20.- Games with Joint Decisions; Negotiation Equilibrium

- There are many interesting Economic models and real-world situations where negotiating a contract (bargaining) is part of a larger game.
- This chapter describes how to embed contract negotiation into a larger extensive form game.
- This is done by introducing **joint decision nodes** into an extensive form tree.
- We also introduce the concept of **negotiation equilibrium**, where **individual decision nodes** are played according to **sequential rationality** and the outcomes of **joint decisions** (contract negotiation) correspond to the **standard bargaining solution**.

- We insert a negotiation component into a larger extensive form game by including **joint decision nodes**.
- **Joint decision nodes.-** indicate places in the game where multiple players negotiate and establish a contract. By definition, multiple players are supposed to make a decision here, hence the term "joint decision node".
- Joint decision nodes are represented **graphically** through the use of **double circles**, in order to differentiate them from individual decision nodes (the only type we have studied so far).

- Every time we have a **joint decision node**, we also need to indicate clearly the **default outcome** which indicates (as we defined previously) the payoffs to the players if they fail to reach an agreement.

- Let's illustrate things with an example…

- **Example:** Consider a game model of contracting played between a **supplier** firm (player 1) and a **buyer** firm (player 2). The game proceeds as follows:

- **Stage 1.-** Both firms jointly negotiate a contract where the terms are: the price 't' that the buyer will pay for the good, AND the amount of damages 'c' that the supplier must pay if good is "low quality". The default outcome if they fail to reach an agreeement is $(0,0)$.
- **Stage 2.-** If they reach an agreement in stage 1, then in stage 2 the supplier has to decide whether to supply a low-quality or high-quality good.
- A high-quality good can be re-sold in the market in a way that earns a profit of $\$10$ to the buyer and $\$5$ for the supplier. If it is low quality, these benefits are $-\$6$ and $\$10$, respectively.
- If the good is low quality, there is a 50% chance that a court will find out, in which case the supplier must pay the damage award 'c' agreed upon in the first round.

- Therefore:
- The payoffs for both players if the supplier provides a high quality good are:

$10 - t$ for the buyer (player 2)

$5 + t$ for the seller (player 1)

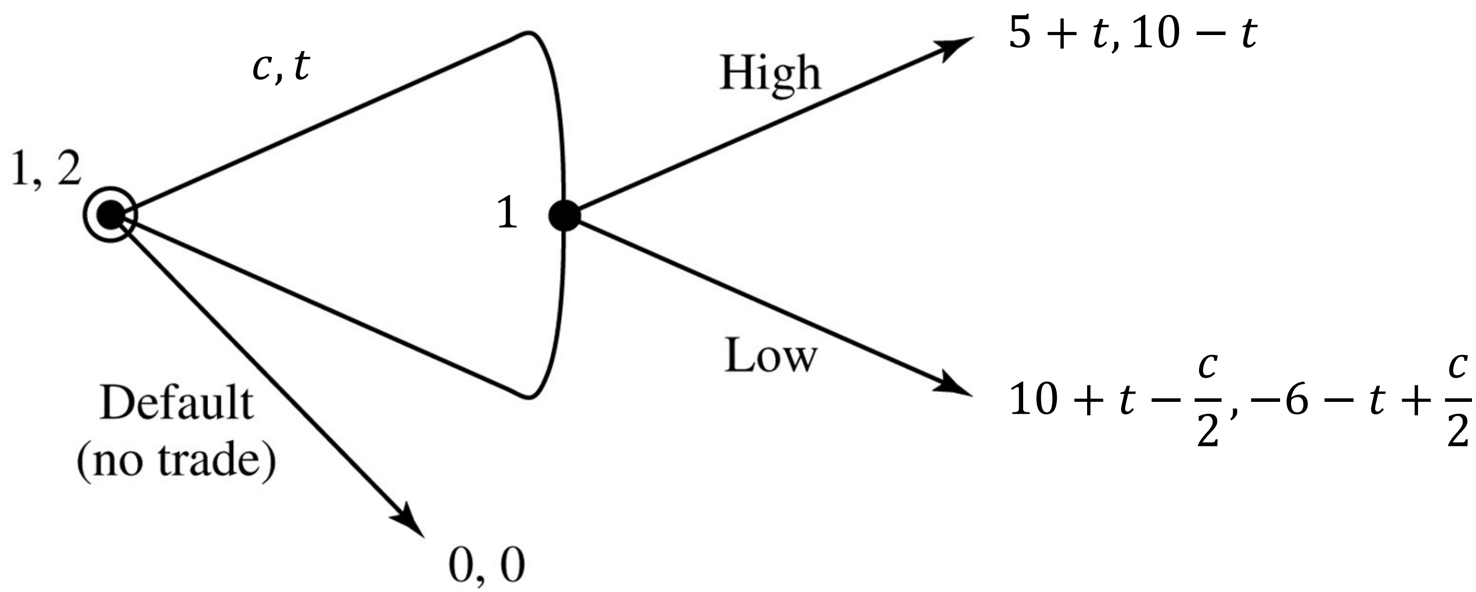
- The expected payoffs for both players if the supplier provides a low quality good are:

$-6 - t + \frac{1}{2} \cdot c$ for the buyer (player 2)

$10 + t - \frac{1}{2} \cdot c$ for the seller (player 1)

– Where "c" is the damage award that they agreed upon in the first-stage contract negotiation and "t" is the price agreed upon.

- This game can be represented as follows, using a joint decision node (payoffs of player 1 are shown first, followed by payoffs of player 2)



- **Note 1:** we always need to indicate which players move in any joint decision node (in this case, both 1 and 2).
- **Note 2:** when we have joint decision nodes, we need to explicitly state the order in which payoffs are expressed numerically, to avoid confusion.

- **Example:** Consider a potential business partnership between players 1 and 2.
- **Stage 1.-** Players 1 and 2 jointly **negotiate a compensation package for player 1**. This package has two components:
  – **Salary**.- Denoted as a monetary transfer 't' from player 2 to player 1.
  – **A performance bonus**.- Denote it as 'z'. This bonus will be **paid if and only if player 1 exerts a high effort** in the second stage of the game (once the business starts operating).

- The default outcome in stage 1 if they fail to reach an agreement is $(0,0)$.
- Stage 2.- If an agreement was reached about t and z in the first stage, then in stage 2 player 1 has to decide whether to exert high or low effort into the business.
- High effort has a monetary cost of \$10,000 to player 1. Exerting low effort has no cost to player 1.
- If player 1 exerts high effort, the business generates a revenue of $\$120{,}000$ to player 2. Low effort generates a revenue of $\$50{,}000$ to player 2. Therefore, effort by player 1 can be verified by player 2 simply by looking at the revenue generated by the business.

- Therefore, if the negotiation is successful, payoffs look as follows:
  - **If player 1 exerts high effort**:

$$Payoff\ to\ player\ 1\ = t + z - 10{,}000$$

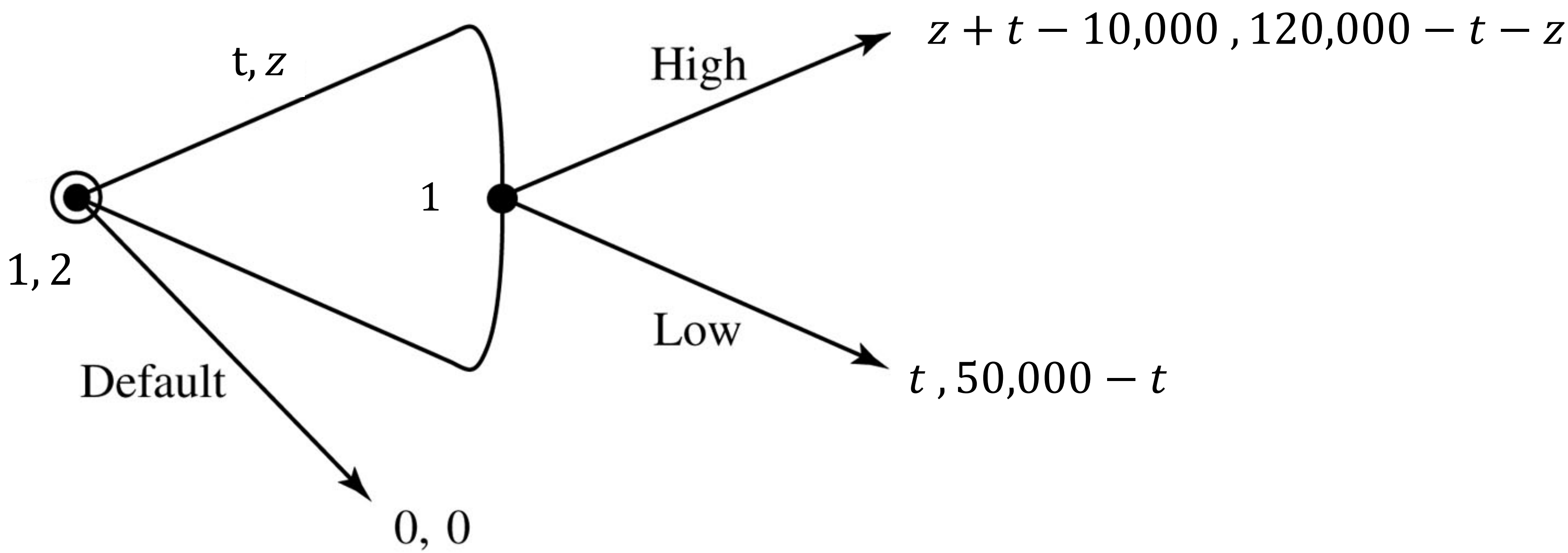
$$Payoff\ to\ player\ 2\ = 120{,}000 - t - z$$

  - **If player 1 exerts low effort**:

$$Payoff\ to\ player\ 1\ = t$$

$$Payoff\ to\ player\ 2\ = 50{,}000 - t$$

- The extensive form can be represented as follows (with the payoffs to player 1 listed first):



- We can have games where some information sets include multiple joint decision nodes…

- Allowing for joint decision nodes requires that we change one of the “tree rules” we studied previously. Specifically, this rule:
- **Tree Rule 4: Information sets belong to individual players only.-** Each information set contains decision nodes for one and one player only.
- <u>Tree Rule 4 needs to be replaced</u>. It is replaced with:
- **Tree Rule 6:** For each information set, all included nodes must be decision nodes <u>for the same subset of players</u>.

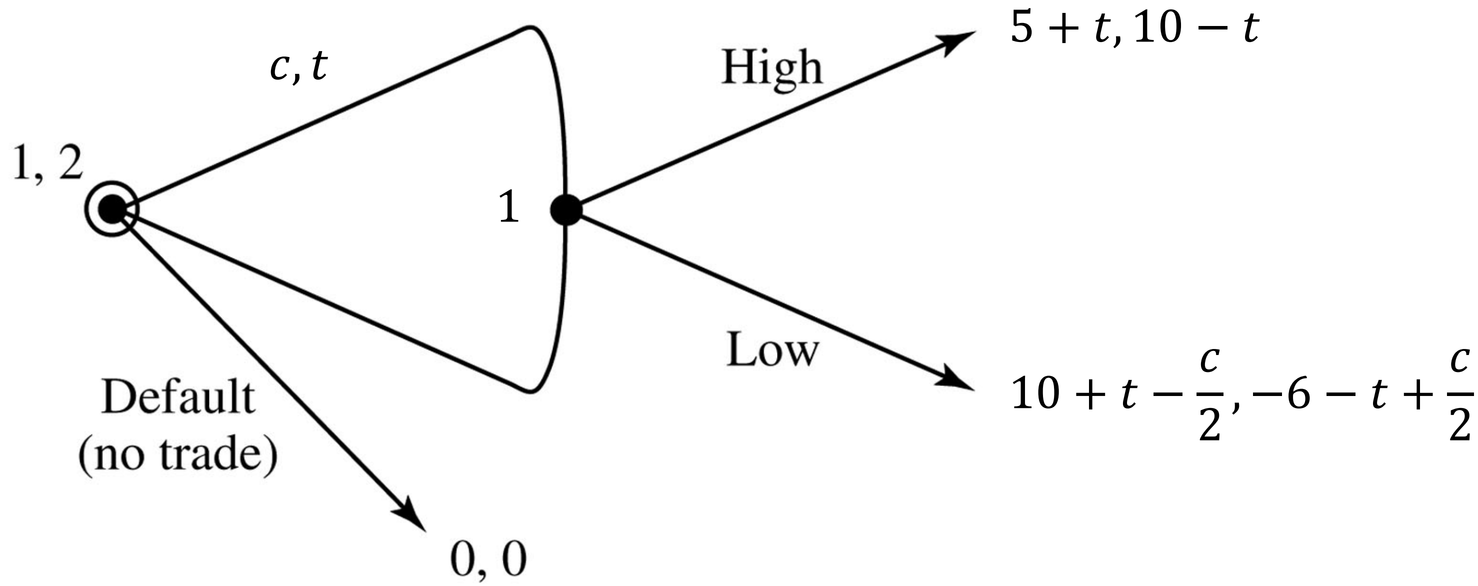
- **Negotiation equilibrium.-** When we have a game that combines contract negotiation (joint decision nodes) with individual decision nodes, we need to clarify what equilibrium concept we want to focus on.
- Here we will focus on a concept called negotiation equilibrium, which **combines individual sequential rationality with the standard bargaining solution**.
- First we need to **define a new concept**: Notice that in games that include individual decisions with joint negotiation, we have combinations of strategies and contract agreements. We refer to such combinations as **regimes**.
- So instead of talking about **strategies**, we talk about **regimes**.

- Thus, a **regime** is a combination of individual strategies and joint decisions.
- **Negotiation equilibrium:** A regime is called a negotiation equilibrium if:

a) Its description of behavior at **individual decision nodes** satisfies **sequential rationality**.

b) Its specification of **joint decisions** is consistent with the **standard bargaining solution**, for given bargaining weights.

- **Question:** Find the negotiation equilibrium in the previous examples…
- Let's start with the first example, about the buyer (player 1) and supplier (player 2):



- The last part of this game is an individual decision node. Therefore, behavior there has to be consistent with sequential rationality.

- Therefore, the supplier (player 1) will provide a high quality good if and only if:

$$5 + t \geq 10 + t - \frac{c}{2}$$

- This simplifies to the condition:

$$c \geq 10$$

- Therefore a high quality good will be provided if and only if the damage awards agreed upon in the first-stage negotiation are **at least $\boldsymbol{c = \$10}$**

- Therefore, sequential rationality implies that the continuation payoffs for both players, as a function of the terms of the contract, "c" and "t“ are:

$$u_2 = \begin{cases} 10 - t \quad if \;\; c \geq 10 \\ -6 + \dfrac{c}{2} - t \quad if \;\; c < 10 \end{cases}$$

$$u_1 = \begin{cases} 5 + t \quad if \;\; c \geq 10 \\ t + 10 - \dfrac{c}{2} \quad if \;\; c < 10 \end{cases}$$

- OK, this characterizes the sequentially rational behavior in the last stage. How about the outcome of the negotiation?
- Negotiation equilibrium predicts that the outcome should be consistent with the **standard bargaining solution**. That is:

a) The negotiation must achieve the most **efficient** outcome. That is, **the contract must maximize the joint surplus.**

b) The surplus must be split according to players' bargaining weights**.**

- From our previous results, we have that the total surplus is given by:

$$V = \begin{cases} 10 - t + 5 + t - (0 + 0) = 15 \;\; if \;\; c \geq 10 \\ -6 + \frac{c}{2} - t + t + 10 - \frac{c}{2} = 4 \;\; if \;\; c < 10 \end{cases}$$

- Therefore, **in order to achieve efficiency, first-stage contract must specify a damage award $\boldsymbol{c \geq 10}$.** In this case it will be sequentially rational for the supplier to provide a high-quality good.

- If $c \geq 10$, a high quality good will be provided in the second stage, and therefore the surplus that will be split between the players is:

$$V = 10 - t + 5 + t - \underbrace{(0 + 0)}_{\text{Default option}} = 15$$

- The standard bargaining solution predicts that this surplus will be split according to the bargaining weights of both players. This will determine the price (transfer) t in the contract.

- Let π_1 denote the bargaining weight of the buyer and let π_2 denote the bargaining weight of the supplier. Recall that the buyer is the one paying "t" to the supplier.
- We can apply the formula for transfers "t" in the standard bargaining solutions (since the player giving the transfer in that formula was also labeled as "player 2" and the one receiving it was labeled as "player 1"):

$$t = \pi_1 \times \left[{v^*}_2 - d_2\right] - \pi_2 \times \left[{v^*}_1 - d_1\right]$$
$$= \pi_1 \times [10 - 0] - \pi_2 \times [5 - 0]$$

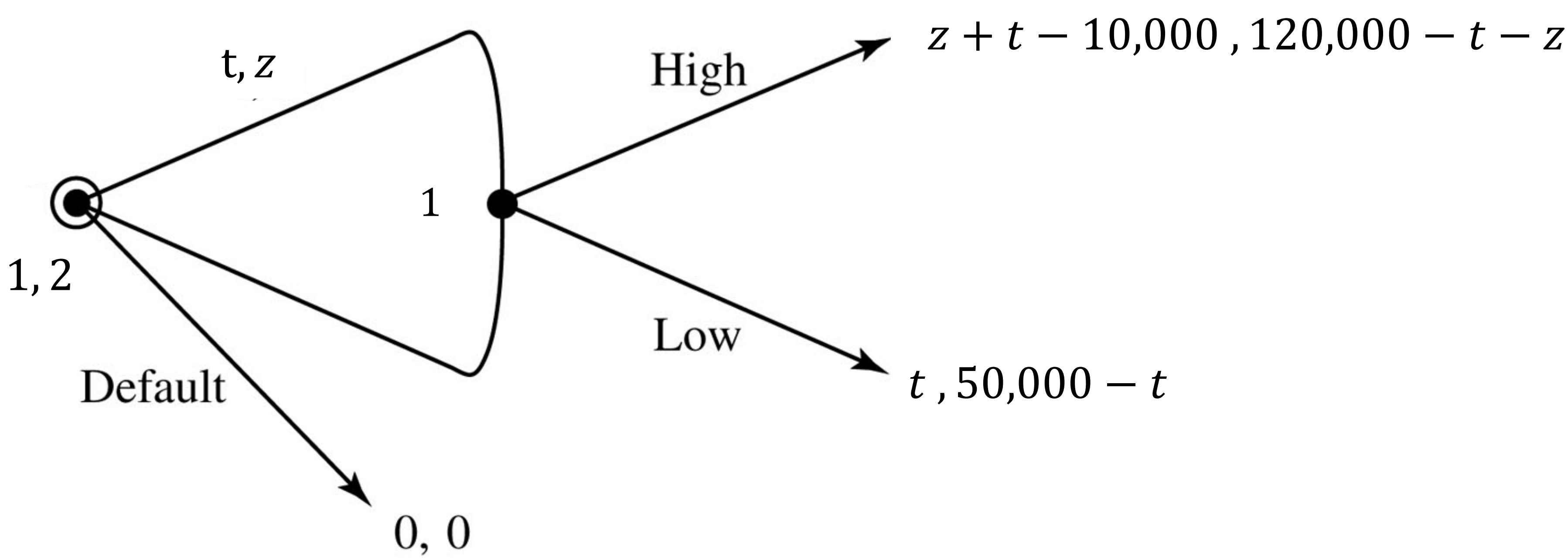
- Therefore, the price for the good must be:

$$\boldsymbol{t = 10 \cdot \pi_1 - 5 \cdot \pi_2}$$

- Therefore, a negotiation equilibrium in this game has the following features:

a) The transaction price is
$$t = 10 \cdot \pi_1 - 5 \cdot \pi_2$$
b) The damages are $c \geq 10$
c) An agreement is reached and the supplier finds it optimal to provide a high quality good.

- **Example:** Find the negotiation equilibrium in the partnership example.
- The extensive form was given by:



- Recall that “t” was the salary received by player and “z” was the bonus if player 1 exerted high effort.

- Player 1 moves in the last decision node. Therefore, player 1 will exert high effort if and only if:

$$z + t - 10{,}000 \geq t$$

- That is, if and only if $\boldsymbol{z \geq 10,000}$
- Therefore, the "effort bonus" z must be at least $\$10{,}000$.
- Note that if player 1 exerts high effort, the joint value of the partnership is:

$$u_1 + u_2 = z + t - 10{,}000 \; + 120{,}000 - t - z$$
$$= 110{,}000$$

- Otherwise if player 1 exerts low effort, the joint value of the partnership would be:

$$u_1 + u_2 = t + 50{,}000 - t = 50{,}000$$

- Therefore, the joint value of the partnership satisfies the following:

$$u_1 + u_2 = \begin{cases} 110{,}000 \quad if \;\; z \geq 10{,}000 \\ 50{,}000 \quad if \;\; z < 10{,}000 \end{cases}$$

- Therefore, **in order to achieve efficiency, the first-stage contract must specify a damage award $\mathbf{z \geq 10,000}$.**

- If $z \geq 10{,}000$, player 1 will exert high effort and therefore the surplus that will be split between the players is:

$$V = z + t - 10{,}000 + 120{,}000 - t - z - \underbrace{(0 + 0)}_{\text{Default option}} = 110{,}000$$

- The standard bargaining solution predicts that this surplus will be split according to the bargaining weights of both players.
- This will help pin down the total payment $z + t$ that player 1 will receive in the contract.

- Let π_1 denote the bargaining weight of the buyer and let π_2 denote the bargaining weight of the supplier. Recall that the buyer is the one paying "t" to the supplier.
- As long as $z \geq 10{,}000$, player 1 will exert high effort and his total remuneration will be $t + z$ (the agreed salary PLUS the bonus). The standard bargaining solution's formula predicts that:

$$t + z = \pi_1 \times \left[v^*{}_2 - d_2\right] - \pi_2 \times \left[v^*{}_1 - d_1\right]$$
$$= \pi_1 \times [120{,}000 - 0] - \pi_2 \times [-10{,}000 - 0]$$

- Therefore, the total remuneration for player 1 must be:

$$\boldsymbol{t + z = 120{,}000 \cdot \pi_1 + 10{,}000 \cdot \pi_2,}$$

with $\boldsymbol{z \geq 10{,}000}$

- For example, suppose the bonus is set simply at $z = 10{,}000$. Then, the salary must be:

$$t = 120{,}000 \cdot \pi_1 + 10{,}000 \cdot \pi_2 - 10{,}000$$

- If both players have equal bargaining power, salary would become:

$$t = 120{,}000 \cdot \frac{1}{2} + 10{,}000 \cdot \frac{1}{2} - 10{,}000 = 55{,}000$$

- Alternatively, the bonus could be set, say at $z = 50{,}000$. In this case, the salary would be:

$$t = 120{,}000 \cdot \pi_1 + 10{,}000 \cdot \pi_2 - 50{,}000$$

- If both players have equal bargaining power, salary would become

$$t = 120{,}000 \cdot \frac{1}{2} + 10{,}000 \cdot \frac{1}{2} - 50{,}000 = 15{,}000$$

- **High powered incentives:** The previous game (and others we have studied before) are an illustration of so-called **high-powered incentives**.
- High powered incentives exist when employee's remuneration is directly tied to a verifiable measure of performance or production.
- High-powered incentives and other forms of flexible pay are observed in the real world.
- Game theory can help us understand why these incentives are a good idea and how they are negotiated.

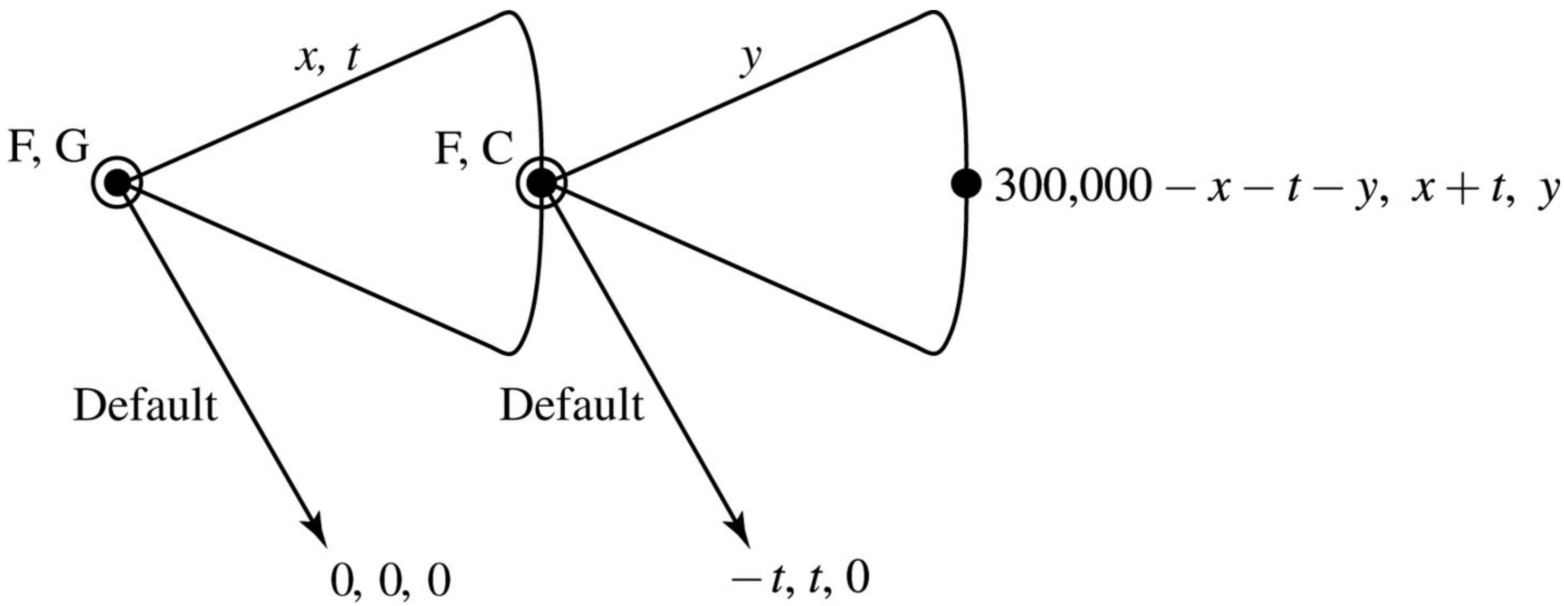
- **Example: A sequential bargaining game.-** Consider a situation in which an employer, "$\boldsymbol{F}$" is looking to hire two candidates, "$\boldsymbol{G}$" and "$\boldsymbol{C}$".
- Suppose F will negotiate the terms of their contracts **sequentially**.
- First, F negotiates with G about the following:
  - Whether G will take the job or not.
  - A payment (salary) "t" that G will receive for certain, regardless of what happens afterwards.
  - An additional payment "x" that G will receive in the event that C also takes the job in the subsequent negotiation.

- Then, once the negotiation between F and G is over, the outcome of this negotiation is perfectly observed by everyone and the negotiation between F and C begins. The terms of this negotiation are:
  – Whether C will take the job or not.
  – The payment (salary) "y" that C will receive

- Suppose the following extensive form summarizes the payoffs for all three players. Payoffs are shown in the following order:

  (Payoff to F, Payoff to G and Payoff to C)

- Extensive form:



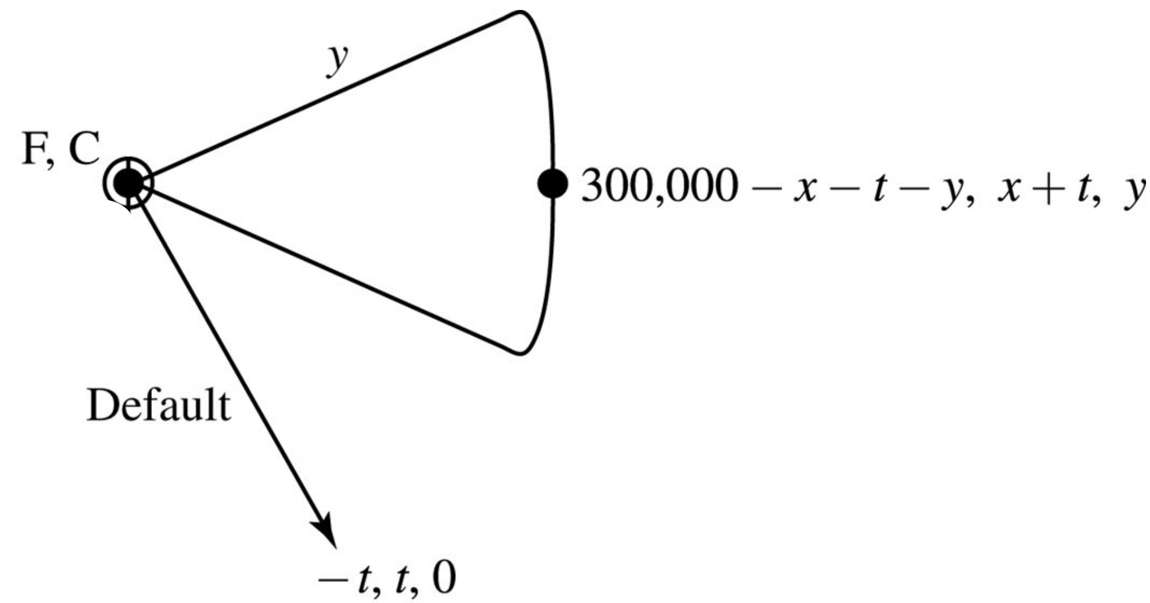
- Notice how the negotiation between F and C takes place only if the first-stage negotiation between F and G was successful.
- Also note that if F does not hire BOTH G and C, then F loses for sure the initial payment "t" made to G and makes no profits whatsoever. F needs BOTH employees to make a profit.

- **Question:** Find the negotiation equilibrium in this game. **Suppose all three players have equal bargaining power**.

- Given the sequential, perfect information nature of this game we start in the last stage subgame, which is the **negotiation between $\boldsymbol{F}$ and $\boldsymbol{C}$**.

- Recall that in this last negotiation the bargaining is over the payment "y", taking "t" and "x" as given.

- Last-stage subgame:



- Recall that the payoffs to F and C are the first and last numerical entries shown above. **Recall that the last-stage bargaining involves F and C only**.

- Note that the default payoffs in this subgame are – t (for "F") and 0 (for "C"). Therefore there will be an agreement ONLY IF:

$$300{,}000 - x - t - y \geq -t \quad \text{AND} \quad y \geq 0$$

- That is: $\mathbf{300,000 - x \geq y}$ AND $\mathbf{y \geq 0}$
- In this case the total joint value of the agreement would be:

$$u_F + u_C = 300{,}000 - x - t - y + y$$

- And the surplus would be:

$$V = u_F + u_C - [d_F + d_C]$$
$$= 300{,}000 - x - t - y - [-t + 0]$$
$$= 300{,}000 - x - y$$

- The standard bargaining solution predicts that the surplus will be split according to players' bargaining weights.
- Since all players have equal bargaining weight, then in the negotiation between F and C the surplus must be split 50%-50% between them.
- Therefore we must have:

$$300{,}000 - x - t = -t + \frac{1}{2} \cdot [300{,}000 - x]$$

AND

$$y = 0 + \frac{1}{2} \cdot [300{,}000 - x]$$

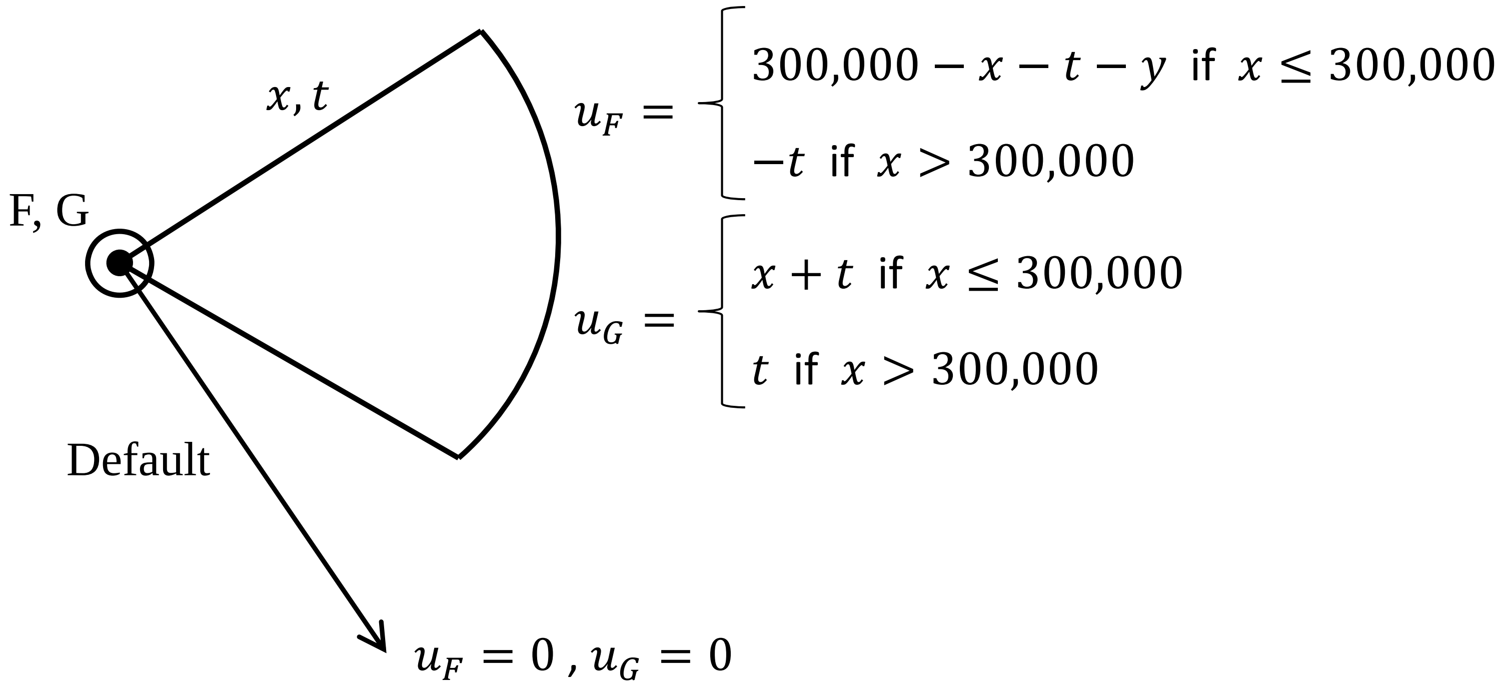
- Recall that the last-stage negotiation is only over the value of “y”, since “x” and “t” were negotiated in the first-stage.
- Therefore, the standard bargaining solution predicts the following:

a) If $x \leq 300{,}000$, then an agreement will be reached in the last stage and it will yield

$$y = \frac{1}{2} \cdot [300{,}000 - x]$$

b) Otherwise if $x > 300{,}000$, then no agreement will be reached in the last stage.

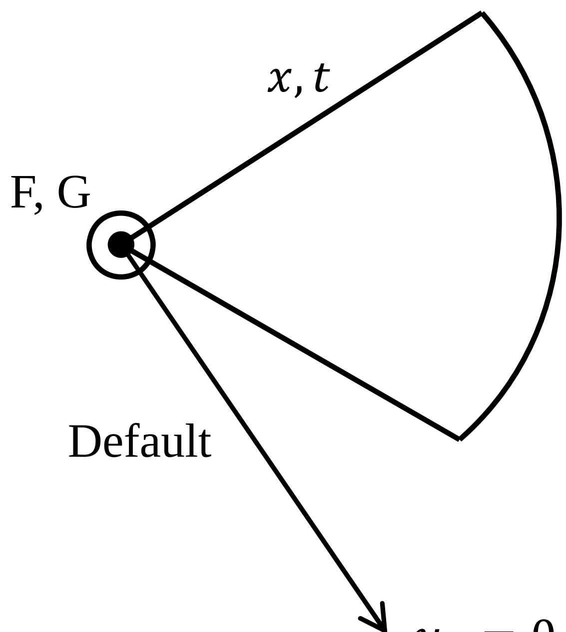
- Therefore the continuation payoffs for the first-stage negotiation are:



- But recall that if $x \leq 300{,}000$ then $y = \frac{1}{2} \cdot [300{,}000 - x]$. Replacing this expression for "y" in the extensive form described above yields…

- Continuation payoffs using the expression

$$y = \frac{1}{2} \cdot [300{,}000 - x]:$$



$$u_F = \begin{cases} 150{,}000 - \frac{x}{2} - t \text{ if } x \leq 300{,}000 \\ -t \text{ if } x > 300{,}000 \end{cases}$$

$$u_G = \begin{cases} x + t \text{ if } x \leq 300{,}000 \\ t \text{ if } x > 300{,}000 \end{cases}$$

$$u_F = 0 \, , u_G = 0$$

- The standard bargaining solution predicts efficiency and a split of the surplus according to bargaining weights.

- The total surplus is given by:

$$V = \begin{cases} 150{,}000 - \frac{x}{2} - t + x + t - [0 + 0] \text{ if } x \leq 300{,}000 \\ -t + t - [0 + 0] \text{ if } x > 300{,}000 \end{cases}$$

- This simplifies to:

$$V = \begin{cases} 150{,}000 + \frac{x}{2} \text{ if } x \leq 300{,}000 \\ 0 \text{ if } x > 300{,}000 \end{cases}$$

- The surplus V is maximized if they choose

$$x = 300{,}000$$

- Therefore efficiency in the standard bargaining solution predicts $\boldsymbol{x^* = 300,000.}$ This yields a surplus of $\boldsymbol{V^* = 150,000 + \frac{x^*}{2} = 300,000}$
- The standard bargaining solution also predicts that the surplus will be split between F and G according to their bargaining weights, which by assumption are equal (both are equal to ½). This will nail down the transfer "t". We must have:

$150{,}000 - \frac{x^*}{2} - t = 0 + \frac{1}{2} \cdot V^*$ (for player "F")

$$x^* + t = 0 + \frac{1}{2} \cdot V^* \text{ (for player "}G\text{")}$$

- Plugging in the values of $x^* = 300{,}000$ and $V^* = 300{,}000$ into either of the two equations described above MUST yield the same solution for "t". Namely:

$$\boldsymbol{t = -150,000}$$

- We are done finding the sequential equilibrium in this game. This equilibrium is described by the regime where:

a) In the first-stage negotiation, an agreement in which where the employer F **receives** $\$150{,}000$ from G and pledges to pay G a transfer of $\$300{,}000$ if C accepts the job in the second-stage negotiation.

b) In the second-stage negotiation an agreement between F and C is reached whereby C accepts to join the project and gets paid zero.

- In the end, F and G end up splitting the total surplus evenly, each earning a payoff of $\$150,000$.

- Recall that all three players have the same bargaining power. However, **negotiating sequentially** instead of simultaneously **allows F to take a bigger piece of the "surplus pie"**. G also has a **first-mover advantage** vis-à-vis C.

- What would the standard bargaining solution predict in a simultaneous negotiation?

- In this case each prospective employee would receive a transfer (salary) labeled simply as "x" and "y" for G and C respectively.
- The total surplus would be:

$$V = 300{,}000 - x - y + x + y - [0 + 0 + 0]$$

- This simplifies to

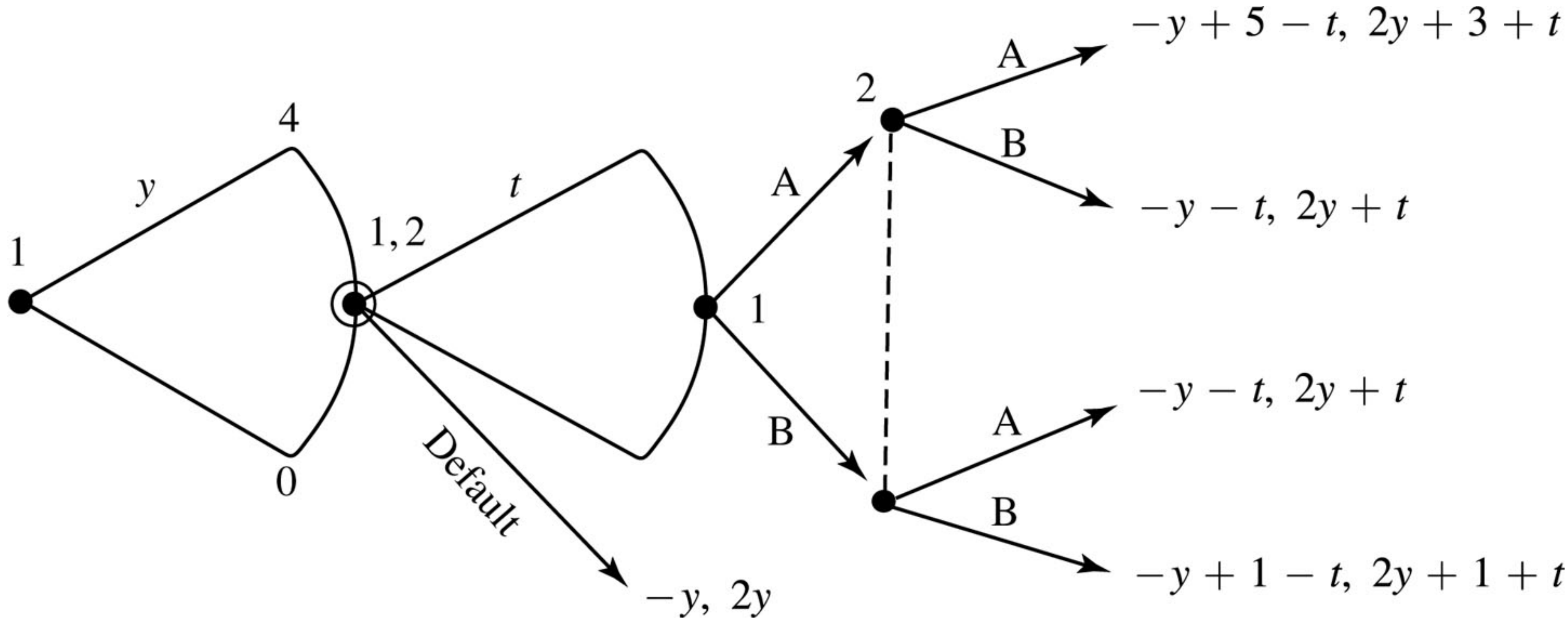
$$V = \$300{,}000$$

- This surplus would be split evenly (since all three players have equal bargaining power). Therefore each would wind up with $\$100{,}000$. That is:

$$x = y = 100{,}000$$

- Contrast this with the negotiation equilibrium in the sequential bargaining case.

- **Example:** Consider the following extensive-form game with joint decisions:



- Suppose $y \in [0,4]$ and let π_1 and π_2 denote players' bargaining weights.

- **Question:** What is the largest value of $y \in [0,4]$ that we can observe in a (pure strategy) negotiation equilibrium of this game?
- To answer this question we need to characterize how (if at all) negotiation equilibria in this game depend on "y" .
- Note that in this game, player 1 first chooses "y" and then, once observing "y", players 1 and 2 jointly negotiate over "t". If an agreement is reached, both players then proceed to play a simultaneous game where they choose either "A" or "B".

- Recall that strategies must specify a complete contingent plan over the tree.
- In particular, the strategies in the last-stage simultaneous subgame can depend explicitly on the value of “y” chosen in the first-stage and also on the value of “t” agreed upon in the second-stage negotiation.
- Let $T_1(y, t)$ and $T_2(y, t)$ denote the strategies used by players 1 and 2 in the last subgame. Note that $T_1(y, t)$ and $T_2(y, t)$ can be either “A” or “B” for each player, but the values of “y” and “t” can dictate which strategies to use.

- **How do negotiation equilibria look like in this game?** Since this is a sequential game, we can start by looking at the subgame that is played in the last-stage. This is the simultaneous game described by the following payoff-matrix:

1 \ 2	A	B
A	$-y+5-t\,,\,2y+3+t$	$-y-t\,,\,2y+t$
B	$-y-t\,,\,2y+t$	$-y+1-t\,,\,2y+1+t$

- This subgame has two pure-strategy equilibria regardless of the specific values of “y” and “t”. These equilibria are indicated below:

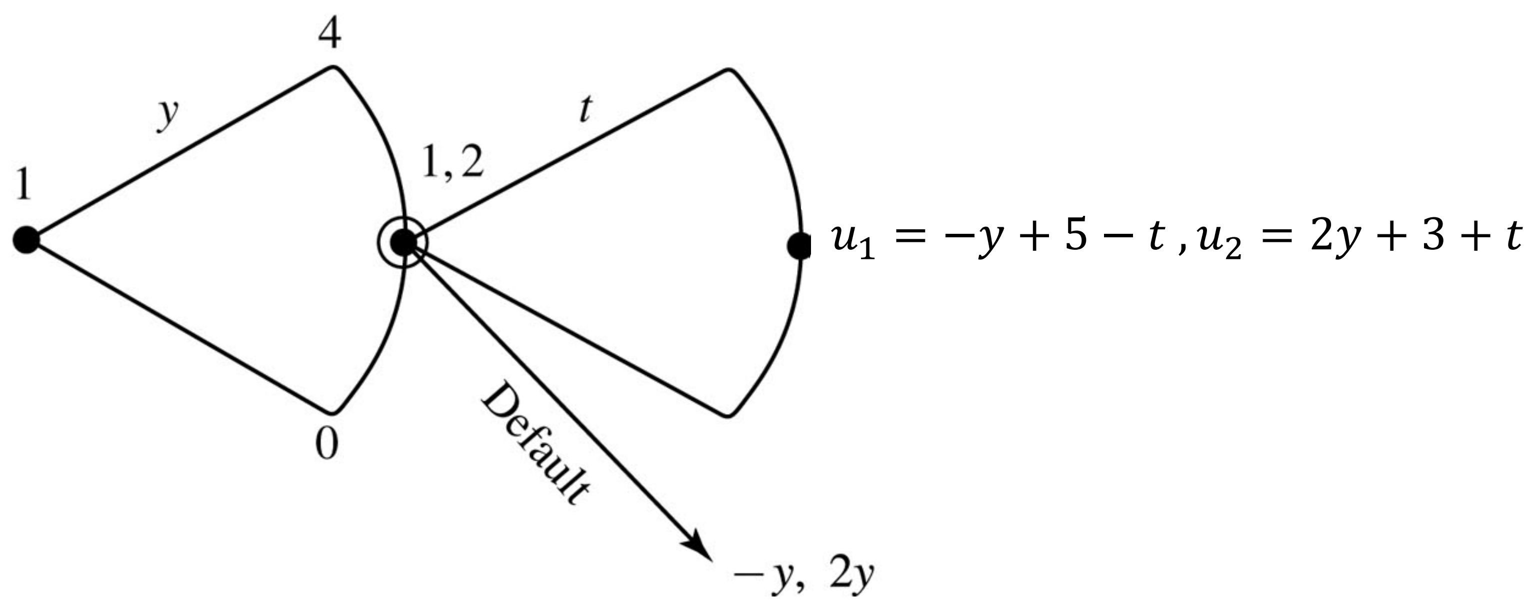
1 \ 2	A	B
A	$-y+5-t\,,\,2y+3+t$	$-y-t\,,\,2y+t$
B	$-y-t\,,\,2y+t$	$-y+1-t\,,\,2y+1+t$

- These Nash equilibria are:

$$(A, A) \quad \text{and} \quad (B, B)$$

- The next step is to examine the continuation payoffs for the second-stage negotiation in both cases: When players play (A, A) and then when players play (B, B).

- **Case 1.- Players play the Nash equilibrium (A, A) in the last-stage subgame.-** In this case the continuation payoffs are simply given by:



- We then proceed to the subgame where they negotiate over “t”.

- The total surplus in this negotiation is:

$$V = -y + 5 - t + 2y + 3 + t - [2y - y]$$

- This simplifies to:

$$V = y + 8 - y = 8$$

- The standard bargaining solution predicts that this surplus will be split between the two players according to their bargaining weights. That is:

$$-y + 5 - t = \underbrace{-y}_{\text{Default payoff for player 1 (see the extensive form tree)}} + \pi_1 \cdot 8 \quad \text{(for player 1)}$$

$$2y + 3 + t = \underbrace{2y}_{\text{Default payoff for player 2 (see the extensive form tree)}} + \pi_2 \cdot 8 \quad \text{(for player 2)}$$

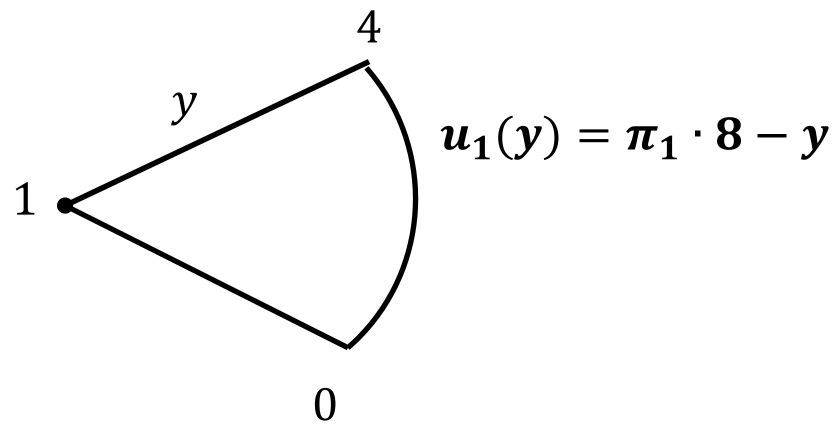
- Recall that $\pi_2 = 1 - \pi_1$. Solving either of the two previous equations yields the same solution. Namely:

$$\boldsymbol{t^* = 5 - \pi_1 \cdot 8}$$

- OK, so we have found the standard bargaining solution to the second-stage negotiation.
- Plugging in the expression $t^* = 5 - \pi_1 \cdot 8$ we can compute the continuation payoffs for "y" in the first-stage of the game. These are given by:

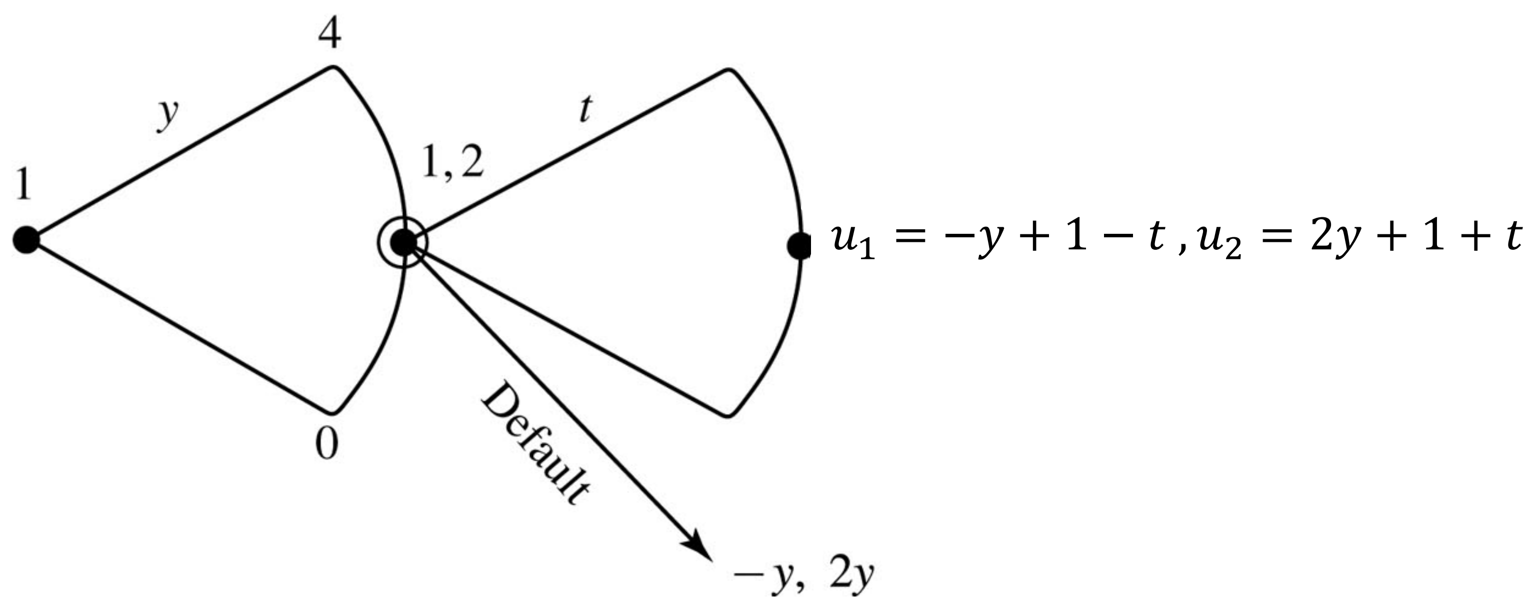
$$u_1(y) = -y + 5 - t^* = \pi_1 \cdot 8 - y$$
$$u_2(y) = 2y + 3 + t^* = 2y + 8 - \pi_1 \cdot 8$$

- In the first-stage of the game, player 1 chooses "y". Therefore only the continuation payoffs of player 1 matter here. We have:



- We will store this expression and make a final comparison once we study the case where players play the Nash equilibrium (B, B) in the last-stage subgame. We will study this case next…

- **Case 2.- Players play the Nash equilibrium $(\boldsymbol{B}, \boldsymbol{B})$ in the last-stage subgame.-** In this case the continuation payoffs are given by:



- We then proceed to the subgame where they negotiate over "t".

- The total surplus in this negotiation is:

$$V = -y + 1 - t + 2y + 1 + t - [2y - y]$$

- This simplifies to:

$$V = y + 2 - y = 2$$

- The standard bargaining solution predicts that this surplus will be split between the two players according to their bargaining weights. That is:

$$-y + 1 - t = \underbrace{-y}_{\text{Default payoff for player 1 (see the extensive form tree)}} + \pi_1 \cdot 2 \quad \text{(for player 1)}$$

$$2y + 1 + t = \underbrace{2y}_{\text{Default payoff for player 2 (see the extensive form tree)}} + \pi_2 \cdot 2 \quad \text{(for player 2)}$$

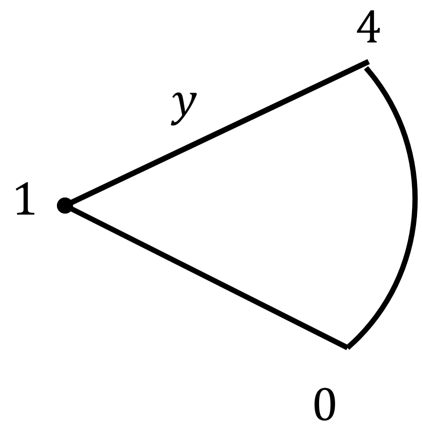
- Solving either of the two previous equations yields the same solution. In this case:

$$\boldsymbol{t^* = 1 - \pi_1 \cdot 2}$$

- OK, so we have found the standard bargaining solution to the second-stage negotiation.
- Plugging in the expression $t^* = 1 - \pi_1 \cdot 2$ we can compute the continuation payoffs for "y" in the first-stage of the game. These are given by:

$$u_1(y) = -y + 1 - t^* = \pi_1 \cdot 2 - y$$
$$u_2(y) = 2y + 1 + t^* = 2y + 2 - \pi_1 \cdot 2$$

- In the first-stage of the game, player 1 chooses "y". Therefore only the continuation payoffs of player 1 matter here. We have:



$$\boldsymbol{u_1(y) = \pi_1 \cdot 2 - y}$$

- OK. Finally we can make a comparison between the continuation payoffs of "y" if players choose (A,A) vs. (B,B) as the Nash equilibrium in the last-stage subgame.

- Continuation payoffs in any negotiation equilibrium for player 1 in the first-stage are:

a) $u_1(y) = \pi_1 \cdot 8 - y$ if they choose Nash equilibrium (A, A) in the last subgame.

b) $u_1(y) = \pi_1 \cdot 2 - y$ if they choose Nash equilibrium (B, B) in the last subgame.

- The question was **"what is the largest value of 'y' that can be observed in a negotiation equilibrium of this game?"**

- Inspecting the continuation payoffs, we can conclude the following:
- Player 1 always wants to choose “y” as small as possible.
- Player 1 can agree to choose a large value of “y” in a negotiation equilibrium ONLY IF choosing a large value of “y” leads to choosing (A, A) in the last-stage subgame.
- This follows because, for any value of “y”, the continuation payoff for player 1 is always higher if they play (A,A) than if they play (B,B) (compare $\pi_1 \cdot 8 - y$ vs. $\pi_1 \cdot 2 - y$).

- Therefore, a large value of "y" can be chosen in the first-stage of a negotiation equilibrium only if the last-stage subgame strategies are of the form:

$$(T_1(y,t), T_2(y,t)) = (A, A) \text{ if } y \geq \bar{y}$$
$$(T_1(y,t), T_2(y,t)) = (B, B) \text{ if } y < \bar{y}$$

- The question then becomes, **how large can $\bar{y}$ be?**
- With strategies as described above, sequential rationality dictates that player 1 behave as follows in stage 1:

"choose $y = 0$ if $\pi_1 \cdot 2 - 0 > \pi_1 \cdot 8 - \bar{y}$"

"choose $y = \bar{y}$ if $\pi_1 \cdot 2 - 0 \leq \pi_1 \cdot 8 - \bar{y}$"

- Therefore, the largest value of $\bar{y}$ that can be chosen in a negotiation equilibrium is the one that satisfies:

$$\pi_1 \cdot 2 - 0 = \pi_1 \cdot 8 - \bar{y}$$

- That is,

$$\bar{y} = \pi_1 \cdot 6$$

- For example, **if both players have equal bargaining power** (that is, $\pi_1 = \frac{1}{2}$) then **the largest value of $\boldsymbol{y}$ that can be chosen in a negotiation equilibrium is $\boldsymbol{y = 3}$.**
- In particular, $\boldsymbol{y = 4}$ **can be chosen in a negotiation equilibrium only if player 1's bargaining weight is $\boldsymbol{\pi_1 \geq \frac{2}{3}}$.**