23.- Collusion, Trade Agreements and Goodwill

- This chapter explores economic examples of repeated games.
- We will focus on the possibility that collusion between duopoly firms may arise if they play a game of quantity competition repeatedly.
- The setting is the usual Cournot duopoly where two firms compete with each other in quantities produced.

- Suppose market price is determined by the demand equation:

$$p = 12 - q_1 - q_2$$

- Suppose both firms have the same cost function:

$$C(q_1) = 2 \cdot q_1$$
$$C(q_2) = 2 \cdot q_2$$

- Suppose both firms compete in quantities produced, but that they compete repeatedly over an infinite horizon.
- The **stage game** therefore consists of quantity competition between these firms. This stage game is repeated infinitely many times.

- As we know, **this stage game has a <u>unique Nash equilibrium</u>: The Cournot equilibrium.**
- In the Cournot model both players simultaneously maximize their profit functions:

$$u_1(q_1, q_2) = (12 - q_1 - q_2) \cdot q_1 - 2 \cdot q_1$$
$$u_2(q_1, q_2) = (12 - q_1 - q_2) \cdot q_2 - 2 \cdot q_2$$

- Best-response functions in the Cournot equilibrium are given as the solutions to the first order conditions:

$$\frac{\partial u_1(q_1, q_2)}{\partial q_1} = 0$$
$$\frac{\partial u_2(q_1, q_2)}{\partial q_2} = 0$$

- Solving, we have that the Cournot quantities produced are:

$$q_1 = q_2 = \frac{10}{3}$$

- And the Cournot profits are:

$$u_1(q_1, q_2) = u_2(q_1, q_2) = \frac{100}{9} \approx 11.11$$

- **The Cournot outcome is the unique Nash equilibrium of the stage game**. In particular, if the game is played only once then this is the only equilibrium outcome we can observe.
- Our question is: **If the game is played repeatedly over an infinite horizon, can collusion be sustained in an SPE?**

- **Collusion:** This would occur if both firms agreed to behave as a single firm with two plants and **choose the quantities q_1 and q_2 that maximizes their joint profits**. **Then they agree to split the joint profits in half.**
- The collusion profit function would therefore be:

$$u_{col}(q_1, q_2) = u_1(q_1, q_2) + u_2(q_1, q_2)$$

- This simplifies to:

$$u_{col}(q_1, q_2) = (12 - q_1 - q_2) \cdot (q_1 + q_2) - 2 \cdot (q_1 + q_2)$$

- The optimal collusion production levels are given by the solution to the first order conditions:

$$\frac{\partial u_{coll}(q_1, q_2)}{\partial q_1} = 0$$

$$\frac{\partial u_{coll}(q_1, q_2)}{\partial q_2} = 0$$

- These simplify to the following expressions:

$$10 - 2 \cdot (q_1 + q_2) \ = 0$$

$$10 - 2 \cdot (q_1 + q_2) \ = 0$$

- They are both the same condition. Therefore, any combination of production levels q_1 and q_2 that satisfy these conditions is optimal for collusion. In particular we can assume that both firms produce the same quantity $(q_1 = q_2)$ which yields the solution:

$$q_1 = q_2 = \frac{5}{2}$$

- Therefore the collusion production levels are:

$$q_1 = q_2 = \frac{5}{2}$$

- And the collusion profits are:

$$u_{col}(q_1, q_2) = u_1\left(\frac{5}{2}, \frac{5}{2}\right) + u_2\left(\frac{5}{2}, \frac{5}{2}\right) = 25$$

- Since they split these profits in half, the collusion profits for each firm are $\frac{\mathbf{25}}{\mathbf{2}} = \mathbf{12.5}$
- **Note that the collusion profits are higher than the Cournot profits: 12.5 vs. 11.11**

- Collusion cannot be sustained if the game is played only once: To see this, note that each firm has an incentive to deviate from collusion in the one-shot version of this game.
- Why? Take any player $i = 1{,}2$ and denote the other player as j. Under collusion, $q_j = \frac{5}{2}$.
- What is the best-response for player i if $q_j = \frac{5}{2}$?
- Player i's profits if $q_j = \frac{5}{2}$ are:

$$u_i\left(q_i, \frac{5}{2}\right) = \left(12 - \frac{5}{2} - q_i\right) \cdot q_i - 2 \cdot q_i$$

- Player i's best response if $q_j = \frac{5}{2}$ is given by the solution to the first order conditions:

$$\frac{\partial u_i\left(q_i, \frac{5}{2}\right)}{\partial q_i} = 0$$

- This simplifies to the condition:

$$\frac{15}{2} - 2 \cdot q_i = 0$$

- Therefore, the best response for player i if $q_j = \frac{5}{2}$ is to produce $q_i = \frac{15}{4} = 3.75$

- The profits for player i would be:

$$u_i\left(\frac{15}{4}, \frac{5}{2}\right) = \frac{225}{16} = 14.0625$$

- **These profits are higher than the collusion profits, which are $\frac{25}{2} = 12.5$ for each firm.**
- **Therefore, each player has an incentive to deviate from the collusion agreement** and produce $q_i = \frac{15}{4}$ instead of the collusion production level $q_i = \frac{15}{2}$.

- Can we sustain collusion if the game is played repeatedly over an infinite horizon?
- We need a **trigger strategy** and we know that the **punishment from deviating from collusion must be a Nash equilibrium of the stage game.**
- But the only Nash equilibrium of the stage game is the Cournot equilibrium. Therefore **the punishment from deviating must be to play the Cournot equilibrium for ever**.

- OK, so we have to put together the following figures:
  - The stage-game profit from cooperation.
  - The stage-game profit from deviation.
  - The stage-game profit from punishment.
- Then we have to use the formula we derived in the previous chapter. Cooperation will be sustained if the discount factor of each firm satisfies:

$$\delta_i \geq \frac{u_i(d_i) - u_i(c)}{u_i(d_i) - u_i(p)}$$

- Where:

  $u_i(d_i)$ =Stage game profits from deviating.

  $u_i(c)$ =Stage game profits from cooperating.

  $u_i(p)$ =Stage game profits from punishment.

- In this case, we have:

$$u_i(d_i) = \frac{225}{16}$$

$$u_i(c) = \frac{25}{2}$$

$$u_i(p) = \frac{100}{9}$$

- Therefore collusion can be sustained in an SPE of this game if both firms' discount factors satisfy:

$$\delta_i \geq \frac{\frac{225}{16} - \frac{25}{2}}{\frac{225}{16} - \frac{100}{9}} = \frac{9}{17}$$

- Therefore we need both firms' discount factors to be at least equal to $\frac{9}{17} \approx 0.529$ in order to sustain collusion as an SPE.

- Price matching guarantees and collusion in the real world: A key to sustain cooperation in a repeated game is that the **punishment threats have to be credible**.
- In a game, this means that **punishment threats need to be Nash equilibria of the stage game**.
- A real world example of credible threats are price matching guarantees between competitors. These price matching guarantees can be seen as implicit, credible threats to initiate a price war if firms deviate from an implicit collusion agreement to keep prices at a certain level.

- The chapter includes also a discussion of trade agreements and goodwill as evidence that reputation has a tangible value in repeated game settings.

- But the discussion of those examples is a bit informal. We will focus only on the collusion example because it can be formalized as a proper game and because it is a continuation of an ongoing example that we have been studying throughout the course.