21.- Unverifiable Investment, Hold Up, Options and Ownership

- This chapter applies the tools of games with joint decisions and negotiation equilibrium to study the **hold-up problem** in economics.
- We first describe what the hold-up problem is, and then some remedies to it.
- The hold-up problem is very important in the Economics field of **modern contract theory**.

- **Hold-up problem:** This is another instance of strategic tension between individual and group interests. Specifically, it arises in the context of **relationship-specific investments**.
- Consider an investor (player "A") and a trading partner (player "B"). A relationship-specific investment will arise if the investment made by "A" is specific to the relationship with "B" so that, if "B" walks away, then "A" will lose the entire investment.
- This occurs in the real world in situations where, for example, investors must make investments that are customized to the special needs of their trading partners.
- For example, machinery or production processes may have to be specifically designed to meet the needs of a trading partner. If the partner walks away, the investment is lost.

- Hold-up arises when part (or all) of the investment return to the investor (player "A") can be appropriated ex-post by his trading partner (player "B").
- If "B" uses his bargaining power to extract a share of the investment returns, then the investor "A" may decide to under-invest. This is called the "hold-up" problem because trading partner "B" may extract part of the returns of "A" by threatening to hold up production.

- **Example:** Consider a model involving a scientist (player "A") whose expertise allows him to design a new medical device. However, in order to produce the device, "A" has to partner with an engineering firm (player "B") which is capable of producing and marketing the device. The timing is as follows:
- **Period 1:** Player "A" must decide whether to invest effort into the design of the device. "A" has three options:
  - **Invest high effort** (at a cost of \$10).
  - **Invest low effort** (at a cost of \$1)
  - **Not invest** (in this case the game ends, both parties walk away with \$0).

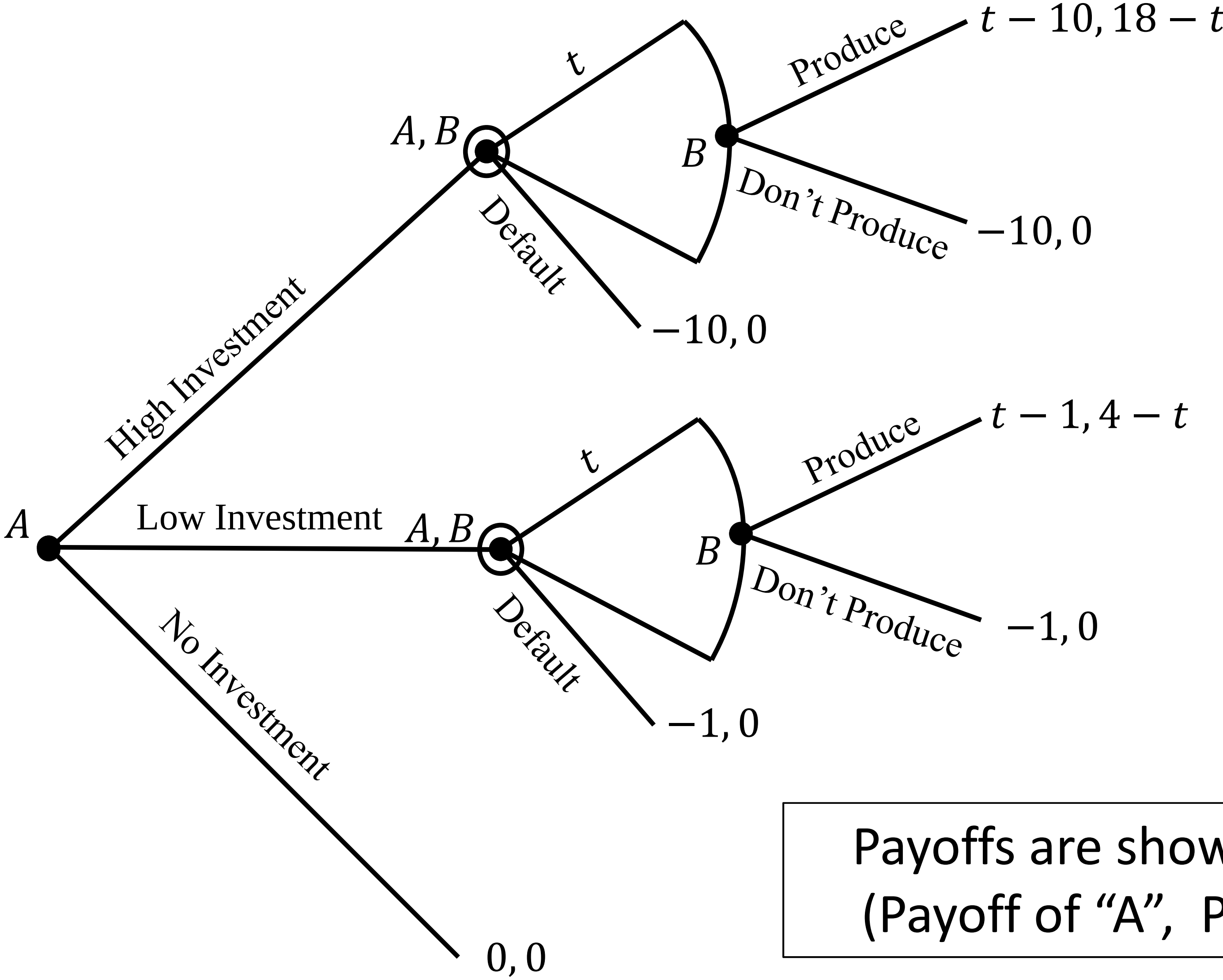
– The investment chosen by "A" is observed by "B" but cannot be verified by any external court.

- **Period 2:** Player "B" observes the investment decision of "A". In period 2, both players negotiate a contract whose terms are the following:
  - A transfer "$\boldsymbol{t}$" that "B" would pay to "A" in the event that "B" decides to produce the device in period 3. This transfer will be paid only if "B" decides to produce the device in period 3.
- **Period 3:** Player "B" decides whether to produce the device or not. The revenue earned by "B" depends on the effort invested by "A" in the design:
- If "A" invested High effort, then the revenue for "B" would be $\$18$.
- If "A" invested Low effort, then the revenue for "B" would only be $\$4$.

- We assume that "B" can produce and market the good only if an agreement was reached with "A" in period 2 about the transfer "t" that would be paid to "A". In other words, we assume that the law forbids "B" to steal the design and produce the good on its own.

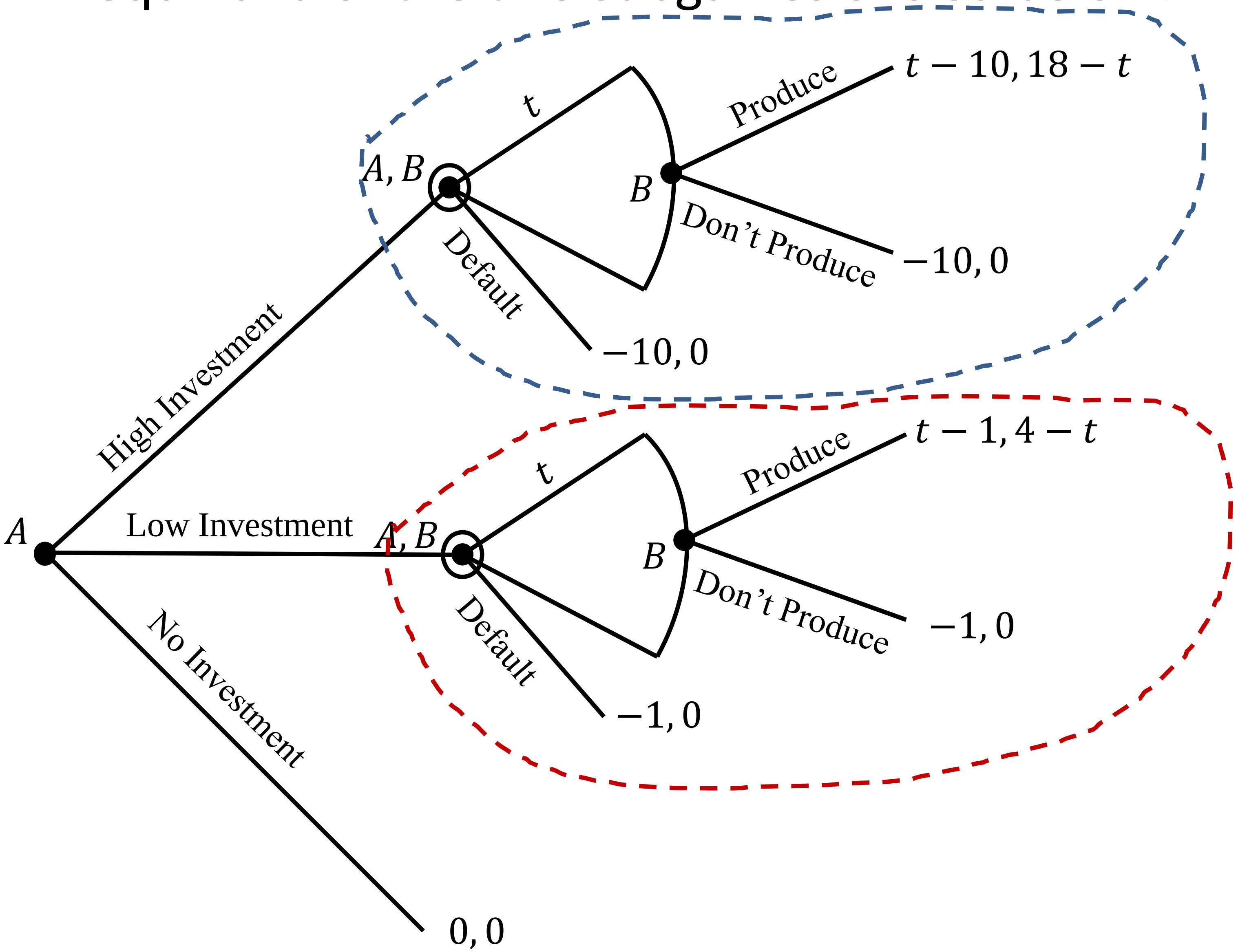
- **Question:** Find the negotiation equilibrium in this game…

- The extensive form of this game looks like this:

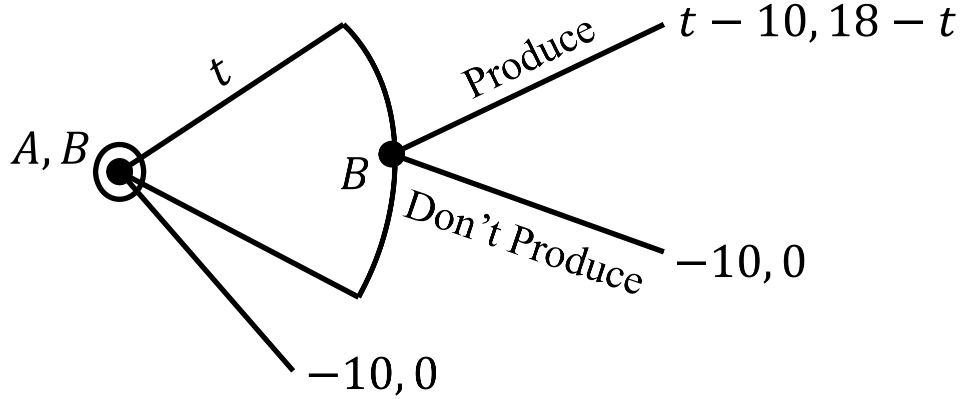


Payoffs are shown as follows:
(Payoff of "A", Payoff of "B")

- To find the negotiation equilibrium we use backward induction. We begin by finding the negotiation equilibria of the two subgames circled below:

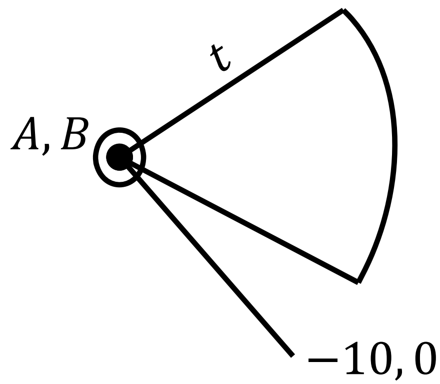


- We begin with the subgame that follows after “high investment”:



- To find the negotiation equilibrium of this subgame we start in the last decision node, where “B” must decide whether to produce the device or not. **Sequential rationality dictates that B will produce the device if and only if $18 - t \geq 0$**

- Therefore the continuation payoffs in this subgame look like this:



$$u_A = \begin{cases} t - 10 & \text{if } t \leq 18 \\ -10 & \text{if } t > 18 \end{cases}$$

$$u_B = \begin{cases} 18 - t & \text{if } t \leq 18 \\ 0 & \text{if } t > 18 \end{cases}$$

- Therefore the surplus in this subgame is given by:

$$V = \begin{cases} t - 10 + 18 - t - [-10 + 0] = 18 & \text{if } t \leq 18 \\ -10 + 0 - [-10 + 0] = 0 & \text{if } t \leq 18 \end{cases}$$

- The standard bargaining solution predicts efficiency. Therefore it **predicts that $\boldsymbol{t \leq 18}$**

- So the surplus in this subgame will be $V^* = 18$. Standard bargaining solution predicts that this surplus will be split according to the players' bargaining weights. That is:

$$t - 10 = -10 + \pi_A \cdot 18 \quad \text{(for player “}A\text{”)}$$
$$18 - t = 0 + \pi_B \cdot 18 \quad \text{(for player “}B\text{”)}$$

- Either of these two equations predicts the same value of "t". Namely:

$$\boldsymbol{t = \pi_A \cdot 18}$$

- Let's **assume that both players have equal bargaining weight**. Then we will have

$$\boldsymbol{t = \pi_A \cdot 18 = \frac{1}{2} \cdot 8 = 9}$$

- Therefore, the negotiation equilibrium in this subgame is such that $t^* = 9$ and player "B" decides to produce the device.

- The continuation payoffs are:

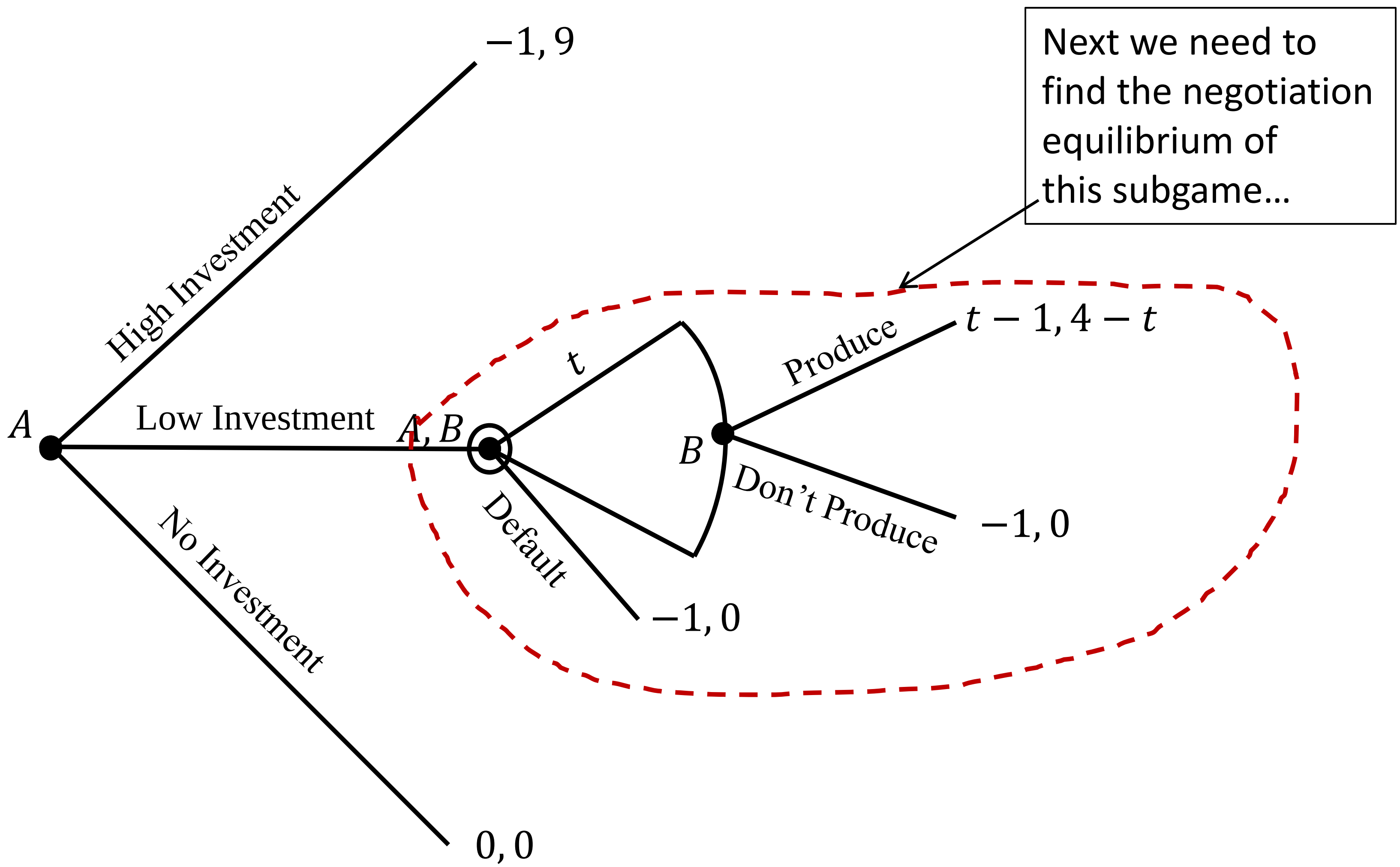
$$u_A = t^* - 10 = 9 - 10 = -1$$

and

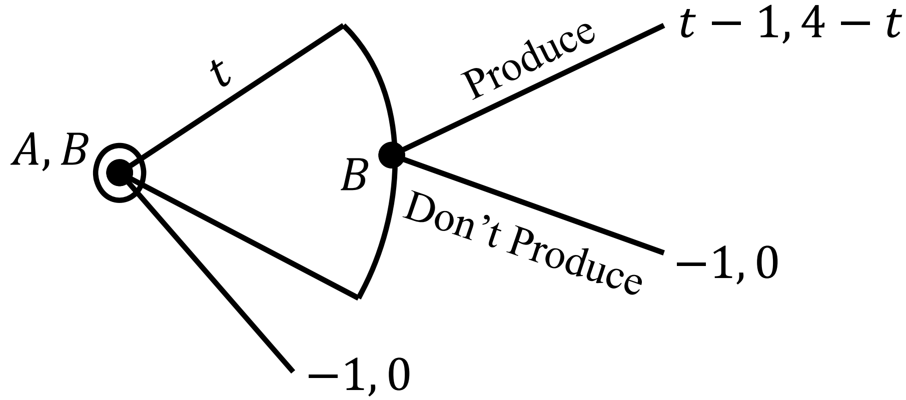
$$u_B = 18 - t^* = 18 - 9 = 9$$

- The above are the continuation payoffs if "A" chooses a High investment.

- Using the continuation payoffs we derived above, backward induction means that the extensive form of the game looks like this:

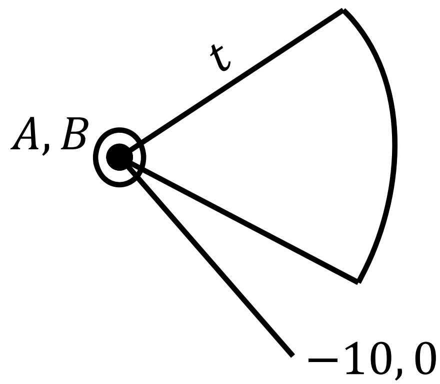


- We now study the subgame that follows after "low investment":



- To find the negotiation equilibrium of this subgame we start in the last decision node, where "B" must decide whether to produce the device or not. **Sequential rationality dictates that B will produce the device if and only if $4 - t \geq 0$**

- Therefore the continuation payoffs in this subgame look like this:



$$u_A = \begin{cases} t - 1 & \text{if } \; t \le 4 \\ -1 & \text{if } \; t > 4 \end{cases}$$

$$u_B = \begin{cases} 4 - t & \text{if } \; t \le 4 \\ 0 & \text{if } \; t > 4 \end{cases}$$

- Therefore the surplus in this subgame is given by:

$$V = \begin{cases} t - 1 + 4 - t - [-1 + 0] = 4 & \text{if } \; t \le 4 \\ -1 + 0 - [-1 + 0] = 0 & \text{if } \; t \le 4 \end{cases}$$

- The standard bargaining solution predicts efficiency. Therefore it **predicts that $\boldsymbol{t \le 4}$**

- So the surplus in this subgame will be $V^* = 4$. Standard bargaining solution predicts that this surplus will be split according to the players' bargaining weights. That is:

$$t - 1 = -1 + \pi_A \cdot 4 \quad \text{(for player "}A\text{")}$$

$$4 - t = 0 + \pi_B \cdot 4 \quad \text{(for player "}B\text{")}$$

- Either of these two equations predicts the same value of "t". Namely:

$$\boldsymbol{t = \pi_A \cdot 4}$$

- We assumed previously that **both players have equal bargaining weight**. Then we will have

$$\boldsymbol{t = \pi_A \cdot 4 = \frac{1}{2} \cdot 4 = 2}$$

- Therefore, the negotiation equilibrium in this subgame is such that $t^* = 2$ and player "B" decides to produce the device.

- The continuation payoffs are:

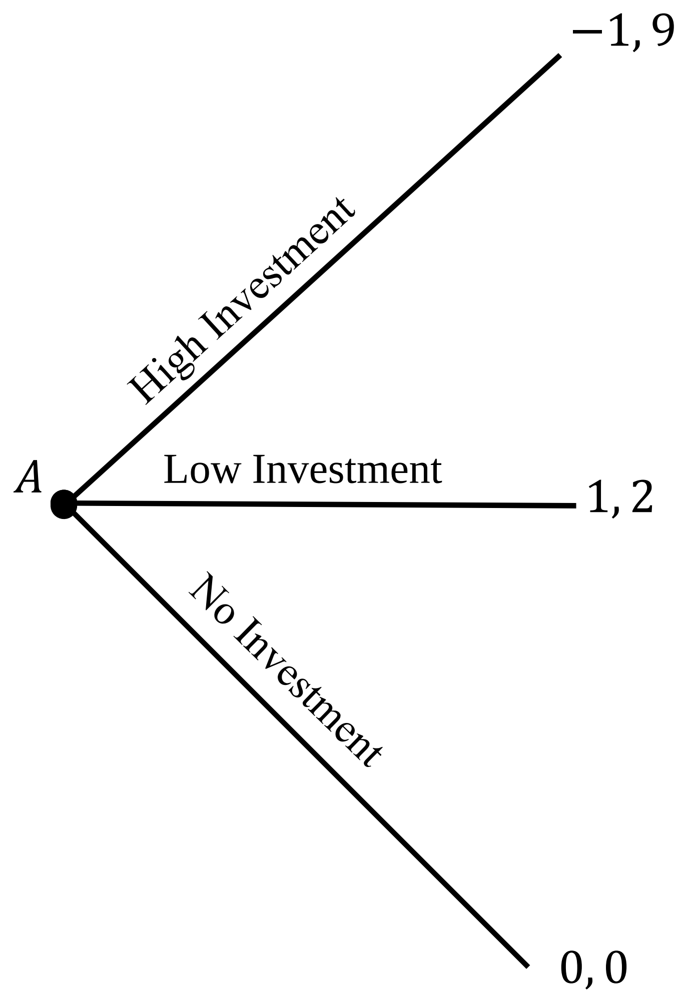
$$u_A = t^* - 1 = 2 - 1 = 1$$

and

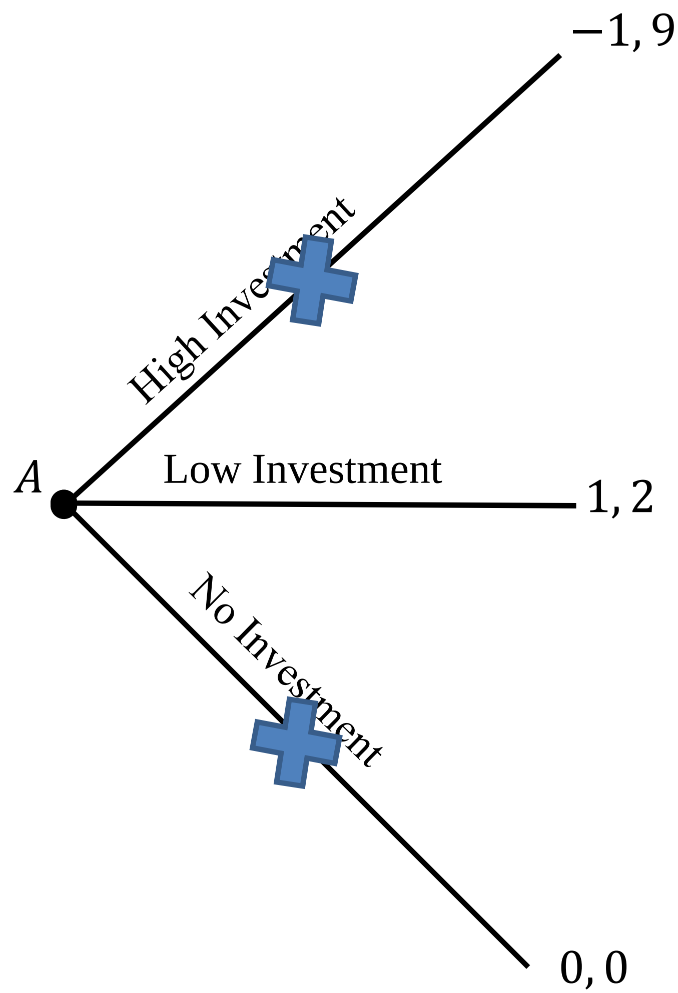
$$u_B = 4 - t^* = 4 - 2 = 2$$

- The above are the continuation payoffs if "A" chooses a Low investment.

- Using the continuation payoffs we derived above, backward induction means that the extensive form of the game looks like this:



- The last step of backward induction is simply to find the investment decision for “A” that yields the largest continuation payoffs:



The optimal choice for “A” is to choose a **Low Investment**.

- The negotiation equilibrium in this game predicts an investment level that is **socially inefficient:** The total surplus with high investment would have been $V = -1 + 9 = 8$**.** In contrast with low investment the surplus is $V = 1 + 2 = 3$.

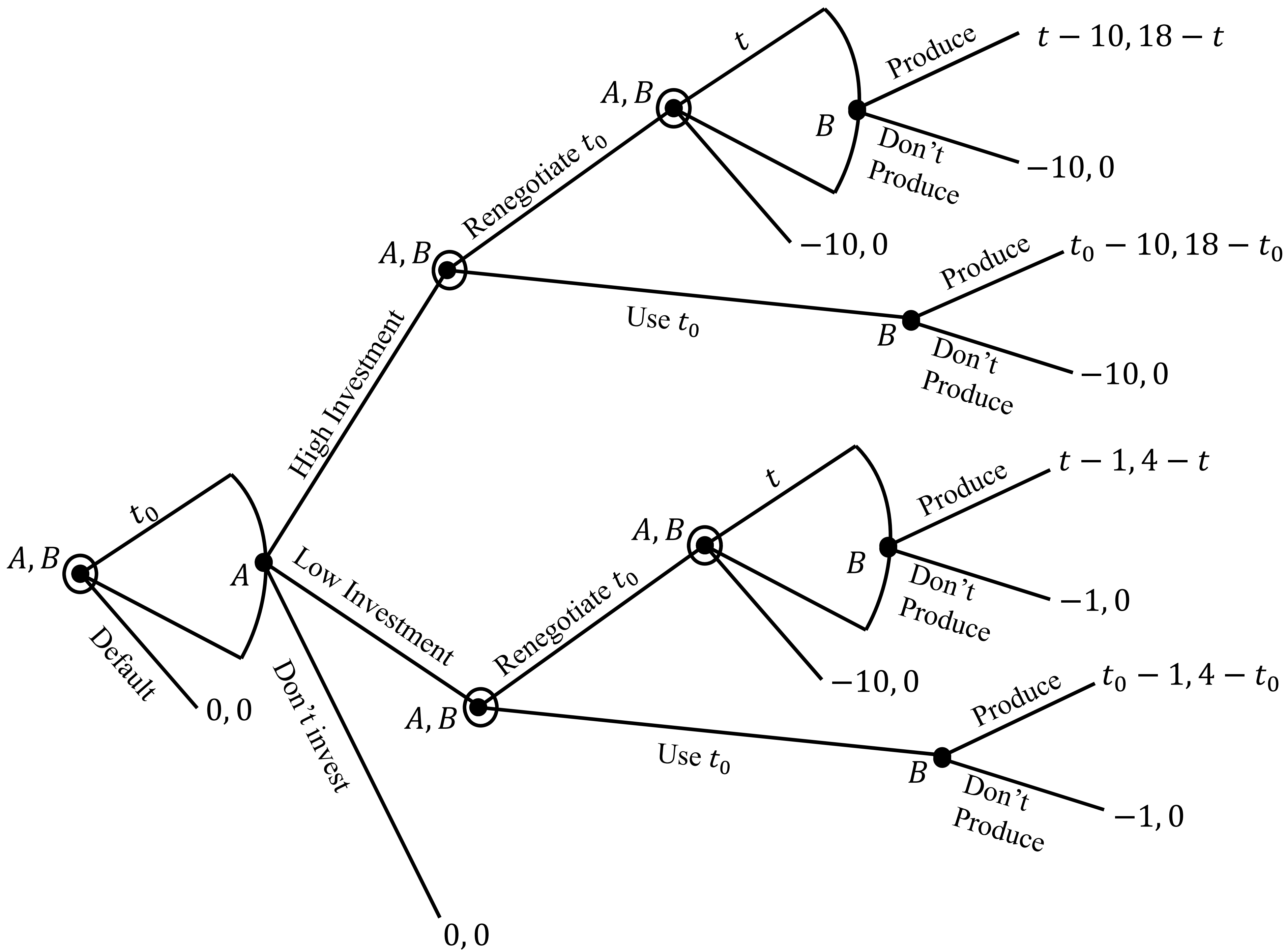
- This suboptimal investment level is a consequence of the **hold-up problem**, which arises because player "B" has the option not to produce, in which case "A" would lose the entire investment made.

- **How to solve the hold-up problem?** This is one of the main questions in modern contract theory.
- Note that the problem arises because players are assumed to negotiate the transaction price "t" only **AFTER** player "A" has made his investment decision.
- Thus, one way to solve the hold-up problem is to allow players the possibility to negotiate the transaction price "t" **BEFORE** player "A" makes his investment decision.
- We can let them re-negotiate the transaction price AFTER the investment decision of "A" is made. Simply allowing them to negotiate a preliminary price BEFORE player "A" makes his investment decision can solve the hold-up problem. This is called **up-front contracting**.

- Specifically, consider the following alternative format of the contract:
- **Period 0:** Players “A” and “B” get together and negotiate a transaction price t_0, which can be re-negotiated in the latter stages of the game.
- **Period 1:** Player “A” makes his investment decision (High, Low or No investment).
- **Period 2:** Players “A” and “B” jointly decide whether to adhere to the initial transaction price t_0 or to renegotiate it to an alternative t
- **Period 3:** Player “B” decides whether to produce the device or not.

- **Question:** Find the negotiation equilibrium in this game. Does high investment occur in equilibrium with up-front contracting?

- As always, the first step is to carefully draw the extensive form of this game…

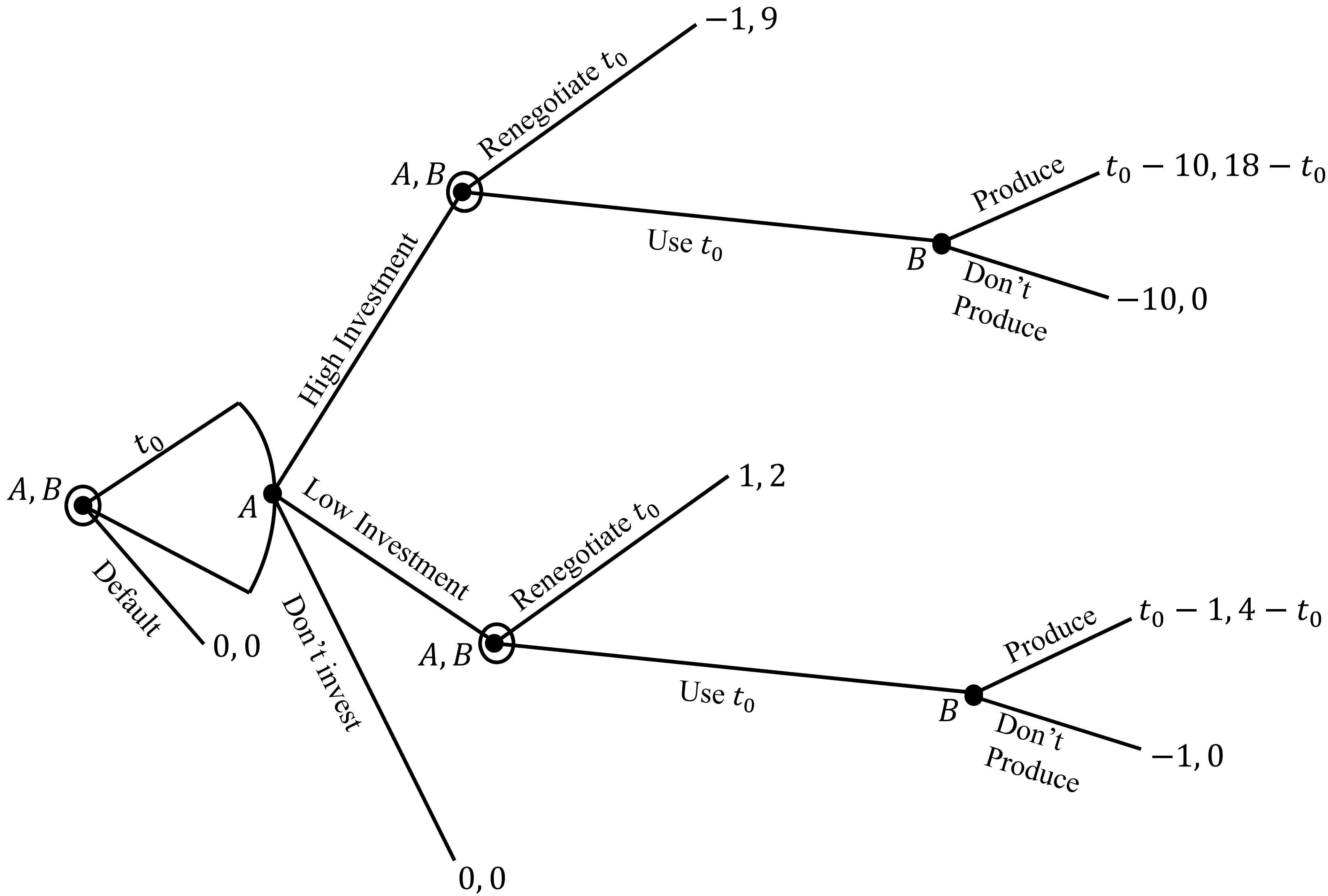


A, B
t₀
Default
0, 0
A
High Investment
Low Investment
Don't invest
0, 0
A, B
Renegotiate t₀
Use t₀
A, B
t
B
Produce
t − 10, 18 − t
Don't Produce
−10, 0
−10, 0
B
Produce
t₀ − 10, 18 − t₀
Don't Produce
−10, 0
A, B
Renegotiate t₀
Use t₀
A, B
t
B
Produce
t − 1, 4 − t
Don't Produce
−1, 0
−10, 0
B
Produce
t₀ − 1, 4 − t₀
Don't Produce
−1, 0

- If they renegotiate in period 2, we already know what will happen in each case since we worked out the details when we analyzed the negotiation equilibrium in the original game.

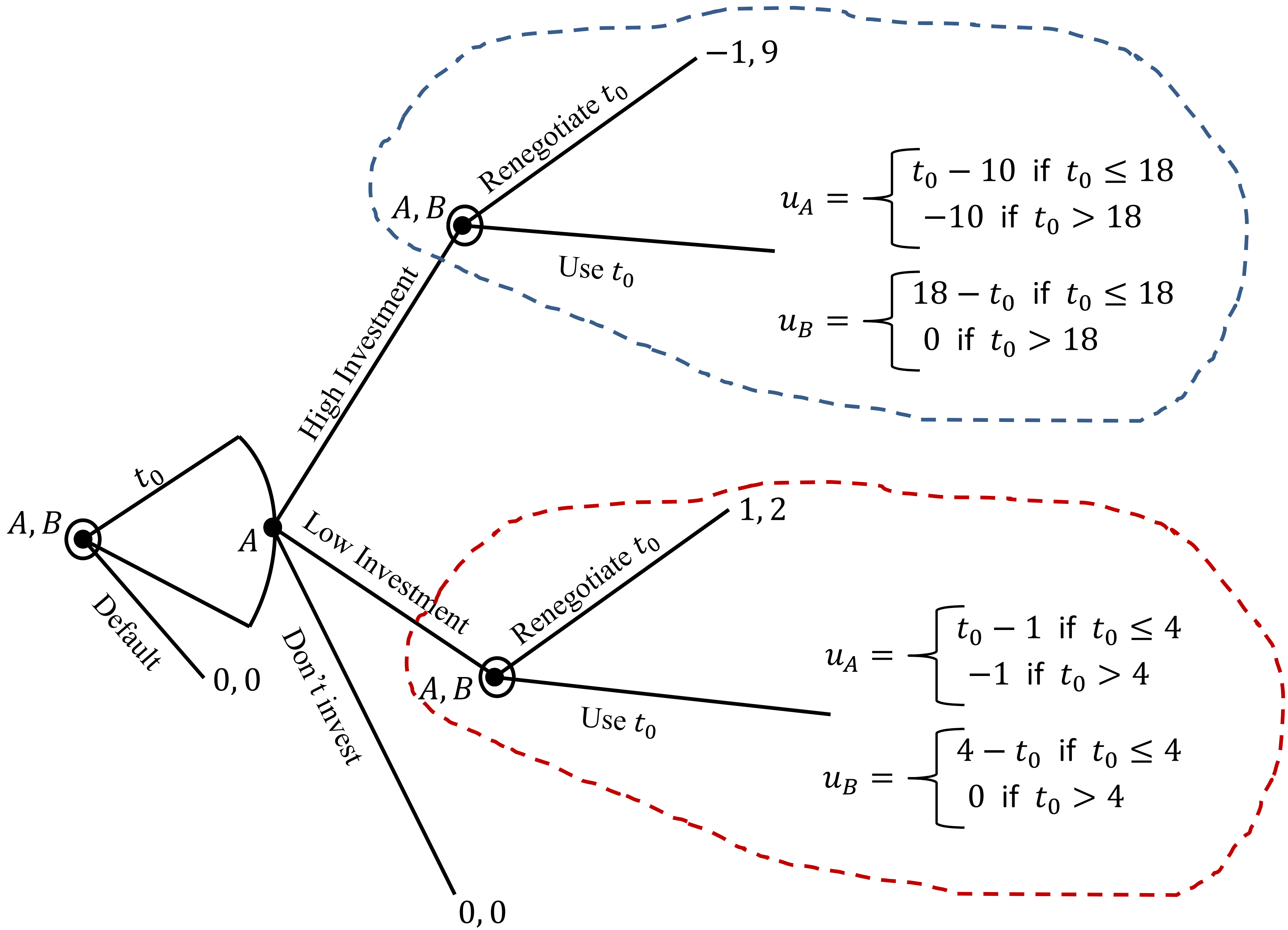
- We can use those results and plug in the continuation payoffs of renegotiating the price t_0 in each case.

- The extensive form looks like this…

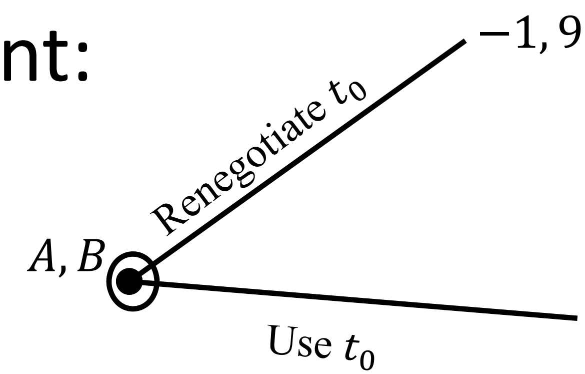


A, B
t0
Default
0, 0
A
High Investment
Low Investment
Don't invest
0, 0
A, B
Renegotiate t0
−1, 9
Use t0
B
Produce
t0 − 10, 18 − t0
Don't Produce
−10, 0
A, B
Renegotiate t0
1, 2
Use t0
B
Produce
t0 − 1, 4 − t0
Don't Produce
−1, 0

- Next we figure out the sequentially rational strategies for player B if players decide to use the original price t_0.
- If investment by "A" is High, then player "B" will choose to produce if and only if $t_0 \leq 18$
- If investment by "A" is Low, then player "B" will choose to produce if and only if $t_0 \leq 4$
- The continuation payoffs are therefore as follows…


A, B
t_0
Default
0, 0
A
High Investment
Low Investment
Don't invest
0, 0
A, B
Renegotiate t_0
−1, 9
Use t_0
$u_A = \begin{cases} t_0 - 10 \text{ if } t_0 \leq 18 \\ -10 \text{ if } t_0 > 18 \end{cases}$
$u_B = \begin{cases} 18 - t_0 \text{ if } t_0 \leq 18 \\ 0 \text{ if } t_0 > 18 \end{cases}$
A, B
Renegotiate t_0
1, 2
Use t_0
$u_A = \begin{cases} t_0 - 1 \text{ if } t_0 \leq 4 \\ -1 \text{ if } t_0 > 4 \end{cases}$
$u_B = \begin{cases} 4 - t_0 \text{ if } t_0 \leq 4 \\ 0 \text{ if } t_0 > 4 \end{cases}$

- OK, now we need to find the negotiation equilibrium in the two subgames circled previously…
- We begin with the one following High investment:



$$u_A = \begin{cases} t_0 - 10 & \text{if } t_0 \leq 18 \\ -10 & \text{if } t_0 > 18 \end{cases}$$

$$u_B = \begin{cases} 18 - t_0 & \text{if } t_0 \leq 18 \\ 0 & \text{if } t_0 > 18 \end{cases}$$

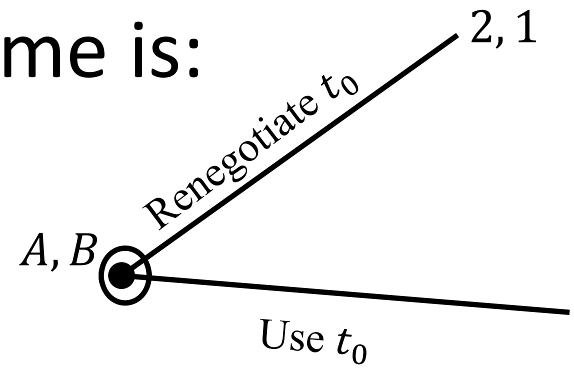
- The standard bargaining solution predicts that efficiency. Note that the surplus if they use t_0 is:

$$V = \begin{cases} t_0 - 10 + 18 - t_0 = 8 & \text{if } t_0 \leq 18 \\ -10 + 0 = -10 & \text{if } t_0 > 18 \end{cases}$$

- On the other hand, the surplus from renegotiating t_0 is simply $V = -1 + 9 = 8$.

- Let us assume that, if players are indifferent between renegotiating t_0 or not, they prefer to stick with t_0. Then, the **standard bargaining solution predicts that, for this subgame: Players will use $\boldsymbol{t_0}$ if and only if $\boldsymbol{t_0 \leq 18}$.**
- Now we apply the same analysis to the subgame that follows after Low investment…

- This subgame is:



$$u_A = \begin{cases} t_0 - 1 \text{ if } t_0 \leq 4 \\ -1 \text{ if } t_0 > 4 \end{cases}$$

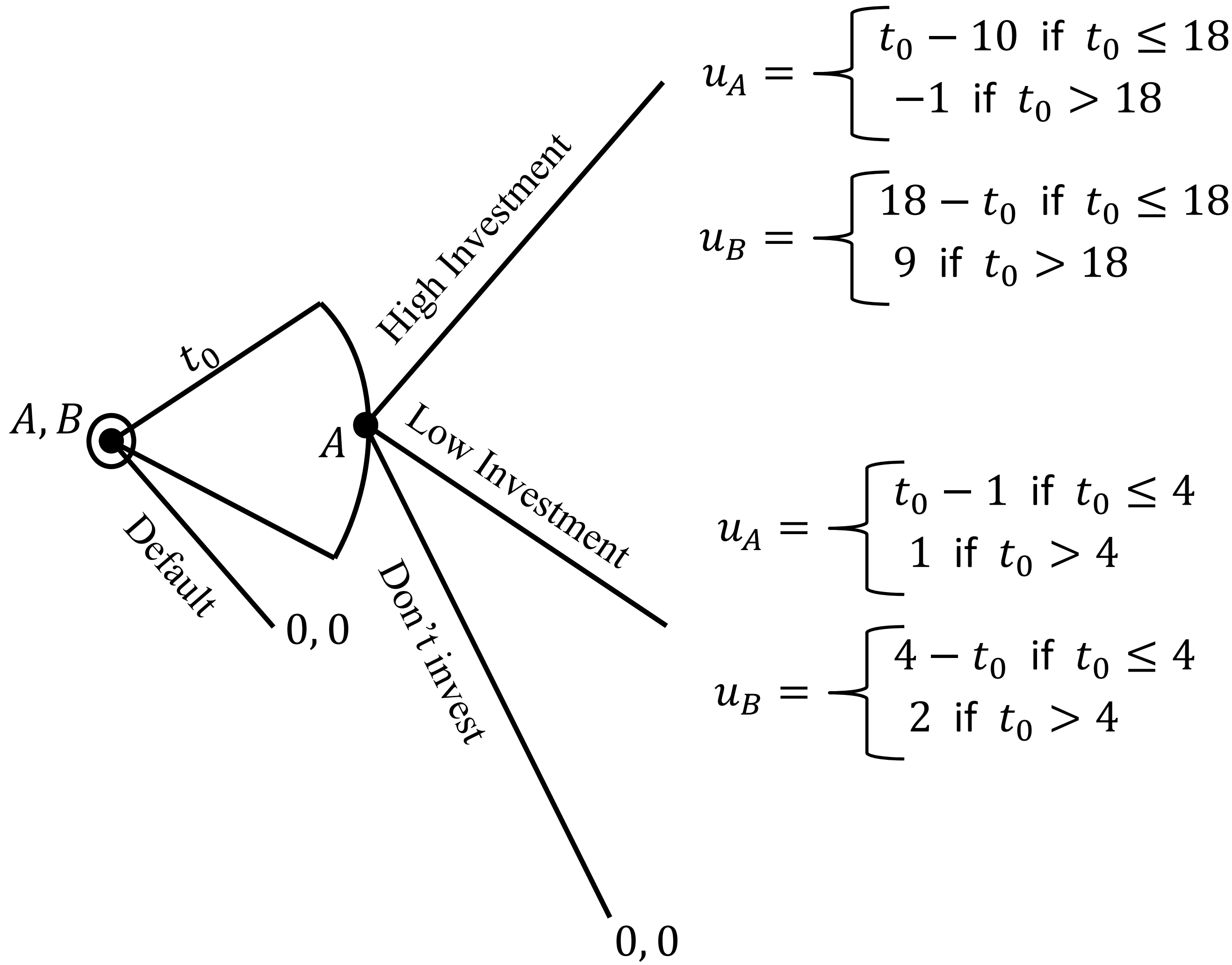
$$u_B = \begin{cases} 4 - t_0 \text{ if } t_0 \leq 4 \\ 0 \text{ if } t_0 > 4 \end{cases}$$

- Again, the standard bargaining solution predicts that efficiency. Note that the surplus if they use t_0 is:

$$V = \begin{cases} t_0 - 1 + 4 - t_0 = 3 \text{ if } t_0 \leq 4 \\ -1 + 0 = -1 \text{ if } t_0 > 4 \end{cases}$$

- And if they renegotiate t_0 the surplus is simply $V = 2 + 1 = 3$

- Therefore, **for this subgame: Players will use $\boldsymbol{t_0}$ if and only if $\boldsymbol{t_0 \leq 4}$.**
- The continuation payoffs look like this:



- Continuing with our backward induction process, we study the sequentially rational strategy for player "A"…
- **If $t_0 \leq 4$:**
- Choosing "High" yields $u_A = t_0 - 10$
- Choosing "Low" yields $u_A = t_0 - 1$
- Choosing "No investment" yields $u_A = 0$
- Therefore, if $t_0 \leq 4$:
- Player "A" will choose "No investment" if $t_0 < 1$
- Player "A" will choose "Low" if $1 \leq t_0 \leq 4$

- **<u>If $4 < t_0 \leq 18$:</u>**
- Choosing "High" yields $u_A = t_0 - 10$
- Choosing "Low" yields $u_A = 1$
- Choosing "No investment" yields $u_A = 0$
- Therefore, if $4 < t_0 \leq 18$:
- Player "A" will choose "Low" if $1 \geq t_0 - 10$. That is, if $t_0 \leq 11$
- Player "A" will choose "High" if $11 < t_0 \leq 18$

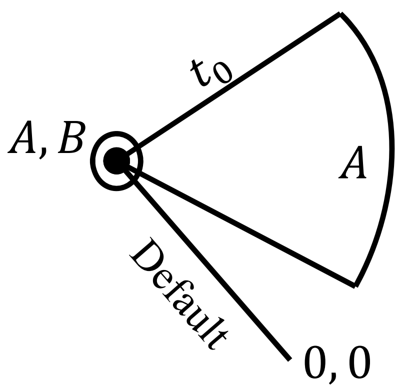
- <u>**If $t_0 > 18$:**</u>
- Choosing "High" yields $u_A = -1$
- Choosing "Low" yields $u_A = 1$
- Choosing "No investment" yields $u_A = 0$
- Therefore, if $t_0 > 18$:
- Player "A" will choose "Low".

- OK, we are ready to summarize these results…

- In summary, the sequentially rational strategy for player “A” is:
- “A” will choose “No investment” if $t_0 < 1$
- “A” will choose “Low” if $1 \leq t_0 \leq 11$
- “A” will choose “High” if $11 < t_0 \leq 18$
- “A” will choose “Low” if $t_0 > 18$

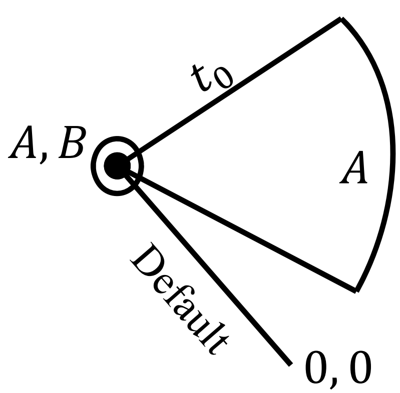
- With these results we can finally go back to the initial node of the game and compute the continuation payoffs…

- Continuation payoffs for the initial (joint decision) node look like this:



- If $t_0 < 1$: $u_A = 0, u_B = 0$
- If $1 \leq t_0 \leq 11$: $u_A = 1, u_B = 2$
- If $11 < t_0 \leq 18$: $u_A = t_0 - 10, u_B = 18 - t_0$
- If $t_0 > 18$: $u_A = 1, u_B = 2$

- Therefore the total surplus “S” of the initial negotiation is:



- If $t_0 < 1$: $S = 0$
- If $1 \leq t_0 \leq 11$: $S = 3$
- If $11 < t_0 \leq 18$: $S = 8$
- If $t_0 > 18$: $S = 3$

- The standard bargaining solution predicts therefore that the negotiation will choose an initial transaction price t_0 that maximizes the joint surplus. That would be a transaction price t_0 such that: $11 < t_0 \leq 18$.
- But what price? The standard bargaining solution predicts that players split the surplus according to their bargaining weights. That is, it predicts that:

$$u_A = t_0 - 10 = 0 + \pi_A \cdot 8$$
$$u_B = 18 - t_0 = 0 + \pi_B \cdot 8$$

- Solving either of these two equations yields:

$$\boldsymbol{t_0 = 10 + \pi_A \cdot 8}$$

- Note that, in order for this price to satisfy $11 < t_0 \leq 18$, we need a minimum amount of bargaining power for player "A". We need: $10 + \pi_A \cdot 8 > 11$. That is, we need:

$$\pi_A > \frac{1}{8}$$

- **Summary of the result:** If player "A" (the investor) has enough bargaining power, then allowing players to pre-negotiate a provisional transaction price t_0 can lead to a negotiation equilibrium that leads "A" to choose a high level of investment, therefore **solving the hold up problem**.

- The previous type of contract is called **up-front contracting**. Therefore up-front contracting is one possible way to solve the hold-up problem.
- Under the conditions described above, up-front contracting can create different incentives for player "B" (the partner) depending on the level of investment chosen by "A". Contracts that create different incentives are called **option contracts**. Thus, under the right circumstances, up-front contracts can become option contracts.

- In addition to up-front contracting (option contracts), the chapter discusses briefly other potential solutions to the hold-up problem.
- In particular, it discusses **asset ownership** as an alternative way to create the right investment incentives.
- Of course, if the investment level and effort could be verified by an external court, the hold-up problem could be solved simply by writing an appropriate contract that can be enforced by an external court. Option contracting and asset ownership are solutions to the hold-up problem when investment is unverifiable by an external court. That's what makes these results interesting to us.