7.- Rationalizability and Iterated Dominance

- In this chapter we will predict rational behavior in games based on the following assumptions:

1. The players in the game are rational and therefore do not play dominated strategies.
2. Each player believes that the other players are rational.
3. Each player believes that the other players believe this, and so on ad infinitum.

- This iterative thinking process will lead to the iterative removal of dominated strategies. This process is **called iterated dominance**.

- The strategies that survive this iterated dominance procedure will be called **rationalizable strategies**.

- To illustrate iterated dominance, consider the following example:

1 \ 2	X	Y	Z
A	3, 3	0, 5	0, 4
B	0, 0	3, 1	1, 2

- First, we note that **strategy "X" is dominated** by "Y" and "Z".
- Rational behavior implies that Player 2 will never choose strategy "X".
- Suppose it is **common knowledge** among the players that both of them are rational.
- This means that **Player 1 knows that Player 2 will never choose "X".**

1 \ 2	X	Y	Z
A	3, 3	0, 5	0, 4
B	0, 0	3, 1	1, 2

- But if Player 1 knows that Player 2 will never choose “X”, then **choosing “A” cannot be justified (rationalized) since it is now dominated by “B” once “X” is ruled out for Player 2**. Therefore Player 2 will never choose “A” if he assumes that Player 1 is rational.

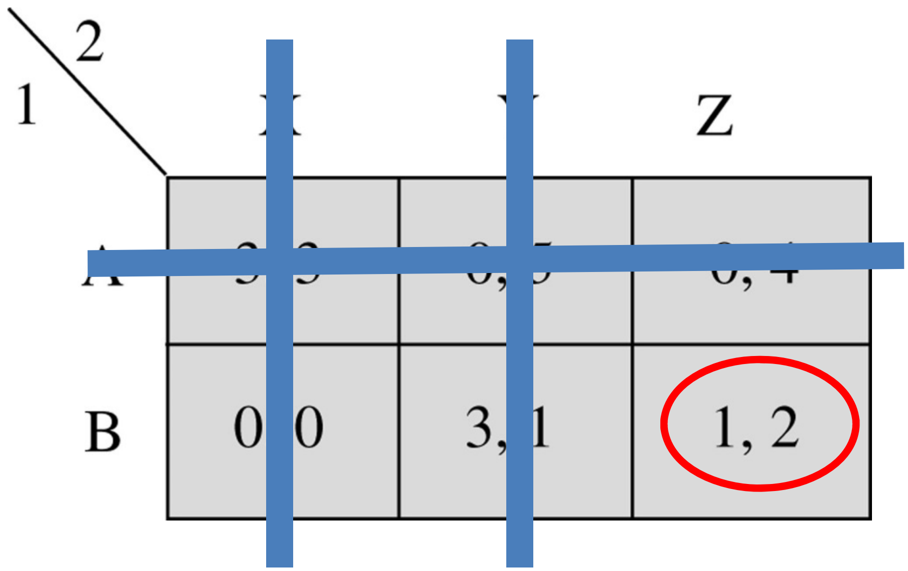
- Suppose Player 2 knows that Player 1 knows that Player 2 is rational. Then, Player 2 knows that Player 1 will never choose “A”.

- Striking out the strategies that cannot be played by rational players, we now have:

1 \ 2	X	Y	Z
A	3, 3	0, 5	0, 4
B	0, 0	3, 1	1, 2

- If Player 2 is rational, he realizes that Player 1 will never choose “A”, then “Y” cannot be rationalized as a best response because it is dominated by “Z”.

- Again, since rational behavior is common knowledge, now Player 1 knows that Player 2 will never choose “Y”. The surviving strategies are:



- That is, the only surviving strategies are “Z” for player 2 and “B” for player 1.

- The only strategy profile that survives this iterative process of deleting dominated strategies is:

$$\boldsymbol{R} = \{\boldsymbol{B}, \boldsymbol{Z}\}$$

- This process of iterative thinking by rational players is called **iterated dominance**.

- In this case, it produced in the end a unique profile of strategies. In other instances it may produce a collection of strategy profiles.

Iterated Dominance. General Steps:

1. Delete all dominated strategies for each player. Let $\boldsymbol{R^1}$ denote the strategy profiles that remain.
2. Take the "reduced" game with only the strategies in R^1. Delete all dominated strategies in this reduced game. Let $\boldsymbol{R^2}$ denote the strategy profiles that remain.
3. Delete all dominated strategies in the reduced game given by R^2. Let $\boldsymbol{R^3}$ denote the strategy profiles that remain.
4. Continue this process to identify smaller sets of strategy profiles $\boldsymbol{R^4}, \boldsymbol{R^5}, \ldots,$ until no more strategies can be deleted. Let $\boldsymbol{R}$ denote the resulting set of strategy profiles.

- The set $\boldsymbol{R}$ is called the set of **rationalizable strategies.** These are strategies that can be justified or *rationalized* by some beliefs that assign positive probability only to strategies of others that survive the process of iterated dominance.

- As we mentioned above, $\boldsymbol{R}$ can include a unique profile of strategies, or it can include multiple profiles.

- Iterated dominance may not yield a unique prediction for the strategies that players may choose in a game.

- Thus, the **reduced game R^k in the k^{th} step of deletion of dominated strategies consists of the set of all undominated strategies from the reduced game R^{k-1}**. We can refer to the original game as R^0.

- For this, **we follow the steps studied in Chapter 6 to characterize the set of undominated strategies in a game**.

- Let us repeat those steps again in the next slide…

- **Steps to characterize the reduced game $\boldsymbol{R^k}$ in the $\boldsymbol{k^{th}}$ step of deletion of dominated strategies:**

1. Take the reduced game R^{k-1} (with the original game being R^0).
2. Find the best responses to pure strategies. These will automatically belong in R^k.
3. Look for strategies that are dominated by other pure strategies. These will automatically be eliminated from R^k.
4. Test each <u>remaining</u> strategy to see if it is dominated by a mixed strategy.
5. Together with the strategies that were identified as best responses in step #2, all strategies that survive steps 3 and 4 will constitute the reduced game R^k.

- **Example:** Find the set R of rationalizable strategies in the following game:

1 \ 2	X	Y	Z
U	5, 1	0, 4	1, 0
M	3, 1	0, 0	3, 5
D	3, 3	4, 4	2, 5

- Step 1: Find all dominated strategies for each player in the original game.-
  - Player 1: “U”, “M” and “D” are best responses to “X”, “Z” and “Y” respectively. Therefore, player 1 does not have dominated strategies in the original game.
  - Player 2: “Y” and “Z” are best responses to “U” and (“M”,”D”) respectively. “X” is not a best response, so we need to check if it is dominated.
  - Using the procedure as in the examples of Chapter 6, we find that “X” is dominated by any mixed strategy that randomizes between “Y” and “Z” as long as

$$\frac{1}{4} < \Pr(Y) < \frac{4}{5}.$$

- Therefore, the reduced game R^1 is given by:

$$R^1 = \{U, M, D\} \times \{Y, Z\}$$

- The matrix form of R^1 is given by:

1 \ 2	X	Y	Z
U	5, 1	0, 4	1, 0
M	3, 1	0, 0	3, 5
D	3, 3	4, 4	2, 5

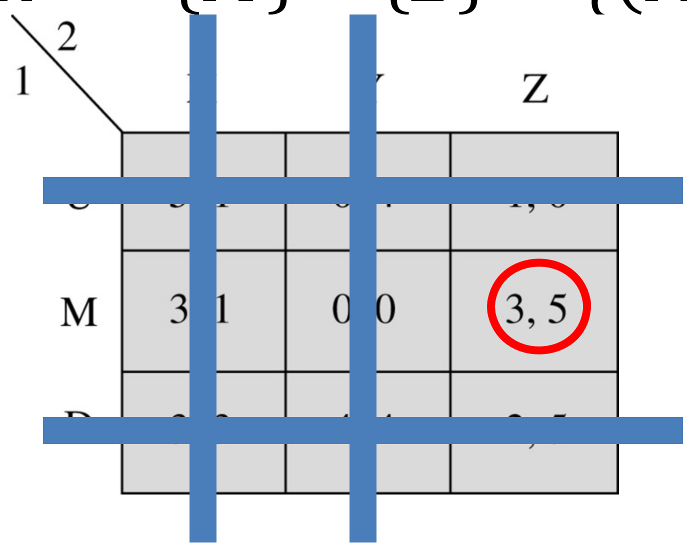
- Step 2: Find all dominated strategies for the reduced game given by R^1.-
  - For Player 1, “U” is now dominated by “M” and by “D” in the reduced game given by R^1.
  - Player 2 does not have a dominated strategy in the reduced game given by R^1.
  - Therefore, $R^2 = \{M, D\} \times \{Y, Z\}$. Reduced game:

1 \ 2	[illegible]	Y	Z
[illegible]	[illegible]	[illegible]	[illegible]
M	3, 1	0, 0	3, 5
D	3, 3	4, 4	2, 5

- Step 3: Find all dominated strategies for the reduced game given by R^2.-
  - Player 1 does not have any dominated strategy in the reduced game given by R^2.
  - For Player 2, “Y” is dominated by “Z” in the reduced game given by R^2.
  - Therefore, $R^3 = \{M, D\} \times \{Z\}$. Reduced game:

1 \ 2	[illegible]	[illegible]	Z
[illegible]	[illegible]	[illegible]	[illegible]
M	3 1	0 0	3, 5
D	3 3	4 4	2, 5

- Step 4: Find all dominated strategies for the reduced game given by R^3.-
  - For Player 1, "D" is dominated by "M" in the reduced game given by R^3.
  - Player 2 has only one remaining strategy ("Z") in the reduced game R^3. Therefore he has no dominated strategies.
  - Therefore, $R^4 = \{M\} \times \{Z\} = \{(M, Z)\}$.



- There are no more dominated strategies that we can eliminate. Therefore, R^4 becomes R. We have:

$$\boldsymbol{R} = \{(\boldsymbol{M}, \boldsymbol{Z})\}$$

- In this case, the **set of rationalizable strategies includes a unique strategy profile. This is the profile $(\boldsymbol{M}, \boldsymbol{Z})$.**

- Other games do not yield a unique element for the set R.

- **Example:** Find the set R of rationalizable strategies in the following game:

1 \ 2	X	Y	Z
U	2, 0	1, 1	4, 2
M	3, 4	1, 2	2, 3
D	1, 3	0, 2	3, 0

(b)

- Step 1: Find all dominated strategies for each player in the original game.-
  - Player 1: “M” is a best response to “X” and “Y”, while “U” is a best response to “Z”. The strategy “D” is dominated by “U”.
  - Player 2: “Z” is a best response to “U”, “X” is a best response to “M” and “D”.
  - We need to check if “Y” is a dominated strategy. Same procedure we followed in Chapter 6 shows that it is NOT a dominated strategy.
  - Therefore:

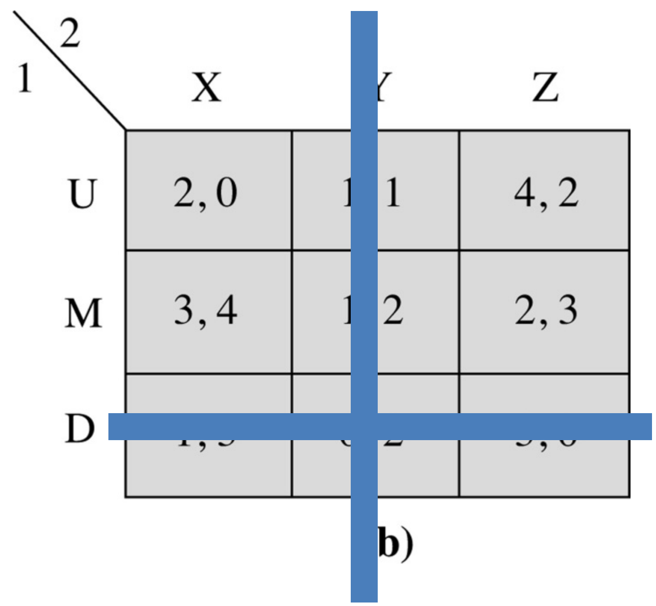
$$R^1 = \{U, M\} \times \{X, Y, Z\}$$

- Matrix form of the reduced game R^1 is:

1 \ 2	X	Y	Z
U	2, 0	1, 1	4, 2
M	3, 4	1, 2	2, 3
D	[illegible]	[illegible]	[illegible]

(b)

- Step 2: Find all dominated strategies for the reduced game given by R^1.-
  - Player 1 has no dominated strategies in the reduced game given by R^1.
  - For Player 2, “Y” is dominated by “Z” in the reduced game given by R^1.
  - Therefore, $R^2 = \{U, M\} \times \{X, Z\}$. Reduced game:



- Step 3: Find all dominated strategies for the reduced game given by R^2.-
  - Player 1 has no dominated strategies in the reduced game given by R^2.
  - Player 2 has no dominated strategies in the reduced game given by R^2.
  - Therefore, no further reduction can be done and we have $R^2 = R$. Therefore,

$$R = \{U, M\} \times \{X, Z\} = \{(U, X), (U, Z), (M, X), (M, Z)\}$$

- The set of rationalizable strategies R includes **four strategy profiles in this case**.

- **Example: Eisner-Katzenberg game (continued):** We pointed out previously that the following strategies are dominated in that game:

K \ E	P	N
LPR	40, 110	80, 0
LPN′	13, 120	80, 0
~~LNR~~	~~0, 140~~	~~0, 0~~
~~LNN′~~	~~0, 140~~	~~0, 0~~
~~SPR~~	~~35, 100~~	~~35, 100~~
~~SPN′~~	~~35, 100~~	~~35, 100~~
~~SNR~~	~~35, 100~~	~~35, 100~~
~~SNN′~~	~~35, 100~~	~~35, 100~~

- Therefore, the reduced game R^1 is given by:

K \ E	P	N
LPR	40, 110	80, 0
LPN′	13, 120	80, 0

- The dominated strategies in R^1 are:

K \ E	P	N
LPR	40, 110	80, 0
LPN′	13, 120	80, 0

- Therefore, the reduced game R^2 is given by:

K \ E	P
LPR	40, 110
LPN′	13, 120

- Eisner has only one remaining strategy "P" in R^2. Katzenberg has a dominated strategy:

K \ E	P
LPR	40, 110
~~LPN′~~	~~13, 120~~

- Therefore, R^3 consists of a unique strategy profile:

$$R^3 = \{(LPR, P)\}$$

- This set cannot be reduced any further. Therefore,

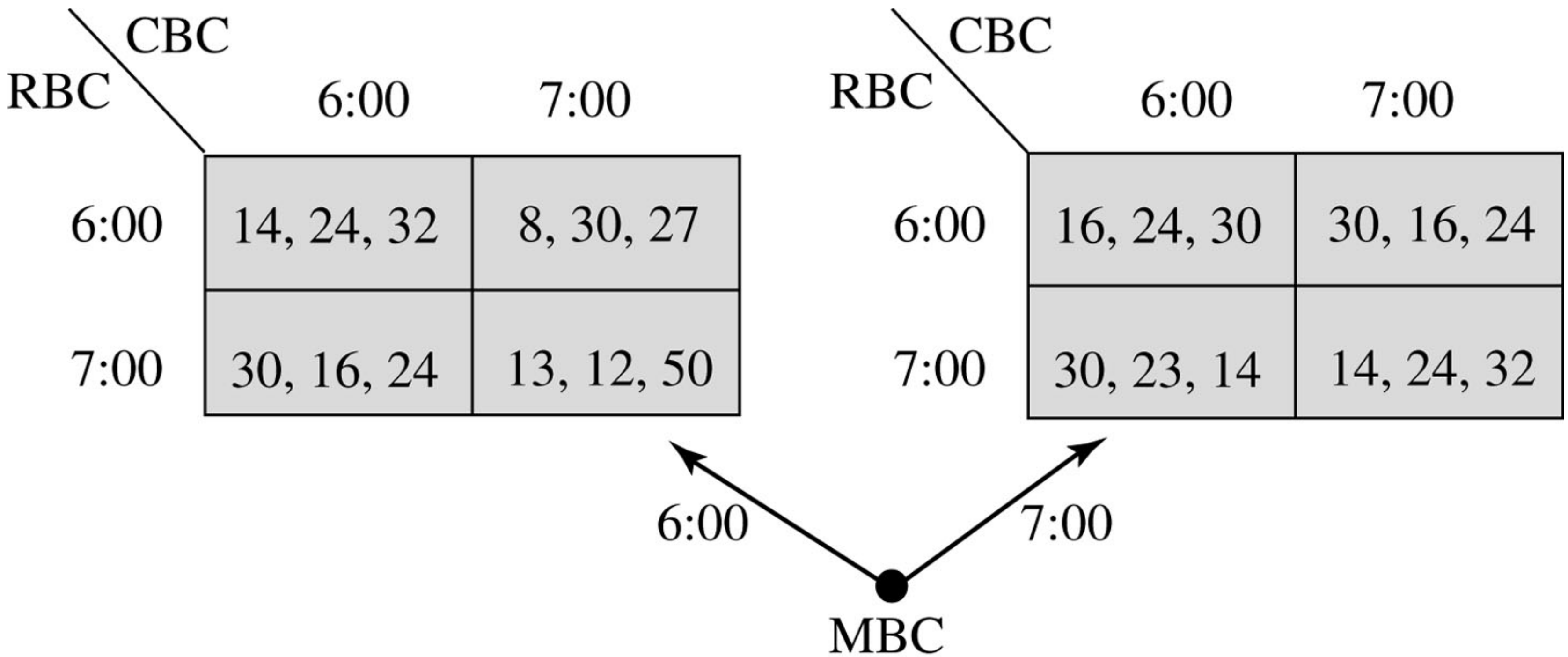
$$R = \{(LPR, P)\}$$

- Rationalizability produces a unique prediction in this game: Katzenberg leaves Disney, produces the movie and releases it early, while Eisner produces his own movie.
- This was the outcome observed in the real world in this particular case.

- **Example. Problem 7.4:**

- Three players: TV affiliates RBC, CBC and MBC.
- They must decide whether to air their evening network news program live at either 6:00PM or 7:00PM.
- Each station wants to maximize its viewing audience.

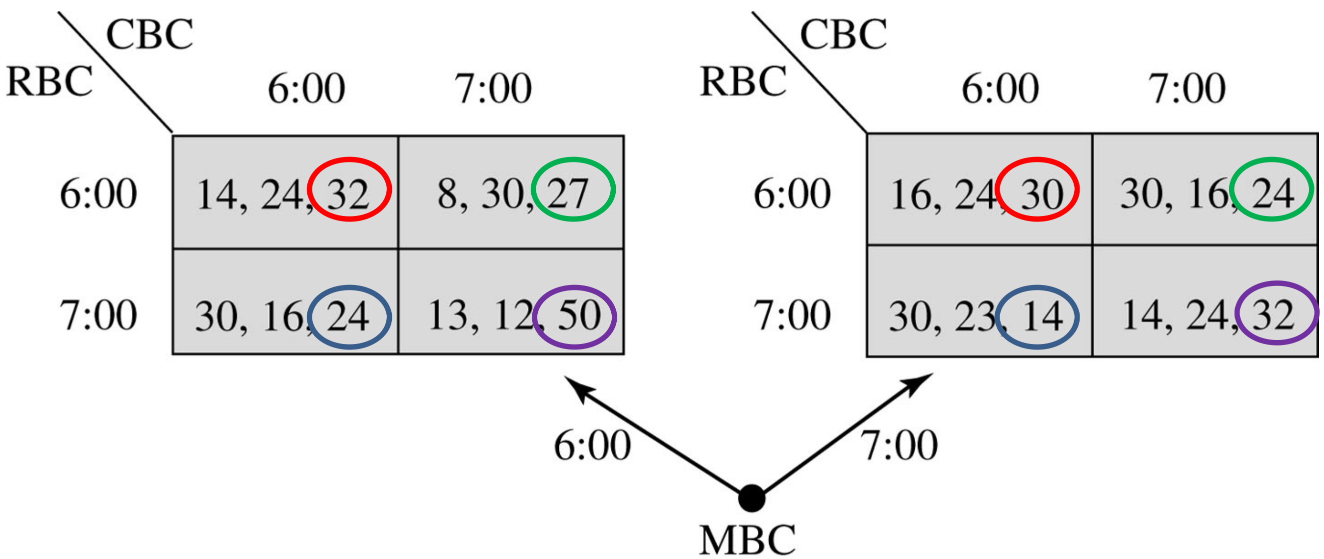
- **Normal-form representation:**



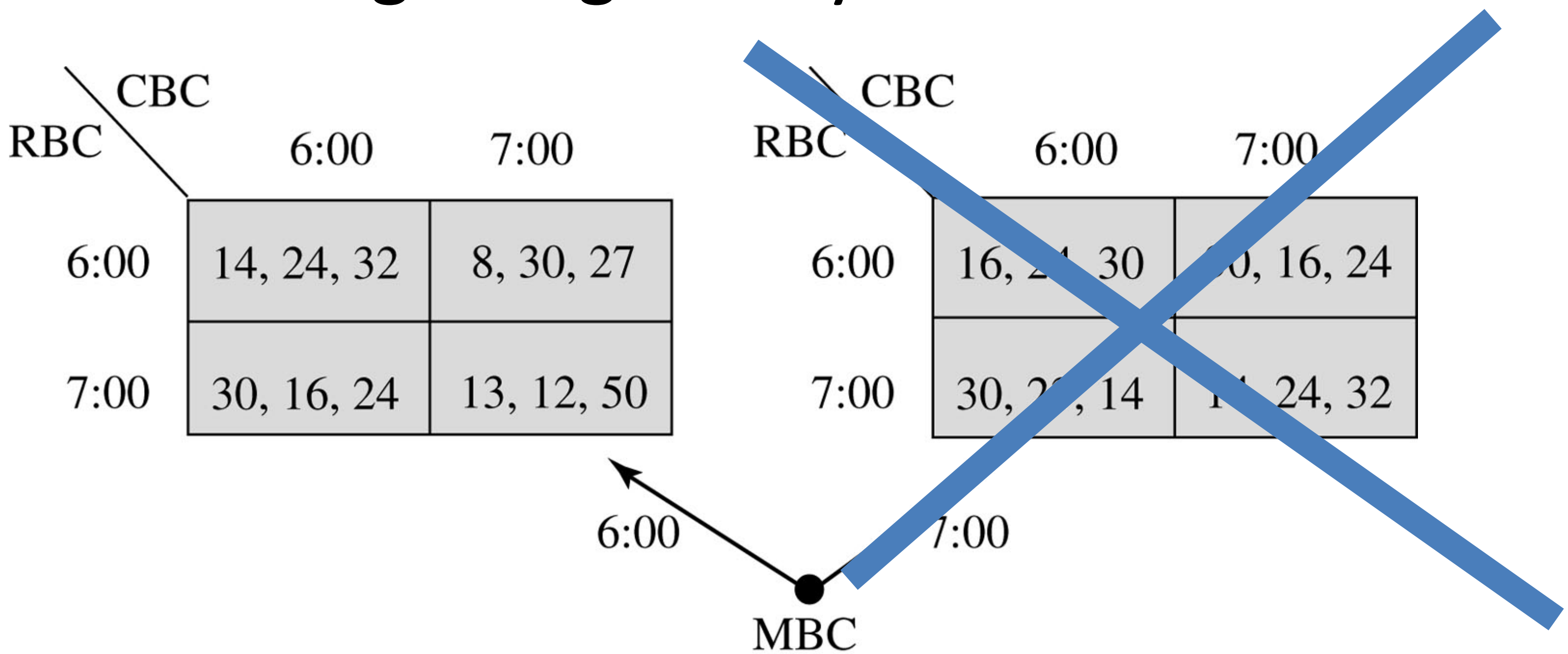
- Order of payoffs: (RBC, CBC, MBC).
- Note that all three players are assumed to choose their strategies simultaneously.

- Find the set of rationalizable strategies, R.
- Step 1: Find all dominated strategies for each player.-
- First note the following:
- A strategy for players **CBC** or **RBC** is dominated only if it is dominated in **both** of the matrix sub-games.
- For example, 6:00 is dominated for RBC in the matrix subgame on the left, but it is not dominated in the one on the right. Therefore 6:00 is NOT dominated for RBC.
- We have that there are **no dominated strategies for either CBC or RBC**.

- To figure out if a strategy is dominated for player **MBC** we need to check if it yields a lower payoff for each possible combination of strategies by **CBC** and **RBC**.
- Graphically, this means we need to make pairwise comparisons between the numbers illustrated in the same color below:



- In each pairwise comparison, choosing 6:00 dominates choosing 7:00 for **MBC**. Therefore **7:00 is a dominated strategy for MBC**.
- We have:
  $$R^1 = \{6{:}00, 7{:}00\} \times \{6{:}00, 7{:}00\} \times \{6{:}00\}$$
- The reduced game given by R^1 looks like this:



- Step 2: Find all dominated strategies for the reduced game given by R^1.-
- Player **MBC** has a unique strategy in R^1 (Namely, 6:00). Therefore it all boils down to looking for dominated strategies for CBC and RBC in the reduced game R^1:

RBC \ CBC	6:00	7:00
6:00	14, 24, 32	8, 30, 27
7:00	30, 16, 24	13, 12, 50

- In this reduced game, 6:00 is dominated by 7:00 for RBC, while CBC has no dominated strategies.
- Therefore,

$$R^2 = \{7{:}00\} \times \{6{:}00, 7{:}00\} \times \{6{:}00\}$$

- The reduced game is:

RBC \ CBC	6:00	7:00
~~6:00~~	~~14, 24, 32~~	~~8, 30, 27~~
7:00	30, 16, 24	13, 12, 50

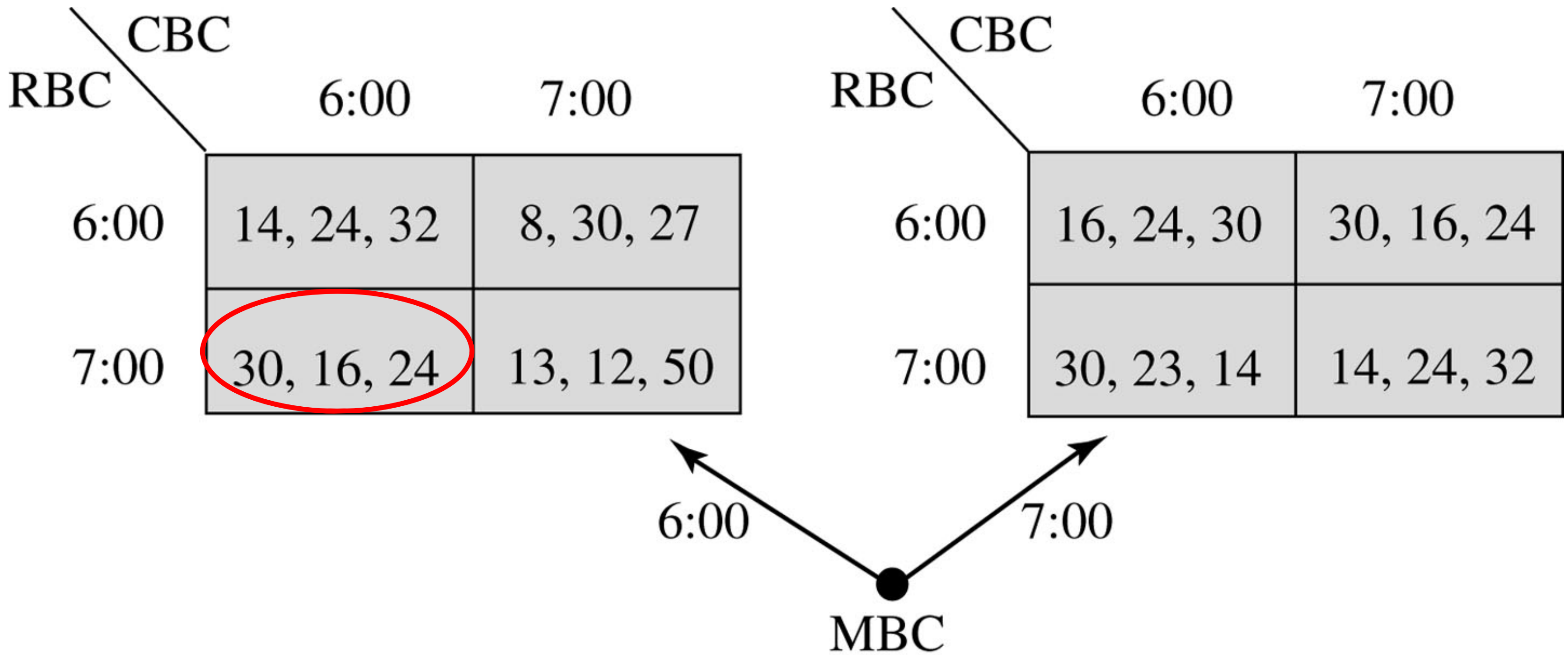
- Step 3: Find all dominated strategies for the reduced game given by R^2.-
- Players **MBC** and **RBC** have a unique strategy in R^2 (Namely, 6:00 and 7:00 respectively).
- Strategy 7:00 is dominated for CBC in the reduced game given by R^2. Therefore,

$$R^3 = \{7{:}00\} \times \{6{:}00\} \times \{6{:}00\}$$
$$= \{(7{:}00, 6{:}00, 6{:}00)\}$$

- This yields a unique strategy profile which cannot be reduced further. Therefore,

$$\mathrm{R} = \{(7{:}00, 6{:}00, 6{:}00)\}$$

- Iterated dominance predicts a unique outcome in this game:



- **"Second Strategic Tension":** Rationalizability only requires that beliefs and behavior be consistent with common knowledge of rationality. However, it does not require beliefs to be consistent with others players' actual behavior. In other words, beliefs can be incorrect and still be consistent with rationalizability.

- The possibility that beliefs are inconsistent with other players' strategies is referred to as *strategic uncertainty* and is identified in the textbook as the "second strategic tension".

- Strategic uncertainty could be resolved through a variety of social institutions, such as contracts, rules, norms or communication between players. These institutions could make beliefs consistent with behavior. The concept of Nash equilibrium (Chapter 9) presupposes this possibility.