Inference in an incomplete information entry game with an incumbent and with beliefs conditioned on unobservable market characteristics

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October 24, 2022

Abstract

We consider a static entry game played between an incumbent and a collection of potential entrants. Entry decisions are made with incomplete information and beliefs are conditioned, at least partially, on a market characteristic that is unobserved by the econometrician. We describe conditions under which, even though the unobserved market characteristic cannot be identified, a subset of parameters of the model can still be identified, including all the strategic-interaction effects. We also characterize testable implications for strategic behavior by the incumbent when this player is able to shift the unobserved market characteristic to deter entry. We present results under Bayesian Nash equilibrium (BNE) and under the weaker behavioral model of iterated elimination of nonrationalizable strategies. Our empirical example analyzes geographic entry decisions in the Mexican internet service provider (ISP) industry. This industry has an incumbent, América Móvil (AMX), which established a widespread geographic presence as a monopolist following the privatization of Telmex in 1990. Our results show significant strategic interaction effects between AMX and its competitors, as well as evidence of strategic behavior by AMX to deter entry and maximize its market share.
Keywords: Inference in discrete games, incomplete information, unobserved market characteristics, entry, Mexican telecommunications.
JEL classification: C01, C31, C35, C57.

1 Introduction

Entry decisions by competing firms have been one of the most important econometric applications of static games. A partial list of examples includes Bresnahan and Reiss (1990), Berry (1992), Seim (2006), Ciliberto and Tamer (2009), Ciliberto, Murry, and Tamer (2020), Ciliberto, Murry,

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and Tamer (2020), Wang (2020), and Fan and Yang (2021). These models have been studied under various assumptions regarding the information available to the players and to the econometrician, the degree of parameterization, the solution concept assumed and the treatment of multiple solutions, among other aspects. Some of this work has produced a number of econometric insights which have spilled over into more general inferential methods, for example, in partially identified models. Different challenges arise depending on the information structure assumed. A particularly outstanding issue involves entry games of incomplete information, where the majority of work assumes that players' beliefs are conditioned on observable covariates to the econometrician, or that they are conditioned on unobservable shocks that can be identified and consistently estimated.

This paper contributes to the econometric literature on incomplete-information entry games. It considers a game where there is an incumbent with presence in all markets along with a collection of potential entrants. Entrants must construct beliefs about the expected behavior of other entrants and about the expected market share of the incumbent which, in turn, depends on the number of entrants in the market. As a result, the game has three different strategic interaction effects. We assume that the game is played with incomplete information but, instead of imposing the usual assumption that beliefs are conditioned only on observable covariates (to the econometrician), we consider a setting where beliefs are also conditioned on an unobserved market-level characteristic which cannot be identified or estimated. We describe conditions under which, in spite of our inability to recover this unobserved characteristic, the strategic interaction effects, along with another subset of parameters of the model, can be identified and consistently estimated. Our results do not rely on any type of distributional assumptions for the unobserved market characteristic. We analyze the game under the assumption of Bayesian Nash equilibrium (BNE) as well as under the weaker restriction of iterated elimination of nonrationalizable strategies while allowing for incorrect beliefs. Another contribution of this paper is allowing for the possibility that strategic behavior by the incumbent can shift the unobserved market characteristic to deter entry and maximize this player's expected market share. We describe testable implications of this type of strategic behavior by the incumbent and we propose a corresponding econometric test.

As an empirical example we apply our model to study entry decisions into geographic markets in the Mexican internet service provider (ISP) industry. The structure of this industry fits the description of our model, with an incumbent, América Móvil (AMX) that started as a monopolist following the privatization of Telmex in 1990 and was able to establish a widespread ISP geographic presence by the time competition was encouraged in 2013 through a wide-ranging telecommunications reform. Our results show that all three of the strategic interaction effects in our model are statistically significant. Then, assuming that the unobserved market characteristic is at least partially associated with the existing ISP infrastructure in the market, our results also provide evidence in support of the assertion that AMX has strategically held back the deployment and sharing of infrastructure (which the 2013 Reform requires from AMX) as a mechanism to deter entry and maximize its own market share. These findings are in line with AMX's documented failure to deploy and make available its infrastructure to competitors, as is required by the 2013 Reform, an anticompetitive conduct which resulted in a significant fine by the Mexican government in 2020.

The paper proceeds as follows. Section 2 describes the game, including the players, actions, payoff functions and information. Section 3 characterizes the implications produced by different behavioral assumptions. There, we describe the properties of Bayesian Nash equilibrium (BNE) behavior and those of the weaker solution concept of iterated elimination of nonrationalizable strategies, which is weaker than BNE and allows for incorrect beliefs, and we also describe testable implications that would result from strategic behavior by the incumbent in a setting where this player is capable of shifting the unobserved market characteristic in a way that deters entry and helps maximize the incumbent's market share. Section 4 introduces an invertibility condition and revisits the results from Section 3 under this restriction. Section 5 proposes estimation and inferential methods under the invertibility condition of Section 4. There, we present results under the assumption of BNE and under iterated elimination of nonrationalizable strategies. Section 6 presents our empirical example, where we apply our model to study entry decisions in geographic markets in the Mexican ISP industry. Section 7 concludes.

2 Description of the game

We will refer to each observation of the game as a *market*. We will assume that we observe the realization of the game in $i = 1,\ldots,n$ markets, and that our sample consists of i.i.d realizations of the game that we will describe below. We will begin by describing the players involved, the information they have, and their decision rules.

2.1 Incumbent

Each market has an incumbent, whose presence in the market precedes the game that we will describe below. The incumbent can be the same across all markets (as will be the case in our empirical example), or its identity can differ across markets as long as each incumbent is characterized by the model that we will describe below.

2.2 Potential entrants

We have a collection of P_i potential entrants in market i. These potential entrants will be assumed to be symmetric in a way that we will describe below. We will label each potential entrant as "player p", with $p = 1, \ldots, P_i$. We will focus on cases where P_i is bounded and known to the econometrician. The asymptotics in this paper will be driven by the number of markets observed, n, not by P_i.

2.3 Information observed by the players and by the econometrician

Ours will be a game of incomplete information, with a privately observed payoff shock for each potential entrant and a market-level characteristic that is unobserved by the econometrician but is publicly observed by the players and is therefore used in the construction of their beliefs. We describe the information available to the econometrician and to the players next.

2.3.1 Market characteristics

We will let X_i denote a collection of market-level characteristics that are observable to the econometrician. We will let α_i denote a scalar, market-level characteristic that is unobservable to the econometrician but observed by all players in the game before making their entry decisions.

2.3.2 Player-specific characteristics

We will let $\varepsilon_{p,i}$ denote an idiosyncratic payoff shock specific to player p in market i that is only privately observed. The realization of $\varepsilon_{p,i}$ is unobserved by the econometrician. All incumbent-specific characteristics observed by the econometrician are included in X_i. If there exist player-specific observable characteristics in the data, they can be included in X_i as long as the symmetry in payoff functions described below[1], in equation (2), is preserved. Any player-specific or incumbent-specific characteristics that are unobserved by the econometrician but observed by all players are assumed to be included in α_i.

2.3.3 What the potential entrants observe before making their entry decisions

We will model entry decisions as being simultaneous. As with all existing work that represents entry decisions through a static game, our justification is that the entry decisions observed are the realization of pre-commitments made by firms before being able to observe the entry decisions of others. The realization of the market-level characteristics X_i and α_i as well as the number of potential entrants P_i are observed by all potential entrants before making their choices. The realization of $\varepsilon_{p,i}$ is only privately observed by player p. Thus, the information possessed by

[1]This paper is principally aimed to model entry decisions in situations where there is a *symmetric* collection of potential entrants, whose number can vary across markets, and where entry decisions are driven by market characteristics, incumbent characteristics and a privately observed idiosyncratic payoff shock for each potential entrant. It can accommodate observable, player-specific characteristics as long as the symmetry of payoffs in equation (2) is preserved.

player p before making their entry decision is $(X_i, \alpha_i, P_i, \varepsilon_{p,i})$. Payoff functions (which we will describe below) are also assumed to be known to all players. We will denote

$$W_i \equiv (X_i, \alpha_i, P_i),$$

as the collection of all market variables that are publicly observed by players before making their choices.

2.3.4 What the econometrician observes

There are two outcomes produced by the game. The incumbent's market share, S_i, and the number of potential entrants who choose to enter into the market, C_i. The econometrician observes both, in addition to the observable market characteristics X_i and the number of potential entrants P_i, but is not able to observe α_i or $\varepsilon_{p,i}$ for any p. Thus,

$$Z_i \equiv (S_i, C_i, X_i, P_i),$$

denotes the market characteristics and outcomes observed by the econometrician.

2.4 Incumbent market share

The incumbent's market share S_i is determined by the number of entrants C_i and by the market-level characteristics (X_i, α_i). We will assume a structural relationship of the form

$$S_i = G\Big(X_i'\beta_0^m - \gamma_0 \cdot C_i + \alpha_i\Big), \tag{1}$$

where $G(\cdot) \in [0,1]$. We will refer to the parameter γ_0 as the *competition effect for the incumbent*. Equation (1) is meant to capture the preferences of the representative consumer towards the incumbent in market i. We do not presuppose that $S_i = 1$ if none of the potential entrants enters into the market. That is, we allow for[2] $G(X_i'\beta_0^m + \alpha_i) < 1$. This would be the case if we define market share in terms of *potential* consumers and we allow them to "opt out" entirely, or if we allow for the presence of marginal, non-strategic firms who can provide the service or product in question but are not part of the game modeled here.

2.5 Entry decisions

Let $Y_{p,i} = \mathbb{1}\{$ player p enters into market $i\}$. We will assume that entry decisions are driven by expected-payoff maximization. We will focus on a parametric von Neumann–Morgenstern (vNM)

[2] We will impose this condition in Section 4.

payoff function of the form

$$u_{p,i}(Y_{p,i}, Y_{-p,i}, W_i) = Y_{p,i} \cdot \left(X_i'\beta_0^c - \delta_0 S_i - \Delta_0 \sum_{q \neq p} Y_{q,i} + \eta_0 \alpha_i - \varepsilon_{p,i} \right). \tag{2}$$

This specification assumes that the impact of entry from each opponent is the same, and that it is measured by the parameter Δ_0, which we will refer to as the *entry-competition effect*. The incumbent's market share is also assumed to (potentially) affect entry decisions. This effect is captured by the parameter δ_0 and we will refer to it as the *incumbent market-share effect*.

Players have to make choices simultaneously, therefore they must construct beliefs about the expected entry decisions of others as well as the expected market share of the incumbent. Our payoff function leads to the following expression for player p's *expected payoff function*,

$$\overline{u}_{p,i}(Y_{p,i}, W_i) = Y_{p,i} \cdot \left(X_i'\beta_0^c - \delta_0 \mu_i^p - \Delta_0 \sum_{q \neq p} \pi_{q,i}^p + \eta_0 \alpha_i - \varepsilon_{p,i} \right), \quad \text{where}$$

$$\mu_i^p \equiv \text{ Player } p\text{'s subjective expectation for } S_i, \text{ conditional on } Y_{p,i} = 1.$$

$$\pi_{q,i}^p \equiv \text{ Player } p\text{'s subjective expectation for } Y_{q,i} \ (q \neq p), \text{ conditional on } Y_{p,i} = 1.$$

This leads to the following entry decision rule for each p,

$$Y_{p,i} = \mathbb{1}\left\{ X_i'\beta_0^c - \delta_0 \mu_i^p - \Delta_0 \sum_{q \neq p} \pi_{q,i}^p + \eta_0 \alpha_i - \varepsilon_{p,i} \geq 0 \right\}, \tag{3}$$

Thus, players' beliefs play a crucial role. The following assumption describes our first set of maintained restrictions about beliefs.

Assumption BEL

(i) *Potential entrants' privately observed shocks* $\varepsilon_1, \ldots, \varepsilon_P$ *are iid, with* $\varepsilon_p \perp (\alpha, X)$ *for each* p. *The marginal cdf of* ε_p *will be denoted by* F_ε, *and we will assume that* $F_\varepsilon(\cdot)$ *is a continuous with unbounded support (i.e,* $0 < F_\varepsilon(\epsilon) < 1 \ \forall \ \epsilon \in \mathbb{R}$*). The functional form of* $F_\varepsilon(\cdot)$ *will be assumed to be common knowledge among all the players in the game, along with the functional forms for payoffs and the true parameter values summarized in the decision rule described in* (3).

(ii) *Accordingly, since* ε_p *contains no information about* ε_{-p}, *players' beliefs in market* i *are assumed to be conditioned on* $W_i \equiv (X_i, \alpha_i, P_i)$, *and this fact is common knowledge. Since the decision rule* (3) *is also common knowledge, players' beliefs treat all opponents symmetrically. Therefore, for each* p *and every pair of opponents* $(q, q') \neq p$, *we have* $\pi_{q,i}^p = \pi_{q',i}^p \equiv \pi_i^p$.

From Assumption BEL it follows that player p treats $\sum_{q \neq p} Y_q$ as a Binomial$(P_i - 1, \pi_i^p)$ random variable, which is independent of $Y_{p,i}$ conditional on the information possessed by player p. Let

$$\sigma(P,q,\pi) \equiv \binom{P-1}{q}\pi^q \cdot (1-\pi)^{P-1-q}.$$

For a given $\pi \in [0,1]$, we will denote

$$\mu_S(W_i,\pi) \equiv \sum_{q=0}^{P_i-1} \sigma(P_i,q,\pi)\cdot G\left(X_i'\beta_0^m + \alpha_i - \gamma_0\cdot(q+1)\right). \tag{4}$$

From Assumption BEL, player p's subjective expectations for $\sum_{q\neq p} Y_{q,i}$ and for S_i conditional on $Y_{p,i}=1$ are given, respectively, by $\sum_{q\neq p}\pi_{q,i}^p = (P_i-1)\cdot\pi_i^p$, and $\mu_i^p = \mu_S(W_i,\pi_i^p)$. Note that if $\gamma_0 = 0$ (no competition effect for the incumbent), we have $S_i = G\left(X_i'\beta_0^m + \alpha_i\right)$ and therefore, from (4), for any $\pi \in [0,1]$ we have

$$\mu_S(W_i,\pi) = G\left(X_i'\beta_0^m + \alpha_i\right)\cdot \underbrace{\sum_{q=0}^{P_i-1}\sigma(P_i,q,\pi)}_{=1} = S_i.$$

Thus, since potential entrants observe X_i and α_i and they know $G(\cdot)$, if there is no competition effect for the incumbent, these players can anticipate exactly the incumbent's market share.

2.6 Expected signs of the three strategic effects in our model

Our game consists of three different strategic effects, captured by the parameters γ_0, δ_0 and Δ_0. Our results will not require us to pre-specify in advance their signs; however, the following pattern might describe most empirical applications.

(i) $\boldsymbol{\gamma_0 \geq 0}$: All else constant, the incumbent's market share is nonincreasing in the number of competitors who enter the market.

(ii) $\boldsymbol{\delta_0 \geq 0}$: All else constant, the attractiveness of entry into a market is nonincreasing in the incumbent's expected market share.

(iii) $\boldsymbol{\Delta_0 \geq 0}$: All else constant, the attractiveness of entry into a market is nonincreasing in the the expected number of competitors who will enter.

3 Properties of the game and behavioral implications

In this section we characterize the implications produced by different behavioral assumptions. First, we will describe the properties of Bayesian Nash equilibrium (BNE) behavior. Then, we will focus on the weaker solution concept of iterated elimination of nonrationalizable strategies, which includes BNE as a special case but allows for incorrect beliefs. Finally, we will describe testable implications that would result from strategic behavior by the incumbent in a setting where this player is capable of shifting the unobserved market characteristic α_i to deter entry and maximize his expected market share.

3.1 Bayesian Nash equilibrium (BNE)

By Assumption BEL, beliefs π_i^p are a deterministic function of W_i, and

$$Pr\left(Y_{p,i} = 1 | W_i\right) = F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i, \pi_i^p) - \Delta_0(P_i - 1)\pi_i^p + \eta_0\alpha_i\right).$$

For a given $\pi \in [0,1]$, let $H(W_i, \pi) \equiv \pi - F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i, \pi) - \Delta_0(P_i - 1)\pi + \eta_0\alpha_i\right)$. BNE beliefs in market i are defined as any $\pi \in [0,1]$ that solves the fixed-point system

$$H(W_i, \pi) = 0. \tag{5}$$

Continuity of F_ε, and of $\mu_S(W_i, \pi)$ as a function of $\pi \in [0,1]$, immediately imply the existence of a BNE for any realization of W_i by Brouwer's Fixed Point Theorem (Mas-Colell, Whinston, and Green (1995, Theorem M.I.1)). We will defer a more detailed discussion about multiplicity and uniqueness until Section 4.1. Let us focus next on the notion of a *regular BNE*.

3.1.1 Regular BNE

For any $\pi \in (0,1)$, we have

$$\begin{aligned}
\frac{\partial \mu_S(W_i, \pi)}{\partial \pi} &= \sum_{q=0}^{P_i} \frac{\partial \sigma(P_i, q, \pi)}{\partial \pi} \cdot G\left(X_i'\beta_0^m + \alpha_i - \gamma_0 \cdot (q+1)\right), \quad \text{where} \\
\frac{\partial \sigma(P, q, \pi)}{\partial \pi} &= \binom{P-1}{q} \cdot (P-1) \cdot \frac{\pi^q \cdot (1-\pi)^{P-1-q}}{\pi \cdot (1-\pi)} \cdot \left(\frac{q}{P-1} - \pi\right)
\end{aligned}$$

Let $f_\varepsilon(\cdot)$ denote the density function associated with $F_\varepsilon(\cdot)$. For any $\pi \in (0,1)$,

$$\frac{\partial H(W_i, \pi)}{\partial \pi} = 1 + f_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i, \pi) - \Delta_0(P_i - 1)\pi + \eta_0\alpha_i\right) \cdot \left(\delta_0 \cdot \frac{\partial \mu_S(W_i, \pi)}{\partial \pi} + \Delta_0(P_i - 1)\right).$$

We say that a BNE solution π^* is *regular* if

$$\frac{\partial H(W_i, \pi^*)}{\partial \pi} \neq 0.$$

Regularity of BNE has been assumed in Aradillas-López (2010) and Aradillas-López (2021) in the context of econometric analysis of incomplete-information games. Regularity produces equilibrium beliefs that are well-behaved functions of the model's parameters. This, in turn, can ultimately lead to regular extremum estimators when these beliefs are plugged into a sample objective function. This will be the case in the present paper. An application of the Index Theorem (Mas-Colell, Whinston, and Green (1995, Proposition 17.D.2)) to our model can be used to show that the number of regular BNE is always odd, and therefore that there always exists at least one regular BNE. This will be relevant for our following assumption.

Assumption BNE *All potential entrants use BNE beliefs, which they obtain by solving the BNE system* (5) *given the realization of W_i. When there exist multiple solutions, all potential entrants select a solution using the same equilibrium selection mechanism, denoted by $\mathcal{M}$. This selection mechanism concentrates on regular BNE, meaning that only regular equilibria can be chosen with nonzero probability by $\mathcal{M}$. Finally, the selection mechanism $\mathcal{M}$ is assumed to be independent of ε_p for every p.*

Suppose that, given the realization of W_i, there exist $\mathcal{E}_i$ regular BNE in market i, and label them as $\pi_1^*(W_i),\ldots,\pi_{\mathcal{E}_i}^*(W_i)$. Let $P^{\mathcal{M}}(\ell|W_i) \equiv Pr(\mathcal{M}$ selects $\pi_\ell^*(W_i)\,|W_i)$. From Assumption BNE,

$$Pr(Y_{p,i}=1|W_i) \equiv \tau_Y(W_i) = \sum_{\ell=1}^{\mathcal{E}_i} F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi_\ell^*(W_i)) - \Delta_0(P_i-1)\pi_\ell^*(W_i) + \eta_0\alpha_i\right)\cdot P^{\mathcal{M}}(\ell|W_i)\ \forall\ p \tag{6}$$

3.1.2 The case of a degenerate equilibrium selection mechanism

Suppose the selection mechanism $\mathcal{M}$ is *degenerate*, meaning that it concentrates on a single regular BNE, choosing it with probability one. Denote it by $\pi^*(W_i)$. Equation (6) becomes

$$\tau_Y(W_i) = F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi^*(W_i)) - \Delta_0(P_i-1)\pi^*(W_i) + \eta_0\alpha_i\right). \tag{7}$$

In particular, this is the expression for $\tau_Y(W_i)$ when there exists a unique BNE.

3.1.3 Bounds implied by BNE behavior

Given W_i, let us rank the $\mathcal{E}_i$ existing regular BNE in market i as

$$\underline{\pi}^*(W_i) \equiv \pi_{(1)}^*(W_i) < \pi_{(2)}^*(W_i) < \cdots < \pi_{(\mathcal{E}_i)}^*(W_i) \equiv \overline{\pi}^*(W_i).$$

Since $\pi_{(\ell)}^*(W_i) = F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi_{(\ell)}^*(W_i)) - \Delta_0(P_i-1)\pi_{(\ell)}^*(W_i) + \eta_0\alpha_i\right)$ by definition of the BNE conditions, the following ranking holds,

$$\begin{aligned}&F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi_{(\ell)}^*(W_i)) - \Delta_0(P_i-1)\pi_{(\ell)}^*(W_i) + \eta_0\alpha_i\right)\\ <&F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi_{(\ell')}^*(W_i)) - \Delta_0(P_i-1)\pi_{(\ell')}^*(W_i) + \eta_0\alpha_i\right)\quad \forall\ \ell<\ell'.\end{aligned}$$

Therefore, from Assumption BNE and (6), we have a lower and upper bound for $\tau_Y(W_i)$ given by

$$F_\varepsilon\Big(X_i'\beta_0^c - \delta_0\mu_S(W_i, \underline{\pi}^*(W_i)) - \Delta_0(P_i-1)\underline{\pi}^*(W_i) + \eta_0\alpha_i\Big) \le \tau_Y(W_i),$$
$$F_\varepsilon\Big(X_i'\beta_0^c - \delta_0\mu_S(W_i, \overline{\pi}^*(W_i)) - \Delta_0(P_i-1)\overline{\pi}^*(W_i) + \eta_0\alpha_i\Big) \ge \tau_Y(W_i)$$

3.2 Iterated elimination of nonrationalizable strategies

BNE presupposes that players have correct beliefs. We can relax this assumption and allow players to have incorrect beliefs while placing restrictions on them based, for example, on *iterated elimination of nonrationalizable strategies*. This is weaker than BNE and includes the latter model as a special case. Identification results in this type of behavioral model in discrete games and first-price auctions were studied in Aradillas-López and Tamer (2008) and have been considered in entry models, for example, in Fan and Yang (2021). Iterated eliminaton of nonrationalizable strategies proceeds as follows.

Step 1: Let

$$\underline{F}^1(W_i) \equiv \min_{0\le\pi\le 1} F_\varepsilon\Big(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i\Big),$$
$$\overline{F}^1(W_i) \equiv \max_{0\le\pi\le 1} F_\varepsilon\Big(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i\Big).$$

Common knowledge of the entry decision rule (3) implies that, while beliefs π_i^p may be incorrect, they must satisfy $\underline{F}^1(W_i) \le \pi_i^p \le \overline{F}^1(W_i)$ for each p in market i. All strategies (entry decisions) that are produced by beliefs outside this range are therefore eliminated. Such strategies are nonrationalizable.

Step 2: Let

$$\underline{F}^2(W_i) \equiv \min_{\underline{F}^1(W_i)\le\pi\le\overline{F}^1(W_i)} F_\varepsilon\Big(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i\Big),$$
$$\overline{F}^2(W_i) \equiv \max_{\underline{F}^1(W_i)\le\pi\le\overline{F}^1(W_i)} F_\varepsilon\Big(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i\Big).$$

If each player p assumes that their opponents' strategies are produced by some set of beliefs that satisfy $\underline{F}^1(W_i) \le \pi_i^q \le \overline{F}^1(W_i)$, then beliefs must satisfy $\underline{F}^2(W_i) \le \pi_i^p \le \overline{F}^2(W_i)$ for all p. All strategies that are produced by beliefs outside this range are therefore eliminated. Such strategies are nonrationalizable.

$\vdots$

Step k: Let

$$
\begin{aligned}
\underline{F}^k(W_i) &\equiv \min_{\underline{F}^{k-1}(W_i)\leq\pi\leq\overline{F}^{k-1}(W_i)} F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i\right), \\
\overline{F}^k(W_i) &\equiv \max_{\underline{F}^{k-1}(W_i)\leq\pi\leq\overline{F}^{k-1}(W_i)} F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i\right).
\end{aligned} \tag{8}
$$

If each player p assumes that their opponents' strategies are produced by some set of beliefs that satisfy $\underline{F}^{k-1}(W_i) \leq \pi_i^q \leq \overline{F}^{k-1}(W_i)$, then beliefs must satisfy $\underline{F}^k(W_i) \leq \pi_i^p \leq \overline{F}^k(W_i)$ for all p. All strategies that are produced by beliefs outside this range are therefore eliminated. These strategies are, once again, nonrationalizable.

The bounds are nested, meaning that $[\underline{F}^k(W_i), \overline{F}^k(W_i)] \subseteq [\underline{F}^{k-1}(W_i), \overline{F}^{k-1}(W_i)]$ for any k. Thus, if every player p performs at least $k-1$ steps of iterated elimination of nonrationalizable strategies, we must have $\underline{F}^k(W_i) \leq Pr(Y_{p,i}=1|W_i) \leq \overline{F}^k(W_i)$ for each p and therefore,

$$
P_i \cdot \underline{F}^k(W_i) \leq E[C_i|W_i] \leq P_i \cdot \overline{F}^k(W_i) \tag{9}
$$

BNE beliefs must survive any number of iterated steps of the procedure described above. Otherwise they would be nonrationalizable, which would contradict the definition of a BNE. Therefore,

$$
\left.
\begin{aligned}
F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\overline{\pi}^*(W_i)) - \Delta_0(P_i-1)\overline{\pi}^*(W_i) + \eta_0\alpha_i\right) \leq \overline{F}^k(W_i) \\
F_\varepsilon\left(X_i'\beta_0^c - \delta_0\mu_S(W_i,\underline{\pi}^*(W_i)) - \Delta_0(P_i-1)\underline{\pi}^*(W_i) + \eta_0\alpha_i\right) \geq \underline{F}^k(W_i)
\end{aligned}
\right\} \ \forall\ k
$$

3.3 Allowing for strategic behavior by the incumbent

Of particular interest is allowing for the possibility that the incumbent is capable of shifting the unobserved market characteristic α_i. Suppose we model α_i as $\alpha_i = \alpha(a_i^*, \zeta_i)$, where a_i^* is a (possibly vector-valued) strategy which is chosen optimally by the incumbent and ζ_i includes all other unobserved market covariates that determine α_i. We will treat a_i^* as a continuous strategy in a strategy space $\mathcal{A}$. For a given $a \in \mathcal{A}$, we will denote $\alpha(a,\zeta_i) \equiv \alpha_i(a)$. We will assume that the mapping $\alpha(a,\zeta_i)$ is smooth in a, so that $\frac{\partial\alpha_i(a)}{\partial a} = \frac{\partial\alpha(a,\zeta_i)}{\partial a}$ is well-defined for all $a \in \mathcal{A}$. Suppose we extend our game to *two stages* described as follows.

Stage 1: The incumbent observes the realizations of (X_i, P_i, ζ_i) and chooses a_i^*. As a result, we have $\alpha_i = \alpha(a_i^*, \zeta_i) \equiv \alpha_i(a_i^*)$.

Stage 2: The entry game described previously is played by the potential entrants.

Assumption INC *The incumbent assumes BNE behavior by potential entrants, knows the optimal decision rules and the properties of the equilibrium selection mechanism $\mathcal{M}$ used by the potential entrants* *in stage 2, and uses backward induction to choose a_i^* in stage 1 in order to maximize its expected market share conditional on (X_i, P_i, ζ_i).*

Suppose the function $G(\cdot)$ that determines the incumbent's market share in (1) is differentiable, and let $G'(\cdot) \equiv g(\cdot)$. Then,

$$\frac{\partial \mu_S(W_i, \pi)}{\partial \alpha_i} = \sum_{q=0}^{P_i - 1} \sigma(P_i, q, \pi) \cdot g\left(X_i' \beta_0^m + \alpha_i - \gamma_0 \cdot (q+1)\right)$$

Suppose $\pi^*(W_i)$ is a regular BNE. Then, by the Implicit Function Theorem (IFT),

$$\frac{\partial \pi^*(W_i)}{\partial \alpha_i} = -\left(\frac{\partial H(W_i, \pi^*(W_i))}{\partial \pi}\right)^{-1} \frac{\partial H(W_i, \pi^*(W_i))}{\partial \alpha_i},$$

where $\frac{\partial H(W_i, \pi^*(W_i))}{\partial \pi}$ is as described above, and

$$\frac{\partial H(W_i, \pi^*(W_i))}{\partial \alpha_i} = -f_\varepsilon\left(X_i' \beta_0^c - \delta_0 \mu_S(W_i, \pi^*(W_i)) - \Delta_0(P_i - 1)\pi^*(W_i) + \eta_0 \alpha_i\right) \cdot \left(\eta_0 - \delta_0 \frac{\partial \mu_S(W_i, \pi^*(W_i))}{\partial \alpha_i}\right).$$

According to Assumption INC, using backward induction, the incumbent knows how the game will unfold in stage 2 for any given a. Let $W_i(a) \equiv (X_i, \alpha_i(a), P_i)$ and denote the resulting collection of regular BNE as $\pi_1^*(W_i(a)), \ldots, \pi_{\mathcal{E}_i(a)}^*(W_i(a))$. By regularity, for each ℓ we have

$$\frac{\partial \pi_\ell^*(W_i(a))}{\partial a} = \frac{\partial \pi_\ell^*(W_i(a))}{\partial \alpha_i} \frac{\partial \alpha(a, \zeta_i)}{\partial a} = -\left(\frac{\partial H\left(W_i(a), \pi_\ell^*(W_i(a))\right)}{\partial \pi}\right)^{-1} \frac{\partial H(W_i(a), \pi^*(W_i(a)))}{\partial \alpha_i} \frac{\partial \alpha(a, \zeta_i)}{\partial a}$$

Since the incumbent is assumed to know the selection mechanism $\mathcal{M}$, Assumption INC implies that, for any given a, the *expected market share for the incumbent, conditional on* $W_i(a)$ is

$$\overline{\mu}_S(W_i(a)) \equiv \sum_{\ell=1}^{\mathcal{E}_i(a)} \mu_S(W_i(a), \pi_\ell^*(W_i(a))) \cdot P^{\mathcal{M}}(\ell | W_i(a))$$

Assuming that $P^{\mathcal{M}}(\ell | W_i)$ is differentiable with respect to α_i, we have

$$\frac{\partial P^{\mathcal{M}}(\ell | W_i(a))}{\partial a} = \frac{\partial P^{\mathcal{M}}(\ell | W_i(a))}{\partial \alpha_i} \cdot \frac{\partial \alpha(a, \zeta_i)}{\partial a}, \quad \text{and} \quad \frac{\partial \overline{\mu}_S(W_i(a))}{\partial a} = \frac{\partial \overline{\mu}_S(W_i(a))}{\partial \alpha_i} \cdot \frac{\partial \alpha(a, \zeta_i)}{\partial a},$$

where

$$\frac{\partial \overline{\mu}_S(W_i(a))}{\partial \alpha_i} = \sum_{\ell=1}^{\mathcal{E}_i(a)} \Bigg[\left(\frac{\partial \mu_S(W_i(a), \pi^*_\ell(W_i(a)))}{\partial \alpha_i} + \frac{\partial \mu_S(W_i(a), \pi^*_\ell(W_i(a)))}{\partial \pi} \cdot \frac{\partial \pi^*_\ell(W_i(a))}{\partial \alpha_i} \right) \cdot P^{\mathcal{M}}(\ell | W_i(a)) + \mu_S(W_i(a), \pi^*_\ell(W_i(a))) \cdot \frac{\partial P^{\mathcal{M}}(\ell | W_i(a))}{\partial \alpha_i} \Bigg].$$

Since $W_i(a_i^*) = W_i$, we have $\frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial \alpha_i} = \frac{\partial \overline{\mu}_S(W_i)}{\partial \alpha_i}$ and, therefore,

$$\frac{\partial \overline{\mu}_S(W_i)}{\partial \alpha_i} = \sum_{\ell=1}^{\mathcal{E}_i} \Bigg[\left(\frac{\partial \mu_S(W_i, \pi^*_\ell(W_i))}{\partial \alpha_i} + \frac{\partial \mu_S(W_i, \pi^*_\ell(W_i))}{\partial \pi} \cdot \frac{\partial \pi^*_\ell(W_i)}{\partial \alpha_i} \right) \cdot P^{\mathcal{M}}(\ell | W_i) + \mu_S(W_i, \pi^*_\ell(W_i)) \cdot \frac{\partial P^{\mathcal{M}}(\ell | W_i)}{\partial \alpha_i} \Bigg]. \tag{10}$$

3.3.1 Strategic behavior without "variable costs"

Suppose there are no "variable costs" associated with the choice of a_i^*, so the latter is given by

$$a_i^* = \underset{a \in \mathcal{A}}{\operatorname{argmax}} \; \overline{\mu}_S(W_i(a)).$$

Assume that the strategy space $\mathcal{A}$ is large enough that a_i^* is always an interior solution. Then, a_i^* would satisfy the first-order conditions,

$$\underbrace{\frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial a}}_{\frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial \alpha_i} \cdot \frac{\partial \alpha(a, \zeta_i)}{\partial a}} = 0 \quad \Longrightarrow \quad \frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial \alpha_i} \cdot \frac{\partial \alpha(a_i^*, \zeta_i)}{\partial a} = 0$$

If we assume that $\frac{\partial \alpha(\cdot, \zeta_i)}{\partial a} \neq 0$, so any change in a always shifts α_i, the f.o.c will hold if and only if

$$\underbrace{\frac{\partial \overline{\mu}_S(W_i)}{\partial \alpha_i}}_{\frac{\partial \overline{\mu}(W_i(a_i^*))}{\partial \alpha_i}} = 0.$$

3.3.2 Strategic behavior with "variable costs"

Let us assume that a is a scalar and suppose we allow for the existence of a "variable cost" for the strategy a captured through a "variable cost function" $c_i(a)$, unknown except for the assumption that it is nondecreasing in a. Accordingly, suppose

$$a_i^* = \underset{a \in \mathcal{A}}{\operatorname{argmax}} \left(\overline{\mu}_S(W_i(a)) - c_i(a) \right).$$

Assume once again that $\mathcal{A}$ is large enough that a_i^* is always an interior solution. In this case, we would generalize the f.o.c described previously to the inequality

$$\underbrace{\frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial a}}_{\frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial \alpha_i} \cdot \frac{\partial \alpha(a,\zeta_i)}{\partial a}} \geq 0 \quad \Longrightarrow \quad \frac{\partial \overline{\mu}_S(W_i(a_i^*))}{\partial \alpha_i} \cdot \frac{\partial \alpha(a_i^*, \zeta_i)}{\partial a} \geq 0.$$

In this case, the first-order condition above would be replaced with the following,

$$\frac{\partial \overline{\mu}_S(W_i)}{\partial \alpha_i} \begin{cases} \geq 0 & \text{if we assume that } \alpha_i \text{ is nondecreasing in } a_i \\ \leq 0 & \text{if we assume that } \alpha_i \text{ is nonincreasing in } a_i \end{cases}$$

Using the above conditions to test for the presence of strategic behavior by the incumbent requires us to assume the sign of $\frac{\partial \alpha(\cdot,\zeta_i)}{\partial a}$. That is, the direction in which α_i changes with a_i^*. This, in turn, would require us to settle on a more precise economic interpretation for the strategy a. Once we have assigned an economic meaning to a, we can identify the sign of $\frac{\partial \alpha(\cdot,\zeta_i)}{\partial a}$ by recalling the following.

(i) From (1), keeping all else constant, the incumbent's market share is increasing in α_i.

(ii) From (3), keeping all else constant, an increase in α_i makes entry more attractive for all potential entrants if $\eta_0 \geq 0$ and less attractive if $\eta_0 < 0$.

For example, suppose $\eta_0 > 0$ and that a_i is a measure of infrastructure deployment by the incumbent, and that infrastructure must be made available to potential entrants by law (as will be the case in our empirical example). If we assume that, all else constant, a wider infrastructure would benefit both the incumbent and potential entrants, we would have that α_i is nondecreasing in a_i. In that case, if we allow for variable costs for infrastructure investment, strategic behavior by the incumbent implies $\frac{\partial \overline{\mu}_S(W_i)}{\partial \alpha_i} \geq 0$.

Remark 1 We do not require or presuppose that the incumbent behaves strategically. Maintaining the assumption that entry decisions are taken conditional on α_i and that strategic behavior from the incumbent would take place in a first stage, before entry decisions are made, the BNE analysis in Section 3.1 focuses on the second stage (the entry stage) while the analysis in this section focuses on a hypothetical first-stage where the incumbent does backward induction and chooses its strategy optimally to deter entry and maximize its expected market share once the game ends. Also note that our analysis presupposes that the incumbent assumes BNE behavior from potential entrants in stage 2. We leave the case where the incumbent allows for incorrect beliefs from potential entrants (e.g, as in Section 3.2) for future work.

4 Introducing an invertibility assumption

Since the market-level characteristic α_i is unobserved by the econometrician, we need further assumptions in order to use the results from the previous sections as the basis for inference in this model. To this end we introduce the following condition.

Assumption IG *The function $G(\cdot)$ that characterizes the incumbent's market share in* (1) *is invertible everywhere on* $\mathbb{R}$.

Invertibility of $G(\cdot)$ in equation (1) assumes that, once we aggregate consumers' preferences in market i, the incumbent's market share is a monotonic function of the market-level index $X_i'\beta_0^m - \gamma_0 \cdot C_i + \alpha_i$. Thus, Assumption IG provides an economic interpretation for this index, along with an implicit assumption about consumer preferences. Because we will not model the individual market shares of potential entrants, we will not assume that the system of market shares for all firms in the market is invertible. Consequently, our results will not rely on additional conditions on consumer preferences, such as the connected substitutes condition in Berry, Gandhi, and Haile (2013) which are typically used to justify invertibility of the system of market shares in empirical models of demand. In that literature, similar to our model, these invertibility conditions have been a way to recover unobserved market-level or product-level shocks in consumer demand models with market-level data (see McFadden (1974), Berry, Levinsohn, and Pakes (1995), Berry and Haile (2009), Nevo (2011), Berry, Gandhi, and Haile (2013)).

As our results will show, invertibility of $G(\cdot)$ will allow us to remain agnostic about the exact nature of the unobserved market-level effect α_i. In Section 3.3 we will describe testable implications for the conjecture that α_i reflects a strategic choice by the incumbent. Assumption IG will allow us to characterize testable implications and design an econometric test of strategic behavior by the incumbent under minimal assumptions[3]. Dropping the assumption of invertibility of $G(\cdot)$ could be done, e.g, if we are willing to model α_i *explicitly* as a strategic action by the incumbent. This, in turn, would require us to make additional assumptions about the precise nature of the incumbent's behavior. Specifically, it would require us to make precise assumptions about the incumbent's objective function. Maintaining invertibility as described in Assumption IG will enable us to bypass such assumptions while still being able to test whether there is evidence that the

[3] We will interpret α_i as reflecting, at least partially, an entry deterrence strategy by the incumbent without having to fully characterize the incumbent's objective function.

market-level effect α_i reflects strategic behavior by the incumbent. From Assumption IG,

$$\alpha_i = G^{-1}(S_i) - X_i'\beta_0^m + \gamma_0 \cdot C_i. \tag{11}$$

Below (Assumption E1), we will assume that G has a known functional form. In this case we could consider estimating β_0^m and γ_0, and then recovering α_i, from the equation,

$$G^{-1}(S_i) = X_i'\beta_0^m - \gamma_0 \cdot C_i + \alpha_i, \tag{11'}$$

However, we will *not* assume that we have instruments that would enable us to consistently estimate (11'). From our model, both X_i and C_i are correlated with the latent variable α_i through the entry decisions of potential entrants as well as the possible strategic behavior from the incumbent. Instead of assuming that we can estimate equation (11') consistently in a first step, we will go back to all the expressions described in sections 2 and 3.1 and we will replace α_i with the expression in (11). We will then study which parameters can be identified and the conditions under which we can identify them.

4.1 Analysis of BNE features following invertibility

Using (11) will allow us to express all our functionals in terms of $Z_i \equiv (S_i, C_i, X_i, P_i)$, the market characteristics and outcomes observed by the econometrician. Take a given $\pi \in [0,1]$. Our definition of $\mu_S(W_i, \pi)$ in (4) becomes,

$$\mu_S(W_i, \pi) = \sum_{q=0}^{P_i-1} \sigma(P_i, q, \pi) \cdot G\Big(G^{-1}(S_i) + \gamma_0 \cdot (C_i - (q+1))\Big) \equiv \mu_S(Z_i, \pi, \gamma_0)$$

From here and (11), we have

$$X_i'\beta_0^c - \delta_0\mu_S(W_i,\pi) - \Delta_0(P_i-1)\pi + \eta_0\alpha_i = X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0\mu_S(Z_i,\pi,\gamma_0) - \Delta_0(P_i-1)\pi + \eta_0\gamma_0 C_i,$$

where $\beta_0 \equiv \beta_0^c - \eta_0\beta_0^m$. Group $\theta_0 \equiv (\beta_0', \eta_0, \gamma_0, \delta_0, \Delta_0)'$. We shall describe conditions under which θ_0 can be identified. Notice that identification of θ_0 would imply that, even though β_0^m and β_0^c cannot be separately identified, *we can still identify all the strategic interaction parameters* $(\gamma_0, \delta_0, \Delta_0)$, along with η_0. We will let Θ denote the parameter space for θ_0. Note that if we let d_x denote the dimension of X_i and if we let d_θ denote the dimension of θ_0, then $d_\theta \equiv \dim(\theta_0) = d_x + 4$. For a given $\theta \equiv (\beta', \eta, \gamma, \delta, \Delta)' \in \Theta$ and $\pi \in [0,1]$, define the index,

$$
\begin{aligned}
m(Z_i, \pi, \theta) &\equiv X_i'\beta + \eta G^{-1}(S_i) - \delta \mu_S(Z_i, \pi, \gamma) - \Delta(P_i - 1)\pi + \eta\gamma C_i, \quad \text{where} \\
\mu_S(Z_i, \pi, \gamma) &\equiv \sum_{q=0}^{P_i - 1} \sigma(P_i, q, \pi) \cdot G\Big(G^{-1}(S_i) + \gamma \cdot (C_i - (q+1))\Big).
\end{aligned}
\tag{12}
$$

Let $H(Z_i, \pi, \theta) \equiv \pi - F_\varepsilon(m(Z_i, \pi, \theta))$. Under Assumption IG we have $H(W_i, \pi) = H(Z_i, \pi, \theta_0)$ and the BNE system (5) becomes,

$$
H(Z_i, \pi, \theta_0) = 0. \tag{5'}
$$

Under Assumption IG, the solutions to (5') correspond to the BNE in market i. For any $\pi \in (0,1)$,

$$
\begin{aligned}
\frac{\partial \mu_S(Z_i, \pi, \gamma)}{\partial \pi} &= \sum_{q=0}^{P_i - 1} \frac{\partial \sigma(P_i, q, \pi)}{\partial \pi} \cdot G\Big(G^{-1}(S_i) + \gamma \cdot (C_i - (q+1))\Big), \quad \text{where} \\
\frac{\partial \sigma(P, q, \pi)}{\partial \pi} &= \binom{P-1}{q} \cdot (P-1) \cdot \frac{\pi^q \cdot (1-\pi)^{P-1-q}}{\pi \cdot (1-\pi)} \cdot \left(\frac{q}{P-1} - \pi\right), \\
\frac{\partial H(Z_i, \pi, \theta)}{\partial \pi} &= 1 + f_\varepsilon(m(Z_i, \pi, \theta)) \cdot \left(\delta \cdot \frac{\partial \mu_S(Z_i, \pi, \gamma)}{\partial \pi} + \Delta(P_i - 1)\right)
\end{aligned}
\tag{13}
$$

The cardinality of BNE in market i can be characterized by the properties of $\frac{\partial H(Z_i, \pi, \theta_0)}{\partial \pi}$. A sufficient condition for regular-BNE uniqueness is if the sign of $\frac{\partial H(Z_i, \pi, \theta_0)}{\partial \pi}$ is constant for all $\pi \in [0,1]$. Since $F_\varepsilon \in (0,1)$, we have $H(Z_i, 0, \theta_0) < 0$. Therefore, the sign of $\frac{\partial H(Z_i, \pi, \theta_0)}{\partial \pi}$ is constant for all $\pi \in [0,1]$ if and only if $\frac{\partial H(Z_i, \pi, \theta_0)}{\partial \pi} \geq 0$ for all $\pi \in [0,1]$. Thus, we have the following result.

Result 2 *A sufficient (but not necessary) condition for the game to have a unique regular BNE in market i is if $\frac{\partial H(Z_i, \pi, \theta_0)}{\partial \pi} \geq 0$ for all $\pi \in [0,1]$.*

4.1.1 A graphical illustration of BNE cardinality

Figures 1-3 illustrate the BNE properties of the game assuming $\gamma_0 \geq 0$, $\delta_0 \geq 0$, $\Delta_0 \geq 0$ (the case described in Section 2.6), and assuming that both G and F_ε correspond to the logistic cdf. Each figure pre-specifies a particular value for Z_i and θ_0 and depicts $F_Y(Z_i, \pi, \theta_0) \equiv F_\varepsilon(m(Z_i, \pi, \theta_0))$ along with the 45-degree line for $\pi \in [0,1]$. Any point of crossing between both curves is a BNE. An *irregular* BNE occurs at a point of tangency. While these graphs constitute only an exploratory analysis, we derived the following observations,

(i) As figures 1 and 2 show, multiple BNE requires that the strategic effect Δ_0 (the entry-competition effect on the potential entrants) be sufficiently dominated by both γ_0 (the competition effect on the incumbent) and δ_0 (the effect of the incumbent's market share on entry decisions).

(ii) The curve $F_Y(Z_i, \pi, \theta_0)$ can be non-monotonic over $\pi \in [0,1]$. As Figures 2 and 3 show, this requires that both γ_0 and δ_0 be sufficiently larger than Δ_0 and is more likely to occur when the number of potential entrants P_i is relatively large. BNE uniqueness can still hold.

(iii) As figure 3 shows, larger values of $|X_i'\beta_0|$ are conducive to uniqueness of BNE by reducing the variability of the curve $F_Y(Z_i, \pi, \theta_0)$ over $\pi \in [0,1]$. Thus, under the assumption that $\beta_{\ell,0} \neq 0$ for some covariate $X_{\ell,i}$, we can mitigate the potential presence of multiple BNE by focusing on markets where $|X_{\ell,i}|$ is relatively large.

5 Estimation and inference with invertibility

In this section we will propose estimation and inference procedures under the invertibility condition in Assumption IG. First, we will propose an estimation procedure under the assumption of BNE behavior. Then, we will outline how to conduct inference based on iterated elimination of nonrationalizable strategies.

5.1 Estimation assuming BNE behavior

We will begin with the following assumption.

Assumption E1 *The mapping G has known functional form, with first derivative denoted by g. The distribution F_ε has known functional form, with density function f_ε, which is twice continuously differentiable. Let $\mathcal{X} \subseteq \mathcal{S}_X$ denote a pre-specified range for X_i. The game has a unique BNE whenever $X_i \in \mathcal{X}$.*

Exploring whether the function $G(\cdot)$ can be modeled nonparametrically using, e.g, sieves methods, is left for future research, as it would give rise to econometric issues that would go beyond the intended scope of the paper, which emphasizes the game itself. Focusing on a parametric model is also representative of most of the existing work in the econometric analysis of discrete games, where the identification and inferential challenges arise from the properties of the game itself, even in a fully parameterized setting. BNE uniqueness is testable ex-post. Our inference range $\mathcal{X}$ can be equal to the entire support of X or it can be a subset of it. The choice of $\mathcal{X}$ can be guided by our equilibrium analysis from Section 4.1, where we discussed that realizations of X_i for which $\left|X_i'\beta_0\right|$ is relatively large are conducive to generating a unique BNE (see Figure 3 and our discussion in Section 4.1.1). Thus, if we presuppose that a particular $X_{\ell,i}$ has a nonzero coefficient in β_0, our inference range can be limited to markets where $|X_{\ell,i}|$ is relatively large.

Under Assumption E1, whenever $X_i \in \mathcal{X}$, the game has a unique BNE, which we denote as $\pi^*(Z_i, \theta_0)$. From the description of $\tau_Y(W_i) \equiv Pr(Y_{p,i} = 1|W_i)$ in (7),

$$X_i \in \mathcal{X} \implies \tau_Y(W_i) = F_\varepsilon\left(m(Z_i, \pi^*(Z_i,\theta_0),\theta_0)\right), \quad \text{and} \quad E\left[C_i|W_i\right] = P_i \cdot F_\varepsilon\left(m(Z_i, \pi^*(Z_i,\theta_0),\theta_0)\right).$$

Recall that $W_i \equiv (X_i, \alpha_i, P_i)$. From the above result, for any measurable function ϕ of W_i,

$$E\Big[\mathbb{1}\{X_i \in \mathcal{X}\}\,\phi(W_i)\cdot\Big(C_i - P_i \cdot F_\varepsilon\left(m(Z_i, \pi^*(Z_i, \theta_0), \theta_0)\right)\Big)\Big] = E\Big[\mathbb{1}\{X_i \in \mathcal{X}\}\,\phi(W_i)\cdot\Big(C_i - P_i \cdot \tau_Y(W_i)\Big)\Big] = 0. \tag{14}$$

We propose to estimate θ_0 based on this moment condition.

5.1.1 An estimator for $\boldsymbol{\theta_0}$

We will begin by strenghtening Assumption E1.

Assumption E2 *There exists a neighborhood $\mathcal{N}$ that contains θ_0 such that, for each $\theta \in \mathcal{N}$, there is a unique regular solution in π to $H(Z_i, \pi, \theta) = 0$ whenever $X_i \in \mathcal{X}$.*

For any given θ and each market, BNE uniqueness is a testable condition under our assumptions. In what follows, for any $\theta \in \Theta$ and any market such that $X_i \in \mathcal{X}$ we will let $\pi^*(Z_i, \theta)$ denote a regular BNE selected by the econometrician. By Assumption E2, the game has a unique regular BNE for each $\theta \in \mathcal{N}$, so the mechanism used by the econometrician to select $\pi^*(Z_i, \theta)$ will be asymptotically irrelevant. Recall that $W_i \equiv (X_i, \alpha_i, P_i)$, and that α_i is unobserved. In order to exploit the type of moment restriction described in (14) we need instruments that are functions of W_i. Immediately, we have X_i and P_i at our disposal, since both are components of Z_i, the observable market covariates. Recall that $d_\theta \equiv \dim(\theta_0) = d_x + 4$, where $d_x \equiv \dim(X_i)$. We will construct additional instruments as follows. Let $h_1, \ldots, h_L$ be a collection of pre-specified functions of X_i. For a given $\gamma \in \Theta$, let $t_\ell(Z_i, \gamma) \equiv G^{-1}(S_i) + h_\ell(X_i) + \gamma C_i$. From (11), it follows that

$$t_\ell(Z_i, \gamma) = \underbrace{\alpha_i + X_i'\beta_0^m + h_\ell(X_i)}_{\equiv r_\ell(W_i)} + (\gamma - \gamma_0)\cdot C_i, \quad \text{therefore} \quad t_\ell(Z_i, \gamma_0) = r_\ell(W_i),$$

where $r_\ell(W_i) \equiv \alpha_i + X_i'\beta_0^m + h_\ell(X_i)$. Our collection of instruments will consist of transformations of,

$$\mathcal{I}(Z_i, \gamma) \equiv \Big(X_i'\,,\, P_i\,,\, t_1(Z_i, \gamma)\,,\, \ldots\,,\, t_L(Z_i, \gamma)\Big)' \in \mathbb{R}^{d_\mathcal{I}},$$

where $d_\mathcal{I} \geq d_\theta$. Note that,

$$\mathcal{I}(Z_i, \gamma_0) = \Big(X_i'\,,\, P_i\,,\, r_1(W_i)\,,\, \ldots\,,\, r_L(W_i)\Big)' \equiv \mathcal{I}(W_i).$$

Next, we pre-specify a collection of *instrument functions* $\phi \in \mathbb{R}^{d_\phi}$, with $d_\phi \geq d_\theta$. For each one of our $\ell = 1, \ldots, d_\phi$ instrument functions ϕ_ℓ, denote

$$\begin{aligned} M_\ell(\theta) &\equiv E\Big[\mathbb{1}\{X_i \in \mathcal{X}\}\,\phi_\ell(\mathcal{I}(Z_i, \gamma))\cdot\Big(C_i - P_i \cdot F_\varepsilon\left(m(Z_i, \pi^*(Z_i, \theta), \theta)\right)\Big)\Big], \\ M(\theta) &\equiv \Big(M_1(\theta), \ldots, M_{d_\phi}(\theta)\Big)'. \end{aligned} \tag{15}$$

From the above results we have,

$$M(\theta_0) = E\Big[\mathbb{1}\{X_i \in \mathcal{X}\}\,\phi\left(\mathcal{I}(W_i)\right)\cdot\Big(C_i - P_i \cdot \tau_Y(W_i)\Big)\Big] = 0.$$

We propose a GMM estimator based on the above moment conditions and the usual population objective function $Q(\theta) \equiv M(\theta)'\mathcal{W}M(\theta)$, where $\mathcal{W}$ is a prespecified, positive definite weight matrix. For each one of our $\ell = 1,\ldots,d_\phi$ instrument functions ϕ_ℓ, denote

$$\begin{aligned}
\widehat{M}_\ell(\theta) &\equiv \frac{1}{n}\sum_{i=1}^{n}\mathbb{1}\{X_i \in \mathcal{X}\}\,\phi_\ell\left(\mathcal{I}(Z_i,\gamma)\right)\cdot\Big(C_i - P_i\cdot F_\varepsilon\left(m(Z_i,\pi^*(Z_i,\theta),\theta)\right)\Big),\\
\widehat{M}(\theta) &\equiv \Big(\widehat{M}_1(\theta),\ldots,\widehat{M}_{d_\phi}(\theta)\Big)'.
\end{aligned}$$

Our GMM sample objective function is of the usual form, $\widehat{Q}(\theta) \equiv \widehat{M}(\theta)'\widehat{\mathcal{W}}\widehat{M}(\theta)$, where $\widehat{\mathcal{W}} \xrightarrow{p} \mathcal{W}$. Let us describe the expression for $\frac{\partial F_\varepsilon(m(Z_i,\pi^*(Z_i,\theta),\theta))}{\partial\theta}$. From our definition of $m(Z_i,\pi,\theta)$,

$$\underbrace{\frac{\partial m(Z_i,\pi,\theta)}{\partial\theta}}_{d_\theta\times 1} = \begin{pmatrix} X_i \\ G^{-1}(S_i) + \gamma C_i \\ -\delta\frac{\partial\mu_S(Z_i,\pi,\gamma)}{\partial\gamma} + \eta C_i \\ -\mu_S(Z_i,\pi,\gamma) \\ -(P_i - 1)\pi \end{pmatrix}, \quad \underbrace{\frac{\partial m(Z_i,\pi,\theta)}{\partial\pi}}_{1\times 1} = -\delta\frac{\partial\mu_S(Z_i,\pi,\gamma)}{\partial\pi} - \Delta\cdot(P_i - 1).$$

Define

$$\begin{aligned}
\lambda_1(Z_i,\pi,\theta) &\equiv \frac{f_\varepsilon\left(m(Z_i,\pi,\theta)\right)}{1 + f_\varepsilon\left(m(Z_i,\pi,\theta)\right)\times\left(\delta\cdot\frac{\partial\mu_S(Z_i,\pi,\gamma)}{\partial\pi} + \Delta(P_i - 1)\right)},\\
\lambda_2(Z_i,\pi,\theta) &\equiv f_\varepsilon\left(m(Z_i,\pi,\theta)\right)\cdot\left[1 - \lambda_1(Z_i,\pi,\theta)\cdot\left(\delta\frac{\partial\mu_S(Z_i,\pi,\gamma)}{\partial\pi} + \Delta\cdot(P_i - 1)\right)\right].
\end{aligned}$$

By regularity of $\pi^*(Z_i,\theta)$ and the Implicit Function Theorem (IFT), we obtain

$$\underbrace{\frac{\partial\pi^*(Z_i,\theta)}{\partial\theta}}_{d_\theta\times 1} = \frac{\partial m(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\theta}\cdot\lambda_1\left(Z_i,\pi^*(Z_i,\theta),\theta\right).$$

And from here,

$$\begin{aligned}
\underbrace{\frac{\partial F_\varepsilon\left(m(Z_i,\pi^*(Z_i,\theta),\theta)\right)}{\partial\theta}}_{d\times 1} &= f_\varepsilon\left(m(Z_i,\pi^*(Z_i,\theta),\theta)\right)\cdot\left(\frac{\partial m(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\theta} + \frac{\partial m(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\pi}\cdot\frac{\partial\pi^*(Z_i,\theta)}{\partial\theta}\right)\\
&= \frac{\partial m(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\theta}\cdot\lambda_2\left(Z_i,\pi^*(Z_i,\theta),\theta\right).
\end{aligned}$$

We will choose smooth instrument functions, so that the Jacobian $\frac{\phi(\mathcal{I}(Z_i,\gamma))}{\partial\gamma}$ is well-defined for all γ. For the ℓ^{th} instrument function, let

$$\underbrace{J_{\ell,\gamma}(Z_i,\theta)}_{d_\theta\times 1} \equiv \begin{pmatrix} \underbrace{0}_{(d_x+1)\times 1} \\ \underbrace{\mathbb{1}\{X_i\in\mathcal{X}\}\frac{\partial\phi_\ell(\mathcal{I}(Z_i,\gamma))}{\partial\gamma}\cdot(C_i-P_i\cdot F_\varepsilon(m(Z_i,\pi^*(Z_i,\theta),\theta)))}_{1\times 1} \\ \underbrace{0}_{2\times 1}\end{pmatrix}$$

Let $M_\ell(\theta)$ be as described in (15). We have,

$$\underbrace{\frac{\partial M_\ell(\theta)}{\partial\theta}}_{d_\theta\times 1} = E\left[J_{\ell,\gamma}(Z_i,\theta)-\frac{\partial m(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\theta}\cdot\mathbb{1}\{X_i\in\mathcal{X}\}\phi_\ell(\mathcal{I}(Z_i,\gamma))P_i\lambda_2(Z_i,\pi^*(Z_i,\theta),\theta)\right],$$

$$\underbrace{\frac{\partial M(\theta)}{\partial\theta}}_{d_\phi\times d_\theta} = \left(\frac{\partial M_1(\theta)}{\partial\theta},\cdots,\frac{\partial M_{d_\phi}(\theta)}{\partial\theta}\right)'$$

Our sample-moment Jacobians are,

$$\underbrace{\frac{\partial\widehat{M}_\ell(\theta)}{\partial\theta}}_{d\times 1} = \frac{1}{n}\sum_{i=1}^{n}\left(J_{\ell,\gamma}(Z_i,\theta)-\frac{\partial m(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\theta}\cdot\mathbb{1}\{X_i\in\mathcal{X}\}\phi_\ell(\mathcal{I}(Z_i,\gamma))P_i\lambda_2(Z_i,\pi^*(Z_i,\theta),\theta)\right),$$

$$\underbrace{\frac{\partial\widehat{M}(\theta)}{\partial\theta}}_{d_\phi\times d_\theta} = \left(\frac{\partial\widehat{M}_1(\theta)}{\partial\theta},\cdots,\frac{\partial\widehat{M}_{d_\phi}(\theta)}{\partial\theta}\right)'$$

Assumption E3

(i) We have $\delta_0\neq 0$ and, accordingly, our parameter space satisfies $\delta\neq 0\ \forall\ \theta\in\Theta$.

(ii) For each $\theta\in\Theta:\theta\neq\theta_0$, we have $Pr\left(m(Z_i,\pi^*(Z_i,\theta),\theta)\neq m(Z_i,\pi^*(Z_i,\theta_0),\theta_0)\ \middle|X_i\in\mathcal{X}\right)>0$, and

$$\begin{aligned}&E\left[\phi_\ell(\mathcal{I}(Z_i,\gamma))\cdot\left(C_i-P_i\cdot F_\varepsilon(m(Z_i,\pi^*(Z_i,\theta),\theta))\right)\middle|X_i\in\mathcal{X}\right]\\ \neq &E\left[\phi_\ell(\mathcal{I}(Z_i,\gamma_0))\cdot\left(C_i-P_i\cdot F_\varepsilon(m(Z_i,\pi^*(Z_i,\theta_0),\theta_0))\right)\middle|X_i\in\mathcal{X}\right],\end{aligned}$$

for some $\ell=1,\ldots,d_\phi$.

(iii) (A full rank condition).- The unobserved market characteristic α_i is continuously distributed conditional on X_i and the incumbent's market share S_i is continuously distributed on $(0,1)$ conditional on X_i. Now, let

$$\underbrace{\rho(Z_i,\theta)}_{d_\theta\times 1}\equiv\begin{pmatrix} X_i \\ G^{-1}(S_i) \\ \frac{\partial\mu_S(Z_i,\pi^*(Z_i,\theta),\theta)}{\partial\gamma}-\frac{\partial\phi_\ell(\mathcal{I}(Z_i,\gamma))}{\partial\gamma}\cdot\left(C_i-P_i\cdot F_\varepsilon\left(m(Z_i,\pi^*(Z_i,\theta),\theta)\right)\right) \\ \mu_S(Z_i,\pi^*(Z_i,\theta),\gamma) \\ \pi^*(Z_i,\theta)\end{pmatrix}$$

Then, for each $\ell=1,\ldots,d_\phi$ and each $\theta\in\Theta$, there does not exist a proper linear subspace of the support of $\rho(Z_i,\theta)$ that contains $\rho(Z_i,\theta)$ w.p.1. (conditional on $X_i\in\mathcal{X}$). In particular, if we define

$$\underbrace{\vartheta(Z_i,\theta)}_{d_\theta\times 1}\equiv\begin{pmatrix} X_i \\ G^{-1}(S_i) \\ g(G^{-1}(S_i))-\frac{\partial\phi_\ell(\mathcal{I}(Z_i,\gamma))}{\partial\gamma}\cdot\left(C_i-P_i\cdot F_\varepsilon\left(m(Z_i,\pi^*(Z_i,\theta),\theta)\right)\right) \\ S_i \\ \pi^*(Z_i,\theta)\end{pmatrix},$$

then, for each $\ell=1,\ldots,d_\phi$ and each $\theta\in\Theta$ such that $\gamma=0$, there does not exist a proper linear subspace of the support of $\vartheta(Z_i,\theta)$ that contains $\vartheta(Z_i,\theta)$ w.p.1. (conditional on $X_i\in\mathcal{X}$).

(iv) The following variance matrix is invertible

$$\Omega_{\mathcal{X}}\equiv E\left[\mathbb{1}\{X_i\in\mathcal{X}\}\,\phi\left(\mathcal{I}(W_i)\right)\phi\left(\mathcal{I}(W_i)\right)'(C_i-P_i\tau_Y(W_i))^2\right].$$

Part (ii) of Assumption E3 is sufficient for identification of θ_0 as the unique minimizer of $Q(\theta)$. Denote $H_0\equiv\frac{\partial M(\theta_0)}{\partial\theta}$. Parts (i) and (iii) of Assumption E3 are sufficient to ensure that the rank$(H_0)=d_\theta$ and therefore, that the $d_\theta\times d_\theta$ matrix $H_0'\mathcal{W}H_0$ has full rank. The assumption that $\delta_0\neq 0$ could be replaced with a different restriction on the strategic-parameter space, but inspection of the elements in H_0 shows that we cannot have rank$(H_0)=d_\theta$ when all strategic parameters are simultaneously equal to zero. Of course, the hypothesis that $\delta_0=0$ is testable (and we will fail to reject it in our empirical example). Under conditions (i)-(iii) of Assumption E3, with probability approaching one, the GMM estimator $\widehat{\theta}$ is characterized by the first-order conditions, $\frac{\partial\widehat{M}(\widehat{\theta})}{\partial\theta}'\widehat{\mathcal{W}}\widehat{M}(\widehat{\theta})=0$, and from here we obtain the linear representation result,

$$\begin{gathered}\widehat{\theta}=\theta_0+\frac{1}{n}\sum_{i=1}^{n}\psi_i^\theta+o_p\left(\frac{1}{n^{1/2}}\right),\quad\text{where}\\ \psi_i^\theta=\left(H_0'\mathcal{W}H_0\right)^{-1}H_0'\mathcal{W}\mathbb{1}\{X_i\in\mathcal{X}\}\,\phi\left(\mathcal{I}(W_i)\right)\cdot\Big(C_i-P_i\cdot\tau_Y(W_i)\Big),\end{gathered}\tag{16}$$

and therefore,

$$\sqrt{n}\left(\widehat{\theta}-\theta_0\right) \xrightarrow{d} \mathcal{N}\left(0, (H_0'\mathcal{W}H_0)^{-1} H_0'\mathcal{W}\Omega_{\mathcal{X}}\mathcal{W}H_0 (H_0'\mathcal{W}H_0)^{-1}\right)$$

From part (iv) of Assumption E3, the efficient choice of the weight matrix is $\mathcal{W} = \Omega_{\mathcal{X}}^{-1}$ (see Newey and McFadden (1994, Section 4.3)), which simplifies the asymptotic variance to $\left(H_0'\Omega_{\mathcal{X}}^{-1}H_0\right)^{-1}$.

5.1.2 What can we infer about β_0^c and β_0^m?

As we have detailed, under our data assumptions we can only identify $\beta_0 \equiv \beta_0^c - \eta_0\beta_0^m$. However, the fact that η_0 can be identified means that we can test the joint null hypothesis $H_0 : \beta_0^c = \beta_0^m$, $\eta_0 = 1$ by testing the joint null hypothesis[4] $H_0 : \beta_0 = 0$, $\eta_0 = 1$. This is an economically relevant conjecture because, in this case, the index of market characteristics $X_i'\beta_0^m + \eta_i$ which determines the incumbent's market share in (1) is the same as the index of market characteristics $X_i'\beta_0^c + \eta_0 \cdot \alpha_i$ which determines the vNM payoff functions of potential entrants in (2).

5.1.3 Testing for strategic behavior from the incumbent

Suppose the incumbent behaves strategically in the manner described in Section 3.3. Under Assumption E1, BNE uniqueness whenever $X_i \in \mathcal{X}$ simplifies equation (10) to,

$$\frac{\partial \overline{\mu}_S(W_i)}{\partial \alpha_i} = \left(\frac{\partial \mu_S(W_i, \pi^*(W_i))}{\partial \alpha_i} + \frac{\partial \mu_S(W_i, \pi^*(W_i))}{\partial \pi} \cdot \frac{\partial \pi^*(W_i)}{\partial \alpha_i}\right).$$

The above expression simplifies to the following under Assumption IG. Let

$$\begin{aligned}
\frac{\partial \mu_S(Z_i, \pi^*(Z_i,\theta), \gamma)}{\partial \alpha_i} &\equiv \sum_{q=0}^{P_i-1} \sigma(P_i, q, \pi^*(Z_i,\theta)) \cdot g\left(G^{-1}(S_i) + \gamma \cdot (C_i - (q+1))\right), \\
\frac{\partial H(Z_i, \pi^*(Z_i,\theta), \theta)}{\partial \alpha_i} &\equiv -f_{\varepsilon}\left(m(Z_i, \pi^*(Z_i,\theta), \theta)\right) \cdot \left(\eta - \delta \frac{\partial \mu_S(Z_i, \pi^*(Z_i,\theta), \gamma)}{\partial \alpha_i}\right), \\
\frac{\partial \pi^*(Z_i,\theta)}{\partial \alpha_i} &\equiv -\left(\frac{\partial H\left(Z_i, \pi^*(Z_i,\theta), \theta\right)}{\partial \pi}\right)^{-1} \frac{\partial H(Z_i, \pi^*(Z_i,\theta), \theta)}{\partial \alpha_i},
\end{aligned}$$

where $\frac{\partial H(Z_i,\pi,\theta)}{\partial \pi}$ is given in (13). Let $\frac{\partial \mu_S(Z_i,\pi,\gamma)}{\partial \pi}$ be as described in (13). Under Assumptions IG and E1, $X_i \in \mathcal{X}$ implies $\frac{\partial \mu_S(W_i,\pi^*(W_i))}{\partial \pi} = \frac{\partial \mu_S(Z_i,\pi^*(Z_i,\theta_0),\gamma_0)}{\partial \pi}$, $\frac{\partial \mu_S(W_i,\pi^*(W_i))}{\partial \alpha_i} = \frac{\partial \mu_S(Z_i,\pi^*(Z_i,\theta_0),\gamma_0)}{\partial \alpha_i}$, and $\frac{\partial \pi^*(W_i)}{\partial \alpha_i} = \frac{\partial \pi^*(Z_i,\theta_0)}{\partial \alpha_i}$. Thus, if we let,

$$\frac{\partial \overline{\mu}_S(Z_i,\theta)}{\partial \alpha_i} \equiv \left(\frac{\partial \mu_S(Z_i, \pi^*(Z_i,\theta), \gamma)}{\partial \alpha_i} + \frac{\partial \mu_S(Z_i, \pi^*(Z_i,\theta), \theta)}{\partial \pi} \cdot \frac{\partial \pi^*(Z_i,\theta)}{\partial \alpha_i}\right).$$

[4]This null hypothesis can be tested for the entire parameter vectors β_0^m and β_0^c or for a subset of them.

Then, under Assumptions E1 and IG, $X_i \in \mathcal{X}$ implies $\frac{\partial \overline{\mu}(W_i)}{\partial \alpha_i} = \frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i}$. Thus, from the results from Section 3.3, strategic behavior by the incumbent implies the following.

(i) If there are no variable costs associated with the incumbent's strategy, then $\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i} = 0$.

(ii) If we allow for the existence of variable costs associated with the incumbent's strategy, then

$$\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i} \begin{cases} \geq 0 & \text{if we assume that } \alpha_i \text{ is nondecreasing in } a_i \\ \leq 0 & \text{if we assume that } \alpha_i \text{ is nonincreasing in } a_i \end{cases}$$

Let ζ be a pre-specified, *strictly positive* real-valued function and define

$$T_\zeta(\theta_0) \equiv E\left[\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i} \cdot \zeta(Z_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}\right]$$

It follows from the implications of strategic behavior for the incumbent that,

(i) If there are no variable costs associated with the incumbent's strategy, then

$$T_\zeta(\theta_0) = 0.$$

(ii) If we allow for the existence of variable costs associated with the incumbent's strategy, then

$$\begin{aligned} T_\zeta(\theta_0) \geq 0 & \text{ if we assume that } \alpha_i \text{ is nondecreasing in } a_i \\ T_\zeta(\theta_0) \leq 0 & \text{ if we assume that } \alpha_i \text{ is nonincreasing in } a_i \end{aligned}$$

We can test the above restrictions as follows. Let

$$\widehat{T}_\zeta(\widehat{\theta}) = \frac{1}{n}\sum_{i=1}^{n} \frac{\partial \overline{\mu}_S(Z_i,\widehat{\theta})}{\partial \alpha_i} \zeta(Z_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}$$

From Assumptions ML and ML2, whenever $X_i \in \mathcal{X}$, the Jacobian

$$\frac{\partial}{\partial \theta}\left(\frac{\partial \overline{\mu}_S(Z_i,\theta)}{\partial \alpha_i}\right) = \frac{\partial^2 \overline{\mu}_S(Z_i,\theta)}{\partial \theta \partial \alpha_i}$$

is well defined and continuous in θ for all $\theta \in \mathcal{N}$. Let

$$\underbrace{\mathcal{S}_0}_{d\times 1} \equiv E\left[\frac{\partial^2 \overline{\mu}_S(Z_i,\theta_0)}{\partial \theta \partial \alpha_i} \zeta(Z_i) \cdot \mathbb{1}\{X_i \in \mathcal{X}\}\right].$$

Under the assumptions leading to (16),

$$\begin{aligned}\widehat{T_\zeta}(\widehat{\theta}) &= T_\zeta(\theta_0) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i}\cdot \zeta(Z_i)\cdot \mathbb{1}\{X_i \in \mathcal{X}\} - T_\zeta(\theta_0)\right) + \mathcal{S}_0'\left(\widehat{\theta}-\theta_0\right) + o_p\left(\frac{1}{\sqrt{n}}\right)\\ &= T_\zeta(\theta_0) + \frac{1}{n}\sum_{i=1}^{n}\psi_i^{T_\zeta} + o_p\left(\frac{1}{\sqrt{n}}\right),\\ \text{where}\quad \psi_i^{T_\zeta} &\equiv \left(\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i}\cdot \zeta(Z_i)\cdot \mathbb{1}\{X_i \in \mathcal{X}\} - T_\zeta(\theta_0)\right) + \mathcal{S}_0'\psi_i^{\theta}\end{aligned}$$

and ψ_i^{θ} is the influence function of the GMM estimator $\widehat{\theta}$ described in (16). From here,

$$\sqrt{n}\left(\widehat{T_\zeta}(\widehat{\theta}) - T_\zeta(\theta_0)\right) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\psi_i^{T_\zeta} + o_p(1) \xrightarrow{d} \mathcal{N}\left(0,\sigma_{T_\zeta}^2\right), \quad \text{where} \quad \sigma_{T_\zeta}^2 \equiv E\left[(\psi_i^{T_\zeta})^2\right].$$

Let

$$\begin{aligned}\widehat{\psi}_i^{T_\zeta} &= \left(\frac{\partial \overline{\mu}_S(Z_i,\widehat{\theta})}{\partial \alpha_i}\cdot \zeta(Z_i)\cdot \mathbb{1}\{X_i \in \mathcal{X}\} - \widehat{T_\zeta}(\widehat{\theta})\right) + \widehat{\mathcal{S}}_0'\widehat{\psi}_i^{\theta}, \quad \text{where}\\ \widehat{\mathcal{S}}_0 &= \frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 \overline{\mu}_S(Z_i,\widehat{\theta})}{\partial\theta\partial\alpha_i}\zeta(Z_i)\cdot \mathbb{1}\{X_i \in \mathcal{X}\}, \quad \text{and}\\ \widehat{\psi}_i^{\theta} &= \left(\frac{\partial M(\widehat{\theta})}{\partial\theta}'\widehat{\mathcal{W}}\frac{\partial M(\widehat{\theta})}{\partial\theta}\right)^{-1}\frac{\partial M(\widehat{\theta})}{\partial\theta}'\widehat{\mathcal{W}}\cdot \mathbb{1}\{X_i \in \mathcal{X}\}\phi\left(\mathcal{I}(Z_i,\widehat{\gamma})\right)\cdot\left(C_i - P_i\cdot F_\varepsilon\left(m(Z_i,\pi^*(Z_i,\widehat{\theta}),\widehat{\theta})\right)\right).\end{aligned}$$

We can estimate σ_T^2 with $\widehat{\sigma}_{T_\zeta}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\widehat{\psi}_i^{T_\zeta}\right)^2$. Under the assumptions leading to (16), we have $\widehat{\sigma}_{T_\zeta}^2 \xrightarrow{p} \sigma_{T_\zeta}^2$. Next, take the test-statistic

$$\widehat{t_\zeta} \equiv \frac{\sqrt{n}\cdot\widehat{T_\zeta}(\widehat{\theta})}{\widehat{\sigma}_{T_\zeta}}. \tag{17}$$

Note that,

$$\widehat{t_\zeta} = \frac{\sqrt{n}\cdot T_\zeta(\theta_0)}{\widehat{\sigma}_{T_\zeta}} + \underbrace{\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\psi_i^{T_\zeta}}{\widehat{\sigma}_{T_\zeta}}}_{\xrightarrow{d}\mathcal{N}(0,1)} + o_p(1).$$

Suppose we allow for the existence of variable costs associated with the incumbent's strategy and we maintain that α_i is nondecreasing in a_i. As we described above, this implies the null hypothesis $H_0: T_\zeta(\theta_0) \geq 0$. Let τ denote our target significance level and let z_τ denote the Standard Normal

τ^{th} quantile. Consider a rule that rejects H_0 if $\widehat{t_\zeta} < -z_\tau$. From (17), we have,

$$Pr\Big(\text{ Reject } H_0 \,\Big|T_\zeta(\theta_0) = 0\Big) \longrightarrow \tau,\ Pr\Big(\text{ Reject } H_0 \,\Big|T_\zeta(\theta_0) > 0\Big) \longrightarrow 0,\ Pr\Big(\text{ Reject } H_0 \,\Big|T_\zeta(\theta_0) < 0\Big) \longrightarrow 1$$

Therefore,

$$\lim_{n\to\infty} Pr\Big(\text{ Reject } H_0 \,\Big|T_\zeta(\theta_0) \geq 0\Big) \leq \alpha, \quad \text{and} \quad \lim_{n\to\infty} Pr\Big(\text{ Reject } H_0 \,\Big|T_\zeta(\theta_0) < 0\Big) = 1.$$

If we conjecture that α_i is nonincreasing in a_i (and therefore, $H_0 : T_\zeta(\theta_0) \leq 0$), we would use a rule that rejects the null hypothesis if $\widehat{t_\zeta} > z_\tau$. If we rule out the presence of variable costs associated with the incumbent's strategic action, our null hypothesis would be $H_0 : T_\zeta(\theta_0) = 0$ and we would reject it if $|\widehat{t_\zeta}| > z_{\tau/2}$.

5.2 Inference using iterated elimination of nonrationalizable strategies

In Section 3.2 we considered a weaker behavioral model than BNE where we allowed for incorrect beliefs but we restricted them to be consistent with at least k iterated steps of elimination of nonrationalizable strategies. The invertibility condition described in Assumption IG enables us to re-express the bounds derived in Section 3.2. As we described there, we proceed iteratively as follows.

Step 1: Let

$$\begin{aligned}
\underline{F}^1(Z_i, \theta_0) &\equiv \min_{0\leq\pi\leq 1} F_\varepsilon\Big(X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0\mu_S(Z_i,\pi,\gamma_0) - \Delta_0(P_i-1)\pi + \eta_0\gamma_0 C_i\Big),\\
\overline{F}^1(Z_i, \theta_0) &\equiv \max_{0\leq\pi\leq 1} F_\varepsilon\Big(X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0\mu_S(Z_i,\pi,\gamma_0) - \Delta_0(P_i-1)\pi + \eta_0\gamma_0 C_i\Big).
\end{aligned}$$

While beliefs π_i^p for player p may be incorrect, they must satisfy $\underline{F}^1(Z_i,\theta_0) \leq \pi_i^p \leq \overline{F}^1(Z_i,\theta_0)$ for all p. All strategies that are produced by beliefs outside this range are nonrationalizable.

⋮

Step k: Let

$$\begin{aligned}
\underline{F}^k(Z_i, \theta_0) &\equiv \min_{\underline{F}^{k-1}(Z_i)\leq\pi\leq\overline{F}^{k-1}(Z_i)} F_\varepsilon\Big(X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0\mu_S(Z_i,\pi,\gamma_0) - \Delta_0(P_i-1)\pi + \eta_0\gamma_0 C_i\Big),\\
\overline{F}^k(Z_i, \theta_0) &\equiv \max_{\underline{F}^{k-1}(Z_i)\leq\pi\leq\overline{F}^{k-1}(Z_i)} F_\varepsilon\Big(X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0\mu_S(Z_i,\pi,\gamma_0) - \Delta_0(P_i-1)\pi + \eta_0\gamma_0 C_i\Big).
\end{aligned} \tag{8'}$$

Suppose the invertibility condition in Assumption IG holds. If every player p performs at least

$k-1$ steps of iterated elimination of nonrationalizable strategies, (9) can be re-expressed as

$$P_i \cdot \underline{F}^k(Z_i,\theta_0) \le E[C_i|W_i] \le P_i \cdot \overline{F}^k(Z_i,\theta_0) \tag{9'}$$

Inference for θ_0 can be based on (9') using existing methods for inference with *conditional* moment inequalities. Examples of such methods include Andrews and Shi (2013) and Chernozhukov, Lee, and Rosen (2013). The more recent approach developed in Aradillas-Lopez and Rosen (2021) (henceforth AR21) is particularly well suited to this problem and is computationally easy to implement. We will outline how their method would work in this case. Let f_X denote the joint density function of X_i. For a given k and a given $\theta \in \Theta$ and $x \in \mathcal{S}_X$, define the following density-weighted functionals,

$$\overline{m}^k(x,\theta) \equiv E\Big[C_i - P_i \cdot \overline{F}^k(Z_i,\theta)\Big|X_i = x\Big] \cdot f_X(x),$$
$$\underline{m}^k(x,\theta) \equiv E\Big[P_i \cdot \underline{F}^k(Z_i,\theta) - C_i\Big|X_i = x\Big] \cdot f_X(x).$$

Let $\mathcal{X} \subseteq \mathcal{S}_X$ be a pre-specified *inference range*. If every player p performs at least $k-1$ steps of iterated elimination of nonrationalizable strategies, we can characterize the identified set for θ_0 as

$$\Theta_{I,k} \equiv \Big\{\theta \in \Theta : \overline{m}^k(x,\theta) \le 0 \text{ , } \underline{m}^k(x,\theta) \le 0 \text{ for a.e } x \in \mathcal{X}\Big\}.$$

$\Theta_{I,k}$ can be re-defined in a convenient way. Let

$$\underline{T}^k(\theta) \equiv E_X\Big[\max\Big\{\underline{m}^k(X,\theta), 0\Big\} \cdot \mathbb{1}\{X \in \mathcal{X}\}\Big],$$
$$\overline{T}^k(\theta) \equiv E_X\Big[\max\Big\{\overline{m}^k(X,\theta), 0\Big\} \cdot \mathbb{1}\{X \in \mathcal{X}\}\Big],$$
$$T^k(\theta) \equiv \underline{T}^k(\theta) + \overline{T}^k(\theta)$$

Note that $T^k(\theta) \ge 0$ for all θ and $T^k(\theta) = 0$ if and only if $\theta \in \Theta_{I,k}$. Thus, we can re-express our identified set as

$$\Theta_{I,k} \equiv \Big\{\theta \in \Theta : T^k(\theta) = 0\Big\}.$$

The approach studied in AR21 is based on an estimator of $T^k(\theta)$. AR21 consider kernel-based estimators $\widehat{\underline{m}}^k(x,\theta)$ and $\widehat{\overline{m}}^k(x,\theta)$ for the functionals $\underline{m}^k(x,\theta)$ and $\overline{m}^k(x,\theta)$, respectively. Suppose we can partition X as $X = (X^c, X^d)$, where X^c and X^d denote the elements in X that are jointly continuously and jointly discretely distributed, respectively. Let $r \equiv \dim(X^c)$. For a given $x \equiv (x^c, x^d) \in \mathcal{S}_X$ and $\theta \in \Theta$, let

$$\widehat{\underline{m}}^k(x,\theta) = \frac{1}{n \cdot h_n^r}\sum_{i=1}^{n}\Big(P_i \cdot \underline{F}^k(Z_i,\theta) - C_i\Big) \cdot K\left(\frac{X_i^c - x^c}{h_n}\right) \cdot \mathbb{1}\Big\{X_i^d = x^d\Big\},$$
$$\widehat{\overline{m}}^k(x,\theta) = \frac{1}{n \cdot h_n^r}\sum_{i=1}^{n}\Big(C_i - P_i \cdot \overline{F}^k(Z_i,\theta)\Big) \cdot K\left(\frac{X_i^c - x^c}{h_n}\right) \cdot \mathbb{1}\Big\{X_i^d = x^d\Big\}$$

where, as usual, K is a kernel function and $h_n \searrow 0$ is a bandwidth sequence. Having density-weighted functionals has the advantage of not having to divide by $\widehat{f}_X(x)$ in the construction of our estimators. The approach in AR21 would call for estimators of $\underline{T}^k(\theta)$, $\overline{T}^k(\theta)$ and $T^k(\theta)$ of the form,

$$\begin{aligned}
\widehat{\underline{T}}^k(\theta) &= \frac{1}{n}\sum_{i=1}^{n} \widehat{\underline{m}}^k(X_i,\theta)\cdot \mathbb{1}\Big\{\widehat{\underline{m}}^k(X_i,\theta) \geq -b_n\Big\}\cdot \mathbb{1}\{X_i \in \mathcal{X}\},\\
\widehat{\overline{T}}^k(\theta) &= \frac{1}{n}\sum_{i=1}^{n} \widehat{\overline{m}}^k(X_i,\theta)\cdot \mathbb{1}\Big\{\widehat{\overline{m}}^k(X_i,\theta) \geq -b_n\Big\}\cdot \mathbb{1}\{X_i \in \mathcal{X}\},\\
\widehat{T}^k(\theta) &= \widehat{\underline{T}}^k(\theta) + \widehat{\overline{T}}^k(\theta).
\end{aligned}$$

where $b_n \searrow 0$ is a positive bandwidth sequence. Under the regularity, smoothness, bandwidth convergence and bias-reduction kernel restrictions described in Theorem 3 of AR21, $\widehat{T}^k(\theta)$ satisfies a uniform asymptotically linear representation result of the form,

$$\widehat{T}^k(\theta) = T^k(\theta) + \frac{1}{n}\sum_{i=1}^{n}\psi_n^k(Z_i,\theta) + \xi_n(\theta), \quad \text{where} \quad \sup_{\theta\in\Theta}|\xi_n(\theta)| = o_p(n^{-1/2-\Delta})$$

for some $\Delta > 0$. The proof follows the same steps as those of Theorem 3 in AR21. The expression for the influence function $\psi_n^k(Z_i,\theta)$ is obtained from the Hoeffding decomposition (see Serfling (1980, pages 177-178) or Sherman (1994, equations (6)-(7))) of the U-process involved in the construction of $\widehat{T}^k(\theta)$. It satisfies $E[\psi_n^k(Z_i,\theta)] = 0$ for all $\theta \in \Theta$ and, if we let $\sigma_k^2(\theta) \equiv \lim_{n\to\infty}\text{Var}[\psi_n^k(Z_i,\theta)]$, then $\sigma_k^2(\theta)$ has the following features. Define the following subset of $\Theta_{I,k}$,

$$\Theta_{I,k}^* \equiv \Big\{\theta \in \Theta : \overline{m}^k(x,\theta) < 0\ ,\ \underline{m}^k(x,\theta) < 0 \ \text{ for a.e } x \in \mathcal{X}\Big\}$$

$\Theta_{I,k}^*$ is the collection of all elements in $\Theta_{I,k}$ that satisfy the inequalities as *strict* inequalities w.p.1 over $\mathcal{X}$. Thus, we would have $\theta_0 \in \Theta_{I,k}^*$ if beliefs satisfy

$$Pr\Big(\underline{F}^{k-1}(Z_i,\theta_0) < \pi_i^p < \overline{F}^{k-1}(Z_i,\theta_0) \text{ for all } p = 1,\ldots,P_i\Big) = 1.$$

On the other hand, we would have $\theta_0 \in \Theta_{I,k} \setminus \Theta_{I,k}^*$ if, with positive probability, there exist potential entrants whose beliefs correspond to the most pessimistic or to the most optimistic beliefs consistent with $k-1$ steps of iterated elimination of nonrationalizable strategies. That is, if

$$Pr\Big(\pi_i^p = \underline{F}^{k-1}(Z_i,\theta_0) \ \text{ or } \ \pi_i^p = \overline{F}^{k-1}(Z_i,\theta_0) \ \text{ for some } p = 1,\ldots,P_i\Big) > 0.$$

From the structure of the influence function $\psi_n^k(Z_i,\theta)$, we have $\sigma_k^2(\theta) = 0\ \forall\ \theta \in \Theta_{I,k}^*$, and $\sigma_k^2(\theta) > 0\ \forall\ \theta \in \Theta \setminus \Theta_{I,k}^*$. Thus, under the conditions of Theorem 3 in AR21, the statistic $\widehat{T}^k(\theta)$ would have the following properties,

$$
\begin{aligned}
&(A) \quad \theta \notin \Theta_{I,k} \implies n^{1/2} \cdot \widehat{T}^k(\theta) \xrightarrow{p} +\infty, \\
&(B) \quad \theta \in \Theta^*_{I,k} \implies n^{1/2} \cdot \widehat{T}^k(\theta) \xrightarrow{p} 0, \\
&(C) \quad \theta \in \Theta^*_{I,k} \implies n^{1/2} \cdot \widehat{T}^k(\theta) \xrightarrow{d} \mathcal{N}\left(0, \sigma_k^2(\theta)\right).
\end{aligned}
$$

Derived from these pivotal asymptotic properties, the results in Theorem 3 in AR21 would then call for constructing a confidence set (CS) for θ_0 based on a statistic of the form,

$$
\widehat{t}(\theta) = \frac{\sqrt{n} \cdot \widehat{T}^k(\theta)}{\max\{\widehat{\sigma_k}(\theta)\ ,\ c\}},
$$

where $c > 0$ is a pre-specified (small) constant[5] which is introduced to regularize the variance of our statistic (since we would have $\widehat{\sigma_k}(\theta) \xrightarrow{p} 0$ for any $\theta \in \Theta^*_{I,k}$). For a pre-specified asymptotic target coverage probability $1 - \tau$, our CS would be given by

$$
CS_{1-\tau} = \left\{\theta \in \Theta : \widehat{t}(\theta) < z_{1-\tau}\right\},
$$

where $z_{1-\tau}$ is the $(1-\tau)^{th}$ quantile from the Standard Normal distribution. Under the conditions leading to Theorem 4 in AR21, $CS_{1-\tau}$ satisfies,

$$
\lim_{n\to\infty} \inf_{\theta \in \Theta_{I,k}} Pr(\theta \in CS_{1-\tau}) \geq 1 - \tau, \quad \text{and} \quad \lim_{n\to\infty} Pr(\theta \in CS_{1-\tau}) = 0 \ \forall\ \theta \notin \Theta_{I,k}.
$$

Sequence of alternatives θ_n against which $CS_{1-\tau}$ has nontrivial local asymptotic power can also be characterized using the arguments in Theorem 4 in AR21.

6 Empirical example

We will apply our model to study entry decisions into geographic markets (municipalities) in the Mexican ISP industry under the assumption of BNE behavior and the estimation methodology described in Section 5.1. The Mexican ISP industry in Mexico consists of an incumbent firm, which started as a monopolist with widespread geographic presence, and three main competitors who became potential entrants following a telecommunications reform enacted in 2013.

[5]As is shown in AR21, τ could be replaced with a sequence $c_n \searrow 0$ if we assume that $\sigma_k^2(\theta)$ is bounded away from zero for all $\theta \in \Theta \setminus \Theta^*_{I,k}$.

6.1 Background

The Internet Service Providers (ISP) industry in Mexico has been dominated, since its inception, by the América Móvil (AMX) telecommunications group through its subsidiary[6], Telmex (Teléfonos de México). Telmex was the monopoly, state-controlled telephone company in Mexico until 1990 when it was privatized. Shortly after being privatized, Telmex began providing Internet access as an Internet service provider (ISP). As of 2005, Telmex had an ISP market share of approximately 80%.

Prior to 2013 the telecommunication industry in Mexico remained highly concentrated, with AMX capturing more than 70% of the ISP market. Lack of competition led to consistently high prices and a stagnant penetration. To address this, the Mexican Congress approved a sweeping telecommunications reform in 2013 whose main aim was to promote competition and ensure consumer access to telecommunication services[7]. The reform established a new regulator, the *Instituto Federal de Telecomunicaciones* (IFT), with the power to declare preponderance of the dominant firm (AMX) and impose asymmetrical rules between AMX and its competitors. Among the most important new regulations, *AMX was required to expand and provide its competitors access to its ISP infrastructure at competitive rates*. Having inherited a telecommunications monopoly in 1990, AMX owned the vast majority of the existing ISP infrastructure at the time of the Reform.

While the Reform led to the entry of new firms, the ISP industry in Mexico remains highly concentrated, with AMX as the dominant player and with three other main competitors: Grupo Televisa (TEV), Megacable (MEG) and Total Play (TP). In December of 2013, when the Reform was enacted, AMX had an ISP national market share of 71%, while TEV, MEG and TP had market shares of 13%, 7% and 1%, respectively, with the remaining 8% of the ISP market being split between 10 small ISPs. By Q2 of 2020, AMX had a market share of 48%, while TEV, MEG and TP had market shares of 25%, 16% and 10%, respectively, with the remaining 10% of the market being split between 15 small ISPs. As a regulator, the IFT splits geographic ISP markets into *municipios* (municipalities). Mexico has a total of 2,474 municipalities. Out of this universe, according to the IFT, a total of 1,619 municipalities have at least one ISP provider. While this constitutes about 65% of municipalities, it includes 96% of the population in Mexico. Following the IFT's criteria, from now on, we will refer to a municipality as a (geographic) market. Table 1 summarizes the market presence of each one of the main four players in the ISP industry by Q2 of 2020. As we can see there, the geographic presence of AMX is widespread relative to that of its

[6] In the state of Baja California, AMX operates as an ISP through its subsidiary Telnor, which also operates in the state of Sonora. Telmex operates as an ISP in all states except Baja California.

[7] See, for example:
- Malkin, Elisabeth (2013-03-11). "Mexican Leaders Propose a Telecom Overhaul", *New York Times*. URL: https://www.nytimes.com/2013/03/12/business/global/mexican-plan-would-rein-in-phone-and-tv-providers.html
- Estevez, Dolia (2013-05-01). "Mexico's Congress Passes Monopoly-Busting Telecom Bill, Threatening Tycoon Carlos Slim's Business Empire", *Forbes*. URL: https://www.forbes.com/sites/doliaestevez/2013/05/01/mexicos-congress-passes-monopoly-busting-telecom-bill-threatening-tycoon-carlos-slims-business-empire/?sh=412beef4b073

rivals.

Table 1: Proportion of markets (municipalities) where each one of the main four ISP players had presence by Q2 of 2020.

AMX	TEV	MEG	TP
97.9%	12.4%	17.2%	29.1%

• The proportion shown is with respect to the 1,619 municipalities in Mexico that had at least one ISP in 2020.
• Source: *Instituto Federal de Telecomunicaciones* (IFT).

6.2 Players, sample and data used

We will apply our model to study entry decisions into geographic markets (municipalities) in the Mexican ISP industry under the assumption of BNE behavior and the estimation and inference methodology described in Section 5.1. Since AMX has presence in almost all markets with an ISP, we will treat this firm as the *incumbent*. We will focus on the three main competitors of AMX as the game's potential entrants: TEV, MEG and TP. We will leave the other smaller ISPs (15 of which split 8% of the market in 2020) as non-strategic agents, or at least as agents whose decisions are outside of our main entry game. Note that we will then have $P_i = 3$ in every market. Our results do not rely on variation on P_i, so having a fixed number of players is entirely compatible with our model and the estimator proposed in Section 5.1. We will use our model to study entry decisions observed by Q2 of 2020.

The source of our data for ISP market shares and market presence is the[8] *Banco de Información de Telecomunicaciones* of Mexico's IFT. Smaller markets in Mexico are poor, rural and have very little ISP penetration and our model is perhaps not a good approximation for them. For this reason our analysis will be focused on the universe of markets in Mexico that fit the following criteria: all markets whose median income per-capita was above the 25th percentile of the national income per-capita distribution, whose total population was above the 25th percentile among municipalities, and whose number of establishments per-capita was also above the 25th percentile among municipalities. This criteria defines our population of interest and it yielded a sample of $n = 230$ markets. We did not trim this universe any further, so our inference range $\mathcal{X}$ corresponded to our entire sample.

Our sample represents approximately 89% of the population in Mexico in 2020 and 94% of the total number of ISP access points. Table 2 summarizes the statistical characteristics of the

[8] The data can be downloaded at https://bit.ift.org.mx/BitWebApp/descargaDatos.xhtml under the section "Servicio Fijo de Acceso a Internet".

incumbent's market share and the number of entrants per market in our sample. The average and median incumbent market shares were slightly below 50%, and this market share ranged from a minimum of around 12% to a maximum of around 85%, with 25th and 75 percentiles of 39% and 57%, respectively. With regard to entry decisions, each market had at least one potential entrant and the median number of entrants was 2. There were about 7% of markets with only one entrant and 6% of markets with three entrants. 86.5% of markets had two entrants.

Table 2: Summary statistics in our sample of 230 markets for the incumbent AMX's market share and entry decisions by the three potential entrants: TEV, MEG and TP.

Summary statistics for the incumbent's market share					
Average	Median	Minimum	25th percentile	75th percentile	Maximum
48.1%	46.0%	11.8%	39.0%	56.9%	85.1%

Summary statistics for the number of entrants per market					
Average number of entrants	Median number of entrants	Proportion of markets with no entrants	Proportion of markets with one entrant	Proportion of markets with two entrants	Proportion of markets with three entrants
1.98	2	0%	7.4%	86.5%	6.1%

• Source: *Instituto Federal de Telecomunicaciones* (IFT).

The market-level observable characteristics included in our estimation were the following. $X_{1i} \equiv$ total population in market i, $X_{2i} \equiv$ population density in market i, $X_{3i} \equiv$ number of economic establishments per capita in market i, and $X_{4i} \equiv$ GDP per capita in market i. Our source of economic information was Mexico's INEGI. There are no available player-specific observable characteristics at the market level. We used $X_i = (1, X_{1i}, X_{2i}, X_{3i}, X_{4i})$, which includes an intercept. We applied the GMM estimation procedure described in Section 5.1.1. In our specification, we modeled both $G(\cdot)$ and $F_\varepsilon(\cdot)$ as corresponding to the Logistic cdf. Given our assumptions, the function $G(\cdot)$ is only restricted to be invertible and bounded in $[0,1]$. The parametric specification in an application is a modeling choice for the econometrician. In our empirical example we used a logistic cdf. This is an intuitive choice, and its functional form and properties facilitated the computation of Bayesian Nash equilibria in the implementation of our estimation procedure and counterfactuals. As described in (15), our instruments included X_i along with transformations of the collection of additional instruments, $\mathcal{I}(Z_i, \gamma) = \left(G^{-1}(S_i) + X_{\ell i} X_{ki} + \gamma C_i\right)_{\ell \geq 1, k \geq 1, \ell \neq k}$. Thus, we have six additional instruments and, combined with X_i, we have a total of 10 instruments. We computed the usual two-step efficient GMM with the identity matrix as the weight matrix in the first step (see Newey and McFadden (1994, Section 6)).

6.3 Estimation results

The data and code we used to obtain all the results that follow can be found and downloaded online at `http://www.personal.psu.edu/aza12/data_code_entry_incumbent.html`. As Table 3 shows, all three of our strategic effects are statistically significant in our data, and the same is true for the effect of the unobserved market characteristic α_i on the potential entrants' decisions (measured by η_0). Importantly, the statistical significance of our estimator for δ_0 satisfies part (i) of Assumption E3. Our results also show that $\eta_0 = 1$ is included in a 95% CI for this parameter, suggesting that the effect of the unobserved market characteristic α_i is the same in both the incumbent's market share (equation (1)) and in the potential entrants' vNM payoff functions (equation (2)).

Table 3: Estimation results. Estimates and 95% confidence intervals.

γ_0	δ_0	Δ_0	η_0
6.422	5.891	5.968	0.967
[3.376 , 9.469]	[1.198 , 10.583]	[2.324 , 9.614]	[0.429 , 1.506]

The results in Table 3 suggest that the three strategic effects are very similar in magnitude. This is confirmed when we compute a Wald test for the joint null hypothesis $H_0 : \gamma_0 = \Delta_0$ and $\delta_0 = \Delta_0$. The statistic was equal to 0.056, with a p-value of 0.972, failing to reject the conjecture that all strategic effect coefficients have the same value. As we discussed in Section 4.1, a similar magnitude in all strategic coefficients will be conducive to BNE uniqueness. We will conduct a test for BNE uniqueness in Section 6.3.2 and our results will show that the conditions of Result 2, which suffice for BNE uniqueness, are satisfied in every market in our data.

6.3.1 Testing the joint null hypothesis that $\beta_0^m = \beta_0^c$ and $\eta_0 = 1$

While we cannot separately identify the original slope coefficients β_0^c and β_0^m that determine the incumbent's market share and the entry decisions, we can test the joint null hypothesis that $\beta_0^m = \beta_0^c$ and $\eta_0 = 1$ as we discussed in Section 5.1.2, by testing the null hypothesis that $\beta_0 = 0$ and $\eta_0 = 1$. The Wald statistic for this joint null hypothesis was 41.79, with a p-value of 0.000. However, if we exclude the intercept and we test this for the slope coefficients only, the Wald statistic is 3.64, with a p-value of 0.60. Thus, our results suggest that, up to a constant shift (i.e, an intercept), we have $X_i'\beta_0^c + \eta_0\alpha_i = X_i'\beta_0^m + \alpha_i$, so both the incumbent's market share (equation (1)) and the potential entrants' vNM payoff functions (equation (2)) depend on the same socioeconomic market index.

Construction of confidence intervals for economic quantities of interest

We will construct confidence intervals (CIs) for various economic measures of interest. These are constructed as projections from a confidence set (CS) of our parameter θ_0. Our CS with target coverage probability $1-\tau$ was constructed in the usual way, as

$$CS_{1-\tau} = \left\{\theta \in \Theta : n\left(\widehat{\theta}-\theta\right)' \widehat{\Omega}_\theta^{-1} \left(\widehat{\theta}-\theta\right) \le \chi^2_{9,1-\tau}\right\},$$

where $\chi^2_{9,1-\tau}$ is the $(1-\tau)^{th}$ quantile of the χ^2_9 distribution. Consider next a scalar economic measure of interest, labeled $\widehat{\Gamma}(\theta_0)$. We construct a CI for it by taking the projection of $CS_{1-\tau}$ on to $\widehat{\Gamma}(\theta)$. That is, if we let $\Gamma_L \equiv \min\left\{\widehat{\Gamma}(\theta) : \theta \in CS_{1-\tau}\right\}$ and $\Gamma_U \equiv \max\left\{\widehat{\Gamma}(\theta) : \theta \in CS_{1-\tau}\right\}$, then our CI for $\widehat{\Gamma}(\theta_0)$ with target coverage probability $1-\tau$ is $[\Gamma_L\ ,\Gamma_U]$. The target coverage probability we use in each case is 95%.

6.3.2 Evidence of BNE uniqueness

Our GMM estimator presupposes that each market in our inference range has a unique regular BNE. In our estimation process we did not find any market with multiple equilibria. Our discussion in Section 4.1 suggested that multiple BNE requires that at least one of the strategic parameters γ_0 or δ_0 be greater than Δ_0. However, as we discussed previously, our results cannot reject the null hypothesis that all three parameters have the same value, further building up evidence that our game has a unique BNE in our data. To explore this a bit further, Figure 4 depicts the BNE equilibrium system for several combinations of the incumbent's market share S_i and the number of entrants C_i using our estimation results and evaluating X_i at median(X_i). As we can see in each instance, the slope of the curve $F_Y(Z_i,\pi,\widehat{\theta})$ is negative for all $\pi \in [0,1]$, ensuring the existence of a unique BNE in every instance.

While these results are supportive of BNE uniqueness, Result 2 provides a sufficient condition to *ensure* that every market has a unique BNE. The result states that a sufficient (but not necessary) condition for the game to have a unique regular BNE in market i is if $\frac{\partial H(Z_i,\pi,\theta_0)}{\partial \pi} \ge 0$ for all $\pi \in [0,1]$, where the expression for $\frac{\partial H(Z_i,\pi,\theta_0)}{\partial \pi}$ is given in equation (13). Let

$$\widehat{\mathscr{I}}(\theta) = \frac{1}{n}\sum_{i=1}^{n}\int_0^1 \min\left\{\frac{\partial H(Z_i,\pi,\theta)}{\partial \pi}\ ,0\right\} d\pi.$$

By construction, $\widehat{\mathscr{I}}(\theta) \le 0$ for any θ and $\widehat{\mathscr{I}}(\theta) = 0$ if and only if $\frac{\partial H(Z_i,\pi,\theta)}{\partial \pi} \ge 0$ for all $\pi \in [0,1]$ for each market $i = 1,\ldots,n$. By Result 2, this guarantees that every market in our sample has a unique BNE for θ. Our 95% CI for $\widehat{\mathscr{I}}(\theta_0)$ consisted of the single point $\{0\}$, meaning that we had $\widehat{\mathscr{I}}(\theta) = 0$ for every $\theta \in CS_{0.95}$. Thus, by Result 2, every element of our CS produced a unique BNE in each market in our sample. We conclude that we have robust evidence of BNE uniqueness in our data.

6.4 Testing for evidence of strategic behavior by the incumbent, AMX

6.4.1 Deployment and sharing of infrastructure by AMX

Having inherited a telecommunications monopoly in 1990, AMX owned the vast majority of the existing ISP infrastructure at the time of the telecommunications reform in 2013. The latter required AMX to meet infrastructure deployment benchmarks in order to enhance the penetration of telecommunication services, and to open its infrastructure to industry rivals in order to promote competition. However, AMX's competitors have repeatedly complained that AMX has deliberately failed to meet its deployment commitments and has not provided effective access to this infrastructure. After repeated complains from its competitors, AMX was fined[9] USD 65.3 million by the IFT in January 2020 for "failing to share information about the availability of its telecom infrastructure, such as posts, with competitors". AMX's failure to deploy and share its infrastructure under the terms stipulated in the 2013 Reform have been repeatedly identified as a barrier to entry by its competitors and by industry experts[10]

6.4.2 Implementing the test for strategic behavior described in Section 5.1.3

Unfortunately, there is no source of information about infrastructure deployment and availability at the market level, but we can study whether there is evidence of strategic behavior by AMX within our model by applying the analysis of Section 5.1.3. To this end, let us associate the unobserved incumbent strategy a_i with "infrastructure deployment and sharing" by AMX and let us assume that, all else equal, expanding infrastructure deployment leads to a better quality of service and attracts more consumers. Since our results indicate that $\eta_0 > 0$, this would imply that the unobserved market characteristic α_i is nondecreasing in a_i. Infrastructure deployment is costly, so in our analysis we should allow for the presence of (unobserved) variable costs of deployment.

[9]See:

- https://www.reuters.com/article/us-mexico-americamovil/mexicos-america-movil-fined-by-regulator-calls-it-illegal-and-disproportionate-idUSKBN1ZQ2FI

[10]The telecommunications consulting firm The Competitive Intelligence Unit has pointed out that AMX invested only 6.1% of its total revenue in infrastructure in 2020, while this figure represented an average of 30% for its competitors, and that AMX's share of the total infrastructure investment in the Mexican telecom industry declined from 55% in 2013 to 25% in 2020 even though AMX's share of the industry's total revenues remained above 60% throughout this period. A detailed discussion of infrastructure sharing as a barrier to entry in the telecommunications industry in Mexico can be found in:

- https://publications.iadb.org/publications/english/document/Digital-Transformation-Infrastructure-Sharing-in-Latin-America-and-the-Caribbean.pdf
- https://www.theciu.com/publicaciones-2/2021/10/25/barreras-a-la-competencia-efectiva-en-telecomunicaciones-reconcentracin-de-mercado-desregulacin-y-espectro-oneroso

Strategic behavior by AMX would then imply the inequality

$$\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i} \geq 0.$$

As in Section 5.1.3, let ζ be a pre-specified, strictly positive real-valued function, and define

$$T_{\zeta}(\theta_0) \equiv E\left[\frac{\partial \overline{\mu}_S(Z_i,\theta_0)}{\partial \alpha_i} \cdot \zeta(X_i)\right]$$

From our results in Section 5.1.3, strategic behavior by AMX implies $T_{\zeta}(\theta_0) \geq 0$. We test this hypothesis using the test-statistic described in equation (17) for various choices of the function ζ. The results are presented in Table (4). As we can see there, we failed to reject the null hypothesis of strategic behavior in all cases. Through our game, our results are consistent with the conjecture that AMX behaves strategically in a way that maximizes its market share and deters entry by its rivals.

Table 4: Testing strategic behavior from the incumbent allowing for "variable costs". Test-statistic $\widehat{t_{\zeta}}$ and p-value in parenthesis for $H_0 : T_{\zeta}(\theta_0) \geq 0$ for various choices of the function ζ

$\zeta(X)=1$	$\zeta(X)=\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert$	$\zeta(X)=\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^2$	$\zeta(X)=\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^4$
6.477	6.888	5.936	2.789
(1.000)	(1.000)	(1.000)	(0.997)

$\zeta(X)=\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^6$	$\zeta(X)=\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^8$	$\zeta(X)=\exp\left\{\sum_{\ell=1}^{4} X_\ell\right\}$	$\zeta(X)=\exp\left\{-\sum_{\ell=1}^{4} X_\ell\right\}$
1.558	1.209	6.862	6.014
(0.940)	(0.887)	(1.000)	(1.000)

$\zeta(X)=1+\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert + \lvert\left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^2 + \left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^4 + \left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^6 + \left\lvert\sum_{\ell=1}^{4} X_\ell\right\rvert^8 + \exp\left\{\sum_{\ell=1}^{4} X_\ell\right\} + \exp\left\{-\sum_{\ell=1}^{4} X_\ell\right\}$
1.389
(0.918)

- Test-statistic $\widehat{t_{\zeta}}$ constructed as described in equation (17).
- For a target significance level of τ, the null hypothesis of strategic behavior, $H_0 : T_{\zeta}(\theta_0) \geq 0$ would be rejected if $\widehat{t_{\zeta}} < -z_{1-\tau}$, where $z_{1-\tau}$ is the $N(0,1)$ $(1-\tau)^{th}$ quantile.
- p-values are computed as $\Phi(\widehat{t_{\zeta}})$, where Φ denotes the $N(0,1)$ cdf.

6.5 Counterfactual analysis

Here we will perform two types of counterfactual experiments. Section 6.5.1 analyzes the impact on the probability of entry of an exogenous shock to players' beliefs about the incumbent's expected market share. Section 6.5.2 considers an exogenous increase in the market's unobserved characteristic α_i and recomputes the BNE in each market.

6.5.1 Impact of an exogenous change in the incumbent's expected market share on the probability of entry

Here we measure how entry probabilities would react to an exogenous shock to beliefs about the incumbent's expected market share of $\Delta\mu_S$ across all markets. We consider a scenario where, in each market i, the expected market share of the incumbent suddenly changes from its BNE level $\mu_S(Z_i, \pi^*(Z_i, \theta_0), \gamma_0)$ to $\mu_S(Z_i, \pi^*(Z_i, \theta_0), \gamma_0) + \Delta\mu_S$. Note that this counterfactual considers a perturbation to the current equilibrium brought about by an exogenous shock to beliefs. We construct a confidence interval (CI) for the median change in the probability of entry across all markets. That is, a CI for the median value of

$$F_\varepsilon\Big(X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0(\mu_S(Z_i, \pi^*(Z_i, \theta_0), \gamma_0) + \Delta\mu_S) - \Delta_0(P_i - 1)\pi^*(Z_i, \theta_0) + \eta_0\gamma_0 C_i\Big)$$
$$-F_\varepsilon\Big(X_i'\beta_0 + \eta_0 G^{-1}(S_i) - \delta_0\mu_S(Z_i, \pi^*(Z_i, \theta_0), \gamma_0) - \Delta_0(P_i - 1)\pi^*(Z_i, \theta_0) + \eta_0\gamma_0 C_i\Big).$$

across all markets in our sample. Table 5 presents results for $\Delta\mu_S = 0.01, 0.02, 0.05$ and 0.10. We see there that, even a change of one percentage point can produce a statistically significant decline in the probability of entry, and that the impact in entry probabilities can be more than twice as large as the original change in the incumbent's expected market share. Our results suggest that even moderate increases in players' beliefs about the incumbent's expected market share can significantly deter entry.

Table 5: Counterfactual experiment. 95% confidence intervals for the change in entry probabilities resulting from an exogenous increment of $\Delta\mu_S$ percentage points in the incumbent's expected market share.

Median change across markets for the probability of entry			
$\Delta\mu_S = 0.01$	$\Delta\mu_S = 0.02$	$\Delta\mu_S = 0.05$	$\Delta\mu_S = 0.10$
[-0.023, -0.004]	[-0.045, -0.007]	[-0.113, -0.018]	[-0.223, -0.037]

• Entry probabilities and market share are measured as fractions in $[0, 1]$.

6.5.2 Equilibrium impact of an exogenous change in the market's unobserved characteristic α_i

Here we consider an exogenous increase in the unobserved market characteristic α_i and we recompute the BNE in each market. This is a policy relevant exercise if we assume that α_i is positively associated with the (unobserved) ISP infrastructure available in market i. Under this scenario, an increase in infrastructure deployment would result in a positive shock to α_i. Note first that our inability to estimate α_i precludes us from measuring its scale. However, going back to (11) and expressing

$$\alpha_i = \underbrace{G^{-1}(S_i) + \gamma_0 \cdot C_i}_{\alpha_{a,i}} \underbrace{- X_i' \beta_0^m}_{\alpha_{b,i}} \equiv \alpha_{a,i} + \alpha_{b,i},$$

we see that we can consistently estimate the component $\alpha_{a,i} \equiv G^{-1}(S_i) + \gamma_0 \cdot C_i$. Based on this, we can conduct counterfactual experiments by increasing α_i by a proportion of the component $\alpha_{a,i}$. Our exercise consists of considering an increment in α_i of magnitude $\Delta\alpha_i = \alpha_{a,i} \times \tau$ for different values of τ and re-computing the BNE in each market. The results of our experiment are presented in Table 6 in the form of 95% CIs for the mean value across markets of (a) the change in the equilibrium probability of entry, and (b) the change in the equilibrium expected market share for the incumbent. We find that small increments in α_i result in an increase in the incumbent's market share but the effect in the probability of entry is ambiguous, while moderately large increments in α_i result in an unambiguous increase in the probability of entry and may lead to reductions in the incumbent's market share.

Table 6: Counterfactual experiment. 95% confidence intervals for the equilibrium changes resulting from an increment in the unobserved market characteristic of $\Delta\alpha_i = \alpha_{a,i} \times \tau$.

Mean change in the equilibrium probability of entry			
$\tau = 0.05$	$\tau = 0.15$	$\tau = 0.25$	$\tau = 0.35$
[-0.081, 0.028]	[-0.025, 0.096]	[0.063, 0.197]	[0.149, 0.317]
Mean change in the equilibrium expected market share for the incumbent			
$\tau = 0.05$	$\tau = 0.15$	$\tau = 0.25$	$\tau = 0.35$
[0.012, 0.153]	[0.016, 0.206]	[-0.050, 0.187]	[-0.125, 0.146]

• Entry probabilities and expected market shares are measured as fractions in $[0, 1]$.

If we assume that α_i is positively associated with the ISP infrastructure available in market i, our findings are in line with our analysis of the strategic behavior of AMX in Section 6.4, where we found evidence consistent with the conjecture that AMX has strategically held back the deployment and sharing of infrastructure as a way to deter entry and keep its market share high. As we discussed before, there is documented evidence of this conduct and it has resulted in significant fines imposed on AMX by the Mexican telecommunication authorities.

7 Concluding remarks

An important limitation of most papers that estimate entry games with incomplete information games is the assumption that players' beliefs are conditioned on observable covariates to the econometrician. This paper contributes to the literature by considering a model where beliefs are conditioned on an unobservable market characteristic that cannot be estimated. Our entry game is characterized by the presence of an incumbent and a collection of symmetric potential entrants. We described conditions under which, even though the unobserved characteristic used to construct beliefs cannot be identified, we can still identify and estimate a subset of parameters of the model, including all the strategic-interaction effects. Our results do not rely on a parametric specification of the distribution of the unobserved market characteristic. We also described testable implications that would arise if the incumbent is behaving strategically in a way that shifts the unobserved market characteristic in order to deter entry and maximize its market share. We studied identification under the assumption of Bayesian Nash equilibrium (BNE) behavior and we also discussed inference under the weaker assumption of iterated elimination of nonrationalizable strategies.

Our results relied on the assumption of invertibility of the function $G(\cdot)$. This, in turn, allowed us to recover the unobserved market-level effect α_i. Dropping the assumption of invertibility could be done if we are willing to put more structure on α_i. In the context of our model, the route we would take would be to model α_i explicitly as an (unobserved) strategy of the incumbent. This would require additional assumptions about the nature of this strategic choice; specifically, it would require us to model explicitly the objective function that the incumbent is maximizing. Invertibility of $G(\cdot)$ allowed us to be agnostic about the exact nature of α_i, while being able to construct a test for whether this market-level effect is associated with strategic choices by the incumbent without having to specify a complete behavioral model. Our model relies on the assumption that the unobserved shock is market-specific and observed by all players. A significant extension, which is beyond the scope of this paper, would be to allow for players to observe different shocks, while allowing them to be correlated and leaving their distribution nonparametrically specified. One potential way to approach this problem might be to impose additional structure to the model that may produce, in turn, invertibility conditions that may allow us to recover these shocks nonparametrically in a way that generalizes the approach of this paper. Allowing for players' private information to be correlated in a nonparametric way remains an area of research opportunity in the econometric analysis of incomplete-information games.

As an empirical illustration, we applied our model to study entry decisions into geographic markets in the Mexican ISP industry under the assumption of BNE behavior. The market structure of our empirical example fits the description of our model, with an incumbent (AMX) that started as a national monopoly and was able to solidify a widespread geographic presence by the time competition was encouraged in 2013 through a telecommunications reform in Mexico. Following the Reform, AMX faces competition from three main ISP potential entrants across geographic markets in Mexico. We estimated the parameters of our game and found statistically significant evidence for all three types of strategic-interaction effects present in our model. Our results show that entry decisions are significantly impacted by the incumbent's expected market share, as well as by the expected number of entrants in the market. In turn, we also found that the incumbent's expected market share is significantly impacted by the expected number of entrants in each market.

Assuming that the unobserved market characteristic is positively associated with the ISP infrastructure available in the market, our model also produced statistically significant evidence in support of the assertion that AMX has strategically held back the deployment and sharing of infrastructure to deter entry. These findings are in line with AMX's documented failure to deploy and share its infrastructure with competitors as is required by the 2013 Reform, an anticompetitive conduct which resulted in a significant fine by the Mexican government in 2020. We conducted counterfactual experiments that support the conjecture that increasing the deployment and availability of infrastructure would lead to a higher probability of entry and a decrease in the incumbent's market share, both of which were the main goals of the 2013 telecommunications reform.

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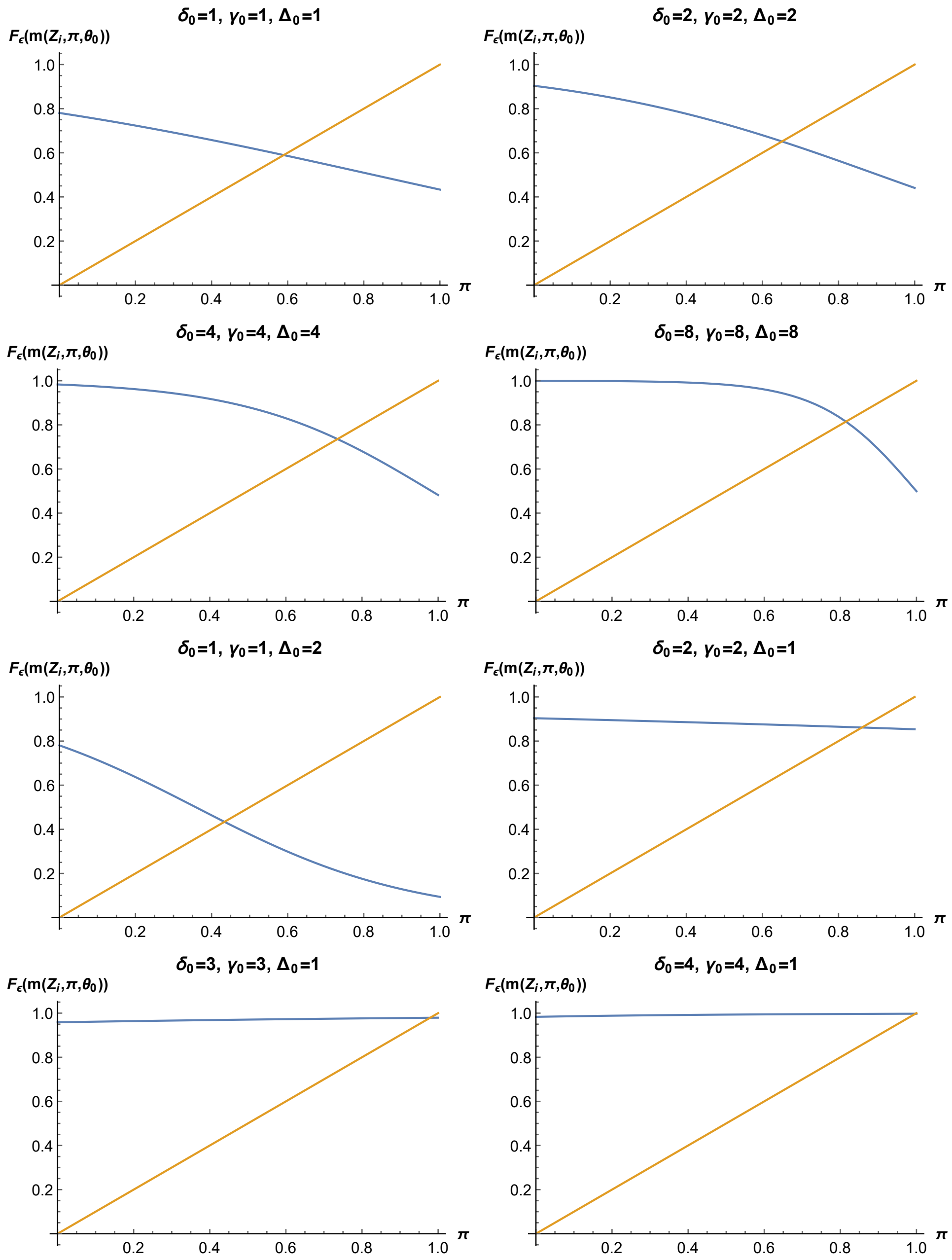
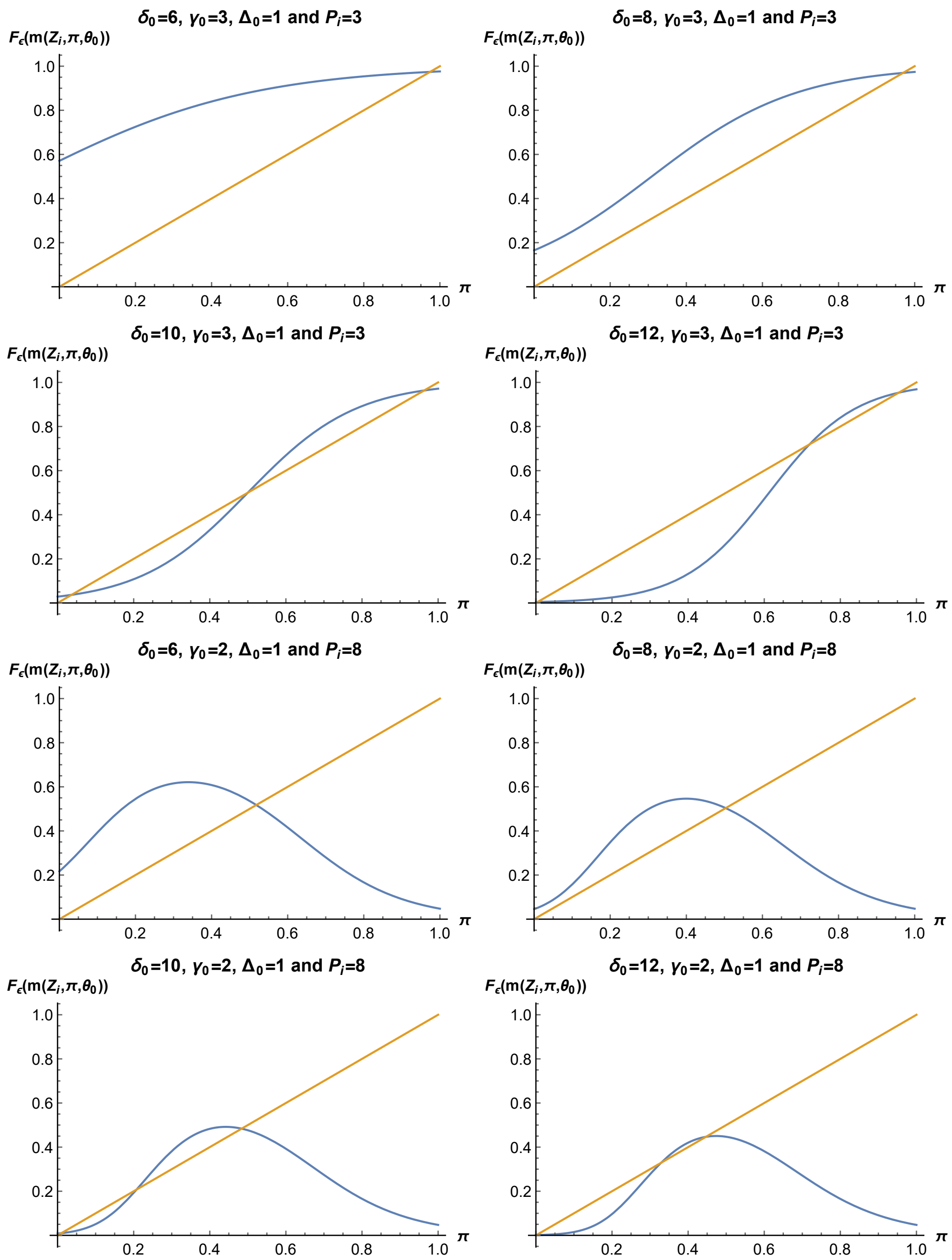
Appendix: Figures

Figure 1: If the strategic effect Δ_0 is not sufficiently dominated by γ_0 and δ_0, this will be conducive to having a unique BNE.



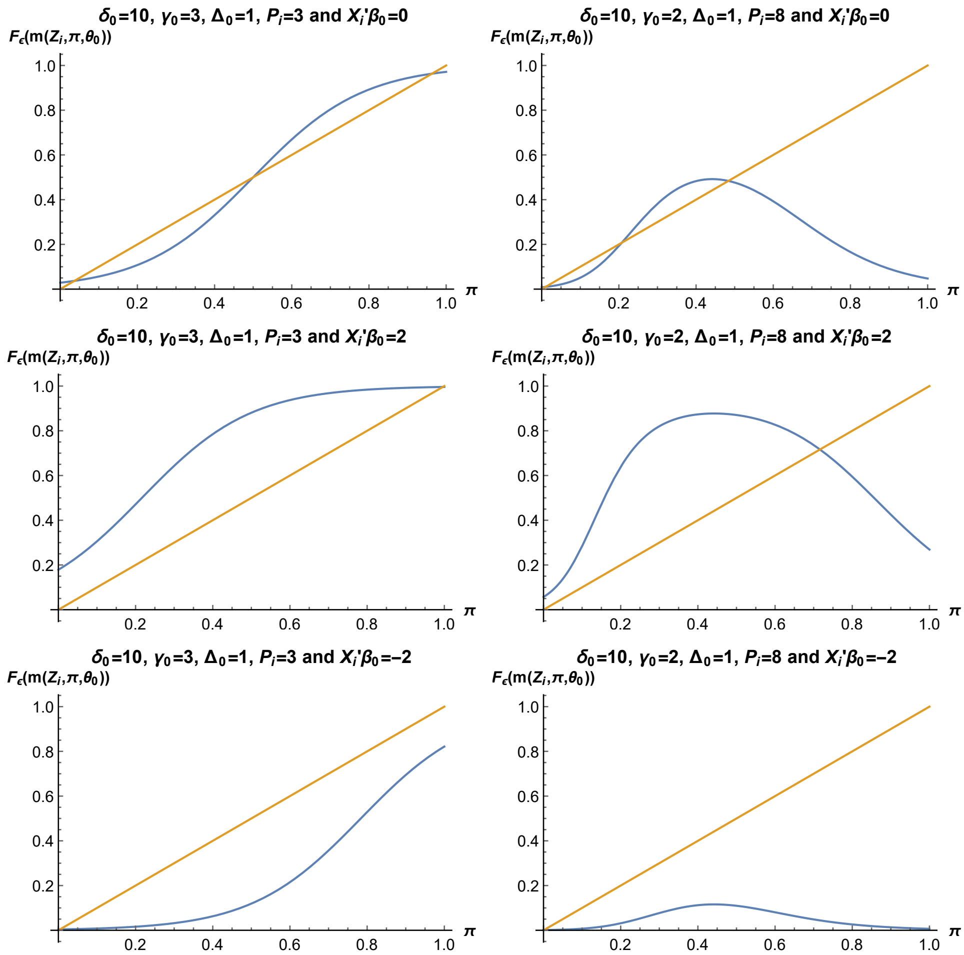
- Each diagram shows the curve $F_\varepsilon(m(Z_i,\pi,\theta_0))$ for $\pi \in [0,1]$ along with the 45-degree line.
- $m(Z_i,\pi,\theta)$ is as defined in (12) and F_ε is the logistic cdf.
- Any point of crossing between both curves constitutes a BNE.
- In all cases shown we have $P_i = 3$, $S_i = 0.5$, $C_i = 2$, $X_i'\beta_0 = 0$ and $\eta_0 = 1$.

Figure 2: Multiple BNE requires that *both* of the strategic effects γ_0 and δ_0 significantly dominate Δ_0. An illustration with $S_i = 0.5$, $C_i = 2$, $\beta_0' X_i = 0$ and $\eta_0 = 1$



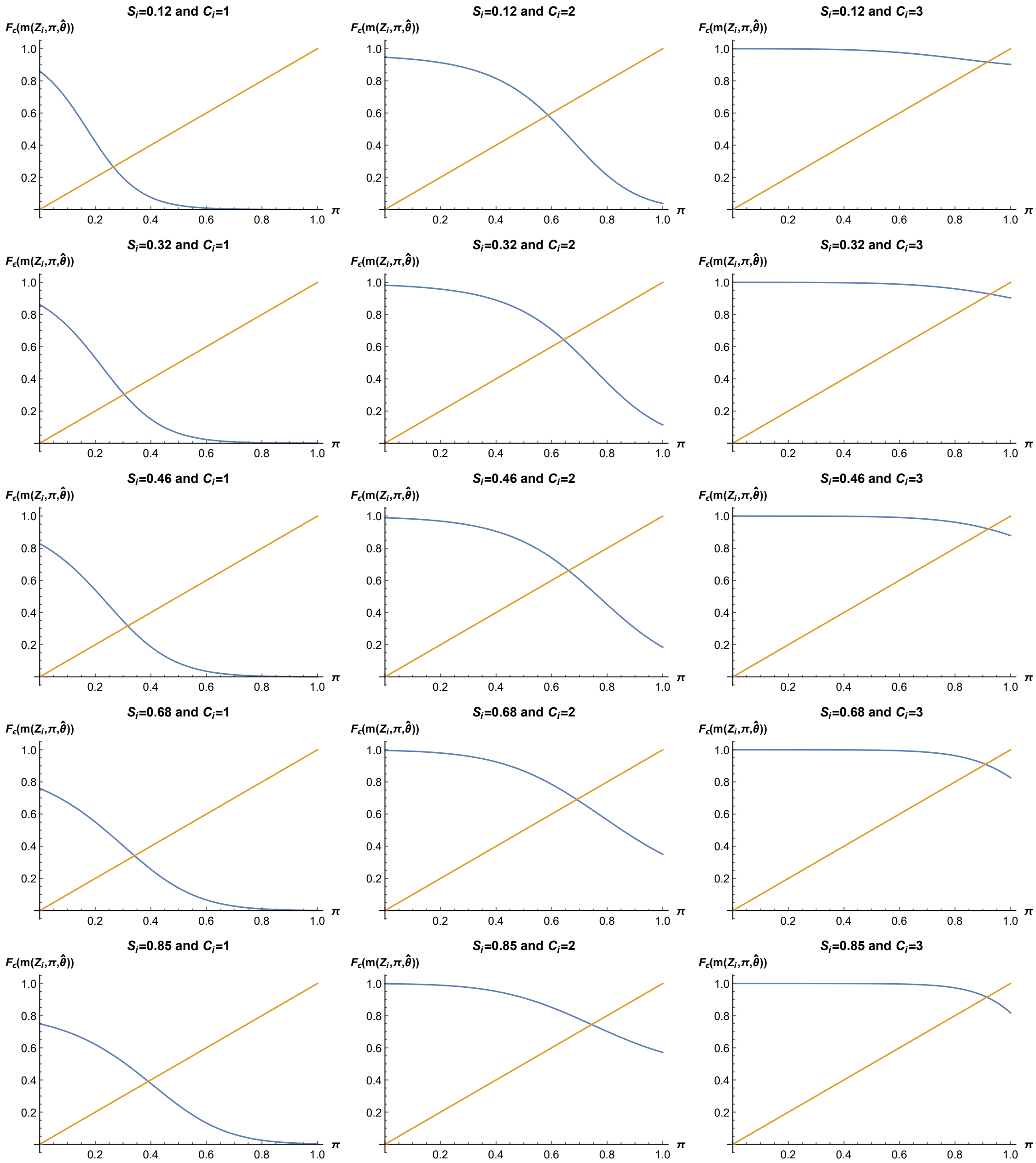
- Each diagram shows the curve $F_\varepsilon(m(Z_i, \pi, \theta_0))$ for $\pi \in [0, 1]$ along with the 45-degree line.
- $m(Z_i, \pi, \theta)$ is as defined in (12) and F_ε is the logistic cdf.
- Any point of crossing between both curves constitutes a BNE.

Figure 3: Larger absolute values of $X_i'\beta_0$ are conducive to generating a unique BNE.



- Each diagram shows the curve $F_\varepsilon(m(Z_i,\pi,\theta_0))$ for $\pi \in [0,1]$ along with the 45-degree line.
- $m(Z_i,\pi,\theta)$ is as defined in (12) and F_ε is the logistic cdf.
- Any point of crossing between both curves constitutes a BNE.
- In all cases shown we have $S_i = 0.5$, $C_i = 2$, and $\eta_0 = 1$.
- The first row has $X_i'\beta_0 = 0$ and shows multiple BNE in two distinct cases.
- The second and third rows show how multiple BNE go away in each of these two cases, when $X_i'\beta_0 = 2$ (second row), and when $X_i'\beta_0 = -2$ (third row).
- Larger absolute values of $X_i'\beta_0$ reduce the variability of the curve $F_Y(Z_i,\pi,\theta_0)$ over $\pi \in [0,1]$, thus producing a unique BNE.

Figure 4: Graphical depiction of the BNE system in our data using our estimation results for $\widehat{\theta}$, with X_i evaluated at median(X_i), for different values of the incumbent's market share S_i and number of entrants C_i.



• Each diagram shows the curve $F_\varepsilon(m(Z_i,\pi,\widehat{\theta}))$ with X_i = median(X_i), for $\pi \in [0,1]$, along with the 45-degree line. $m(Z_i,\pi,\theta)$ is as defined in (12) and F_ε is the logistic cdf.

• Any point of crossing between both curves constitutes a BNE.

• The values use for S_i of 0.12, 0.32, 0.46, 0.68 and 0.85 correspond to the minimum and the maximum, along with the 10^{th}, 50^{th} and 90^{th} quantiles of the incumbent market share observed in the data.