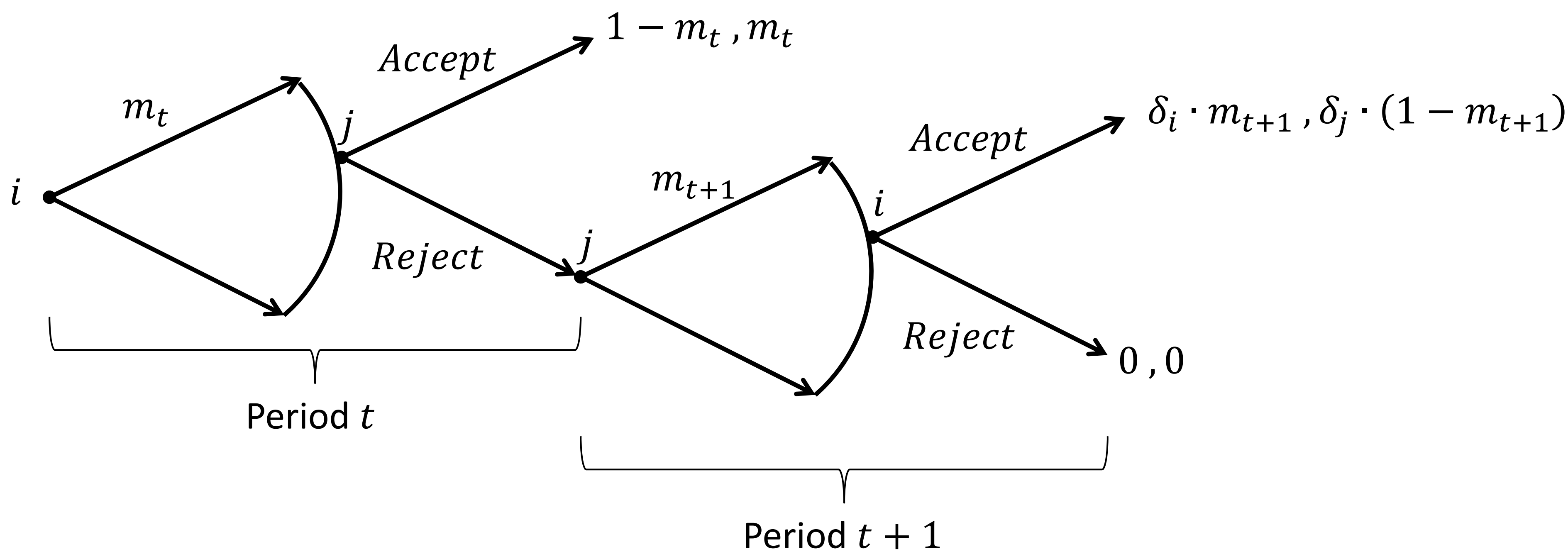
- Therefore, **a dollar received n periods from <u>now</u> is worth δ^{n-1} dollars today**. In economic terms, we say that the **present discounted value** of one dollar received n periods from now is δ^{n-1}.

- **Example: A two-period alternating offer bargaining game.-** Consider a game taking place in periods t and $t+1$, where:
- **In period t:** Player i makes offer $m_t \in [0,1]$ to player j (again, "m" here represents the share of the surplus that will go to the <u>player receiving the offer</u>). If player j accepts the offer, the game ends. Otherwise it continues in period $t+1$.

- **In period $t + 1$:** If player j rejected the previous offer, then player j makes a counteroffer to player i . Label this counteroffer as $m_{t+1} \in [0,1]$. In this case, m_{t+1} is the fraction of the surplus that would go to player i. After observing the counteroffer, player i decides to accept or reject it.
- There are no further negotiations after period $t + 1$. If no agreement is reached, both players earn a payoff of zero.
- The discount factors of players i and j are given by δ_i and δ_j respectively.
- **Question:** Characterize the SPE (subgame perfect equilibrium) of this game…

- The extensive form of this game looks like this:

Note: Payoffs need to be expressed in their present value in order to make valid comparisons across different periods.



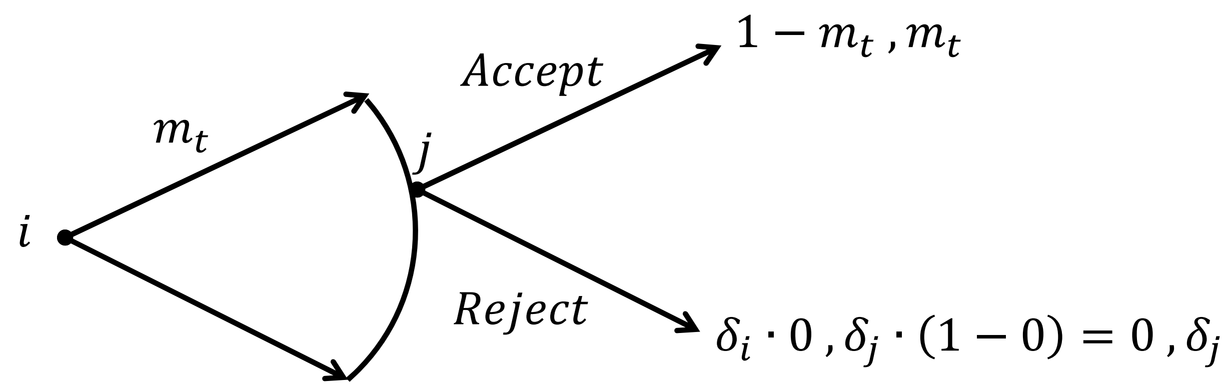
- To find the SPE, we need to start with the game played in period $t + 1$ (the last period of the game).

- The **game played in period t+1 is a one-shot ultimatum game**. We already know what its SPE is (we analyzed it in the previous section).

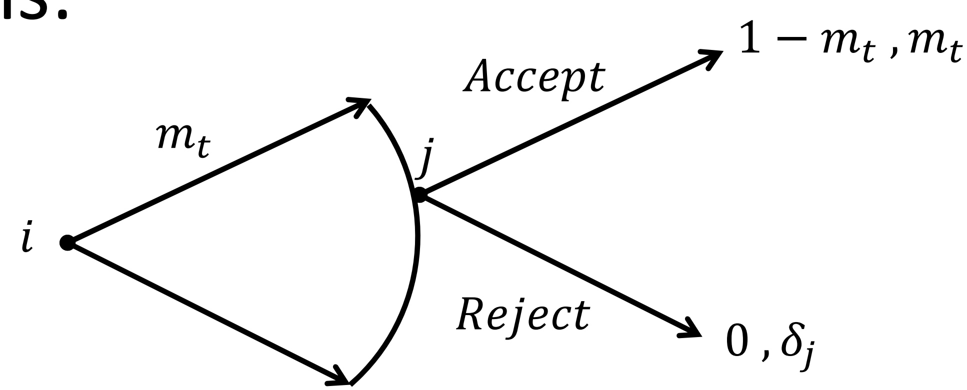
- In the SPE of the game played in period t+1, the proposer (player j) offers a share “m” of zero to the player receiving the offer, and this player accepts. **Therefore, $\boldsymbol{m_{t+1} = 0}$ in any SPE of this game.**

- Therefore, the continuation payoffs of this game look as follows…

- Continuation payoffs in period t:



- That is:



- Given these continuation payoffs, what is the optimal offer m_t for player i in period t?

- Player j knows that if he waits, he will get (in present discounted value) a payoff of δ_j. Therefore, **player j will not accept any offer $\boldsymbol{m_t}$ smaller than $\boldsymbol{\delta_j}$.**
- Since player i wants to make the smallest possible offer that player j will accept, it follows immediately that **the optimal offer for player $\boldsymbol{i}$ is $\boldsymbol{m_t = \delta_j}$.**
- Thus, in any SPE of this game: player i makes offer $m_t = \delta_j$ in period t and this offer is accepted by player j.

- Note that the more patient player j is (that is, the larger his discount factor δ_j is), the larger will be his share of the surplus.

- For example if j is perfectly patient (so that one dollar tomorrow is worth exactly as much as one dollar today), then $m_t = \delta_j = 1$ and player j gets all the surplus.

- If on the other hand player j is perfectly impatient (so that $\delta_j = 0$), then it is player i who gets all the surplus since we would have $m_t = \delta_j = 0$.

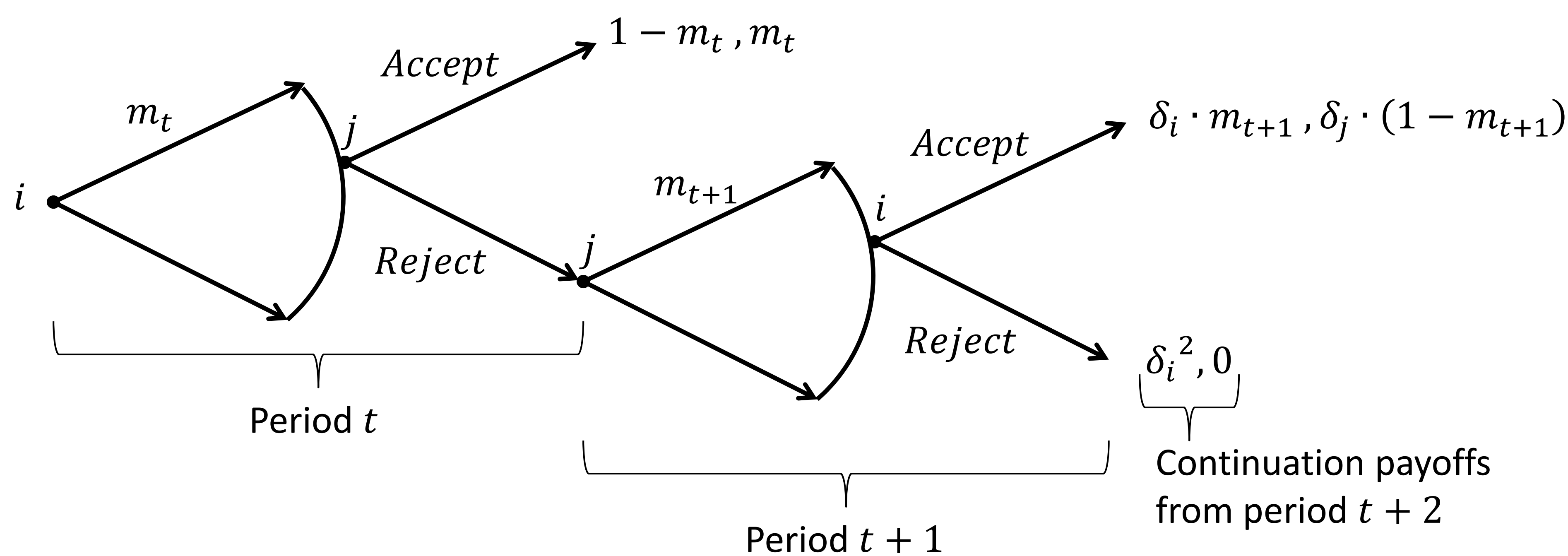
- This shows that **in alternating-offer bargaining games, bargaining power is associated with "patience" as measured by the discount factors of players**.

- **Efficiency of the SPE in the previous game:** The outcome of an alternating offer bargaining model is efficient if an offer is accepted immediately without having to wait and therefore suffer a time-depreciation of the value of the transaction.

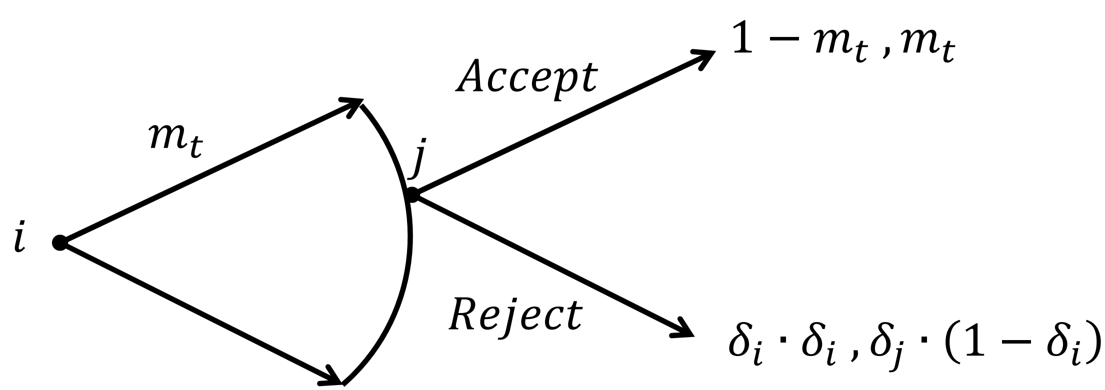
- The subgame perfect equilibrium (SPE) in the previous example leads to an efficient outcome since an offer is accepted immediately instead of having to wait one more period.

- **Three-period alternating offer bargaining game:** To gain more intuition suppose the game now has three possible negotiation periods:
- **In period $\boldsymbol{t}$:** Player i makes offer $m_t \in [0,1]$.
- **In period $\boldsymbol{t+1}$:** If m_t is rejected, then player j makes counter offer $m_{t+1} \in [0,1]$.
- **In period $\boldsymbol{t+2}$:** If m_{t+1} is rejected, then player i makes counter offer $m_{t+2} \in [0,1]$. Game ends in period 3

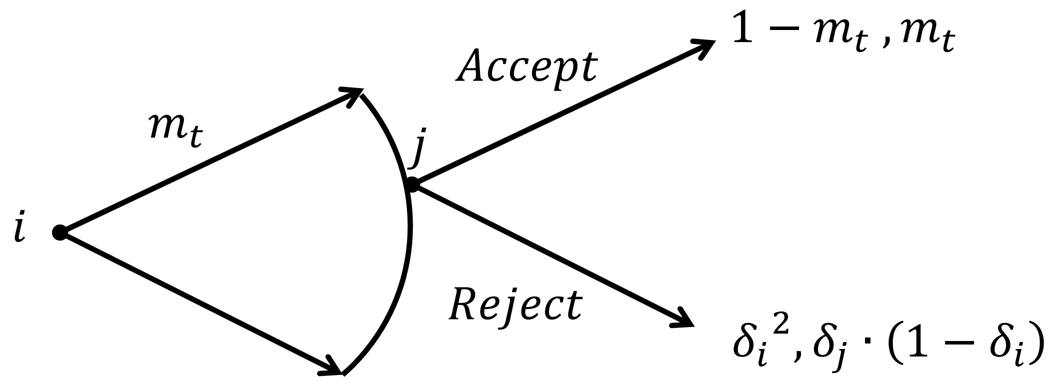
- **Find the SPE of this game:** Again, the final stage game is a one-shot ultimatum game. Its SPE has the offering player (player i) making an offer of $m = 0$. Therefore in any SPE we must have $m_{t+2} = 0$, with this offer being accepted.
- Continuation payoffs in the extensive form now look like this:



- In period $t+1$, it is clear that player i will not accept any offer m_{t+1} such that $\delta_i \cdot m_{t+1} < \delta_i{}^2$. That is, player i will not accept any offer m_{t+1} such that $m_{t+1} < \delta_i$.
- Therefore, the smallest offer that player j can make in period $t+1$ is $\boldsymbol{m_{t+1} = \delta_i}$. This is the optimal offer for player j in period $t+1$.
- Therefore, the **continuation payoffs in period t are**:



- That is, the **continuation payoffs in period t are**:



- We see from here that player j will not accept any offer m_t such that $m_t < \delta_j \cdot (1 - \delta_i)$.
- Therefore, the smallest offer that player i can make that will be accepted is $\boldsymbol{m_t = \delta_j \cdot (1 - \delta_i)}$.
- Therefore, in the SPE of this game, player i makes the offer $\boldsymbol{m_t = \delta_j \cdot (1 - \delta_i)}$ and player j will accept it.

- Again, notice how the proportion of the surplus that goes to the players depends on their relative patience (and also on the specific order in which players move).
- For example, if both players are perfectly patient ($\delta_i = \delta_j = 1$), then $m_t = \delta_j \cdot (1 - \delta_i) = 0$, so player i captures the entire surplus because he is the first proposer.
- In other cases, the relative bargaining powers (the proportion of the surplus each one will appropriate) depends on their relative patience, measured by the relative values of δ_i and δ_j.

- In general: In these sequential counter-offer bargaining models, the SPE is such that the player who is supposed to make an offer at any given period always wants to **make the smallest possible offer that will be accepted**.

- This turns out to be **the offer that makes the other player indifferent between accepting the offer today and waiting one more period**.

- That is, **the offer that gives the other player the present value of his continuation payoffs if he waits one more period**.

- This property of SPE can even help us solve alternating offer bargaining games that take place over an **infinite number of periods**.
- Consider such a game and suppose player 1 makes offers in odd periods and player 2 makes offers in even periods.
- Let us focus on a **stationary SPE** where player 1 makes the same offer every time it is his turn, and player 2 makes the same offer every time it is his turn. Label these offers as m_{odd} and m_{even}.
- In an SPE, these offers must make the other player exactly indifferent between accepting the offer at the time the offer is made OR waiting one more period.

- Suppose player 1 makes the offer m_{odd} in period $t + k$. If the offer is **accepted right then**, then payoff (brought to present discounted value at period "t") to player 2 is ${\delta_2}^k \cdot m_{odd}$. On the other hand, if player 2 waits one more period, then his continuation payoffs are ${\delta_2}^{k+1} \cdot (1 - m_{even})$.

- Therefore, player 2 will be indifferent between accepting m_1 and waiting one more period if and only if $\boldsymbol{{\delta_2}^k \cdot m_{odd} = {\delta_2}^{k+1} \cdot (1 - m_{even})}$. That is, if and only if: $\boldsymbol{m_{odd} = \delta_2 \cdot (1 - m_{even})}$.

- Similarly, player 1 will be indifferent between accepting m_2 and waiting one more period if and only if: $\boldsymbol{m_{even} = \delta_1 \cdot (1 - m_{odd})}$.
- Therefore, the alternating offers m_{even} and m_{odd} are characterized by the pair of equations:

$$m_{odd} = \delta_2 \cdot (1 - m_{even})$$
$$m_{even} = \delta_1 \cdot (1 - m_{odd})$$

- Solving this system of equations yields:

$$m_{odd} = \frac{\delta_2 \cdot (1 - \delta_1)}{1 - \delta_1 \cdot \delta_2}$$
$$m_{even} = \frac{\delta_1 \cdot (1 - \delta_2)}{1 - \delta_1 \cdot \delta_2}$$

- Again, note that the bargaining power of both players depend on their relative discount factors (patience):
  - If δ_1 is large relative to δ_2, then $m_{even} > m_{odd}$ (more bargaining power to player 1).

  - If δ_2 is large relative to δ_1, then $m_{odd} > m_{even}$ (more bargaining power to player 2).

- **Note 1**: As in the finite-period examples, the SPE of this game is such that the first offer is accepted immediately, so that this game does not actually go on forever, it ends in the first period. But we need to specify the strategies for the entire tree of the game. The **SPE leads to an efficient outcome** here, too.

- **Note 2**: If both players are perfectly patient (if $\delta_1 = \delta_2 = 1$) then the infinite-period game does not have a solution (since the denominator in the expressions for m_{even} and m_{odd} would be zero).

- The intuition is that if both players are perfectly patient, then this game cannot have a "stationary" SPE.

- But this is an anomaly since most individuals in the real world are impatient at least to some extent and they use discount factors smaller than 1.