Honoré (1992)

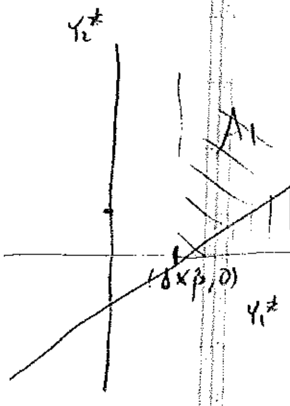
Econ 589

I

i.e., if $\Delta x'\beta \geq 0$:

$$A_1 = \{(Y_1^*, Y_2^*) : Y_1^* > \Delta x'\beta,\ Y_2^* > Y_1^* - \Delta x'\beta\}$$

$$B_1 = \{(Y_1^*, Y_2^*) : Y_1^* > \Delta x'\beta,\ 0 < Y_2^* < Y_1^* - \Delta x'\beta\}$$



$$Y_1^* = \alpha + X_1\beta + \varepsilon_1$$

$$Y_2^* = \alpha + X_2\beta + \varepsilon_2$$

$$Y_1^* > \Delta X'\beta \iff \alpha + X_1\beta + \varepsilon_1 > X_1\beta - X_2\beta$$

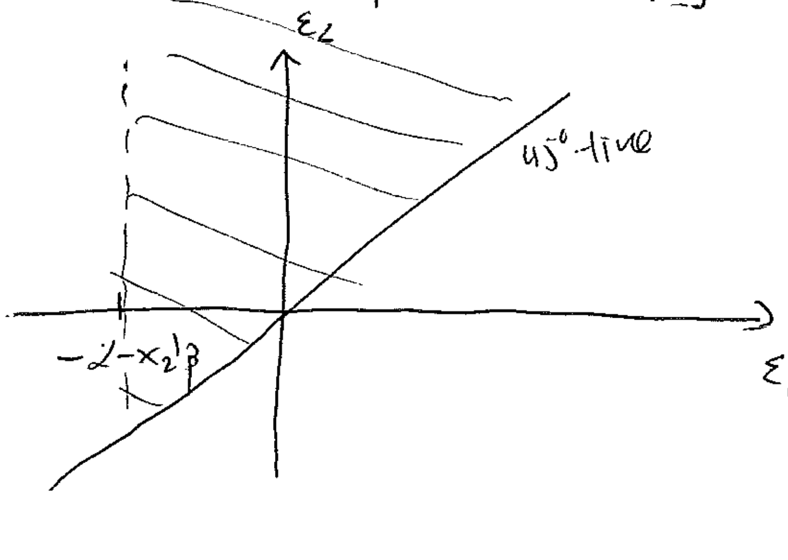
$$\iff \boxed{\varepsilon_1 > -\alpha - X_2\beta}$$

$$Y_2^* > Y_1^* - \Delta X'\beta \iff \cancel{\alpha} + \cancel{X_2\beta} + \varepsilon_2 > \cancel{\alpha} + \cancel{X_1\beta} + \varepsilon_1 - \cancel{X_1\beta} + \cancel{X_2\beta}$$

$$\iff \boxed{\varepsilon_2 > \varepsilon_1}$$

$$\Rightarrow P[(Y_1^*, Y_2^*) \in A_1 \mid X, \alpha] =$$

$$P[\varepsilon_1 > -\alpha - X_2'\beta,\ \varepsilon_2 > \varepsilon_1]$$



II

For B_1:

$$\Pr[0 < Y_2^* < Y_1^* - \Delta X'\beta]$$

$$= \Pr[0 < Y_2^* \quad \text{AND} \quad Y_2^* < Y_1^* - \Delta X'\beta]$$

$$= \Pr[0 < \alpha + X_2'\beta + \varepsilon_2 \quad \text{AND}$$

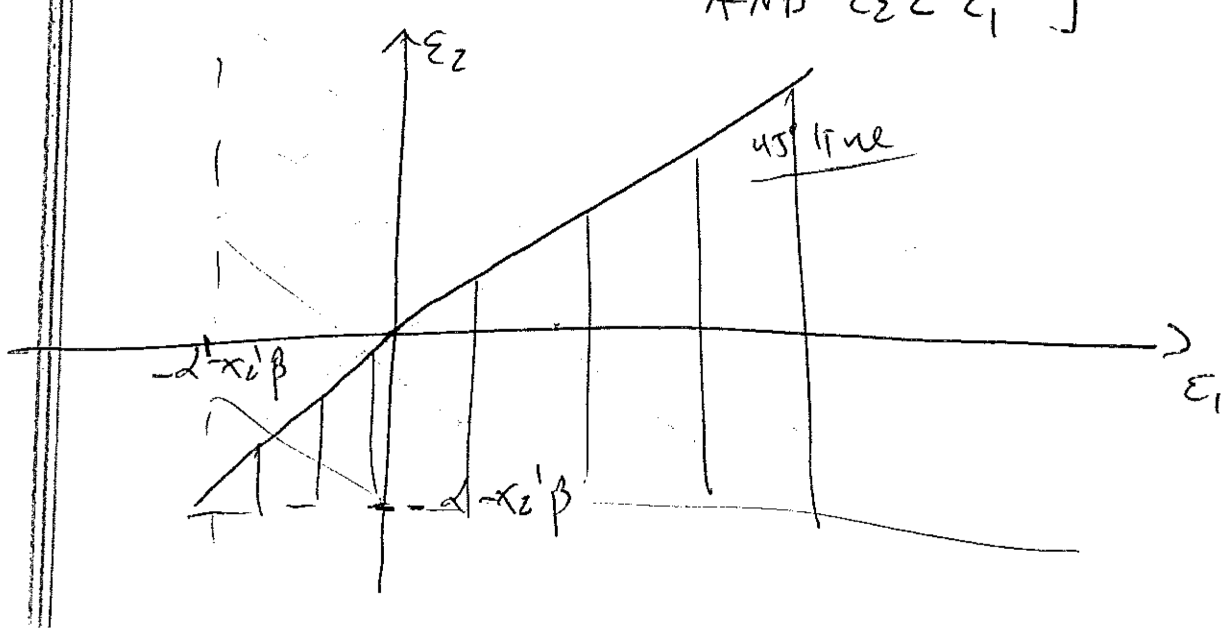
$$\cancel{\alpha} + \cancel{X_2'}\beta + \varepsilon_2 < \cancel{\alpha} + \cancel{X_1'\beta} + \varepsilon_1 - \cancel{X_1'}\beta + \cancel{X_2'\beta}]$$

$$= \Pr[-\alpha - X_2'\beta < \varepsilon_2 \quad \text{AND} \quad \varepsilon_2 < \varepsilon_1]$$

$$\Rightarrow \Pr[(Y_1^*, Y_2^*) \in B_1]$$

$$= \Pr[\varepsilon_1 > -\alpha - X_2\beta \quad \text{AND} \quad \varepsilon_2 > -\alpha - X_2'\beta$$

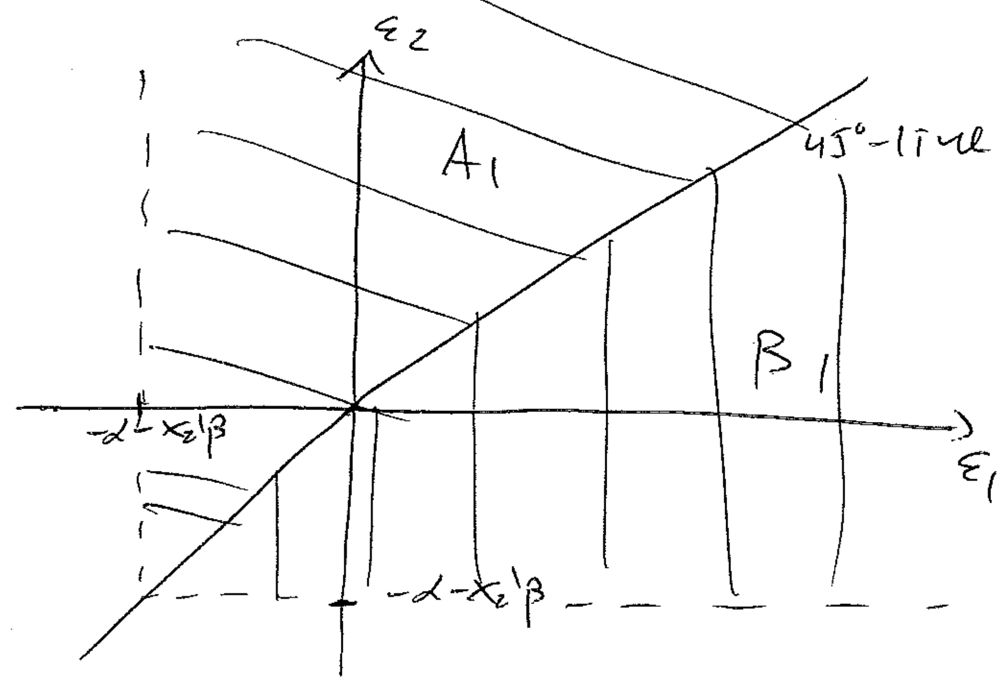
$$\text{AND} \quad \varepsilon_2 < \varepsilon_1]$$

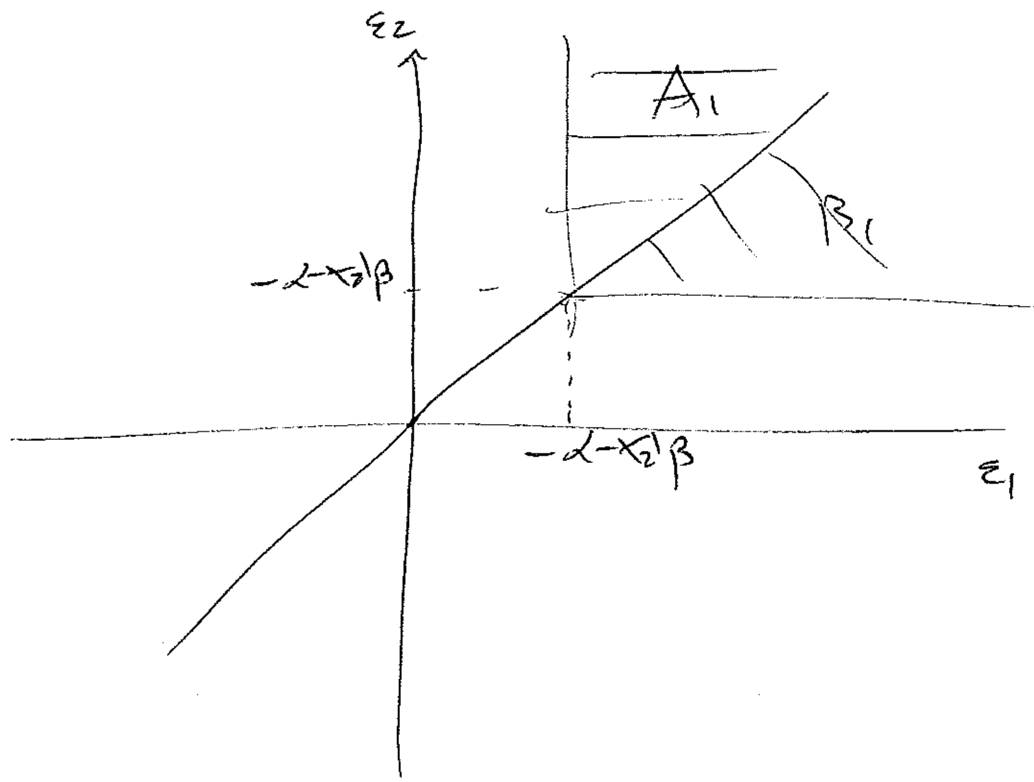


(II)

$\Rightarrow$ if $A_1 = \{(y_1^*, y_2^*) : y_1^* > \Delta x' \beta,\ y_2^* > y_1^* - \Delta x' \beta\}$

$B_1 = \{(y_1^*, y_2^*) : y_1^* > \Delta x' \beta,\ 0 < y_2^* < y_1^* - \Delta x' \beta$





II

if $\Delta x'\beta < 0$:

$A_1 = \{(Y_1^*, Y_2^*) : Y_1^* > 0,\ Y_2^* > Y_1^* - \Delta x'\beta\}$

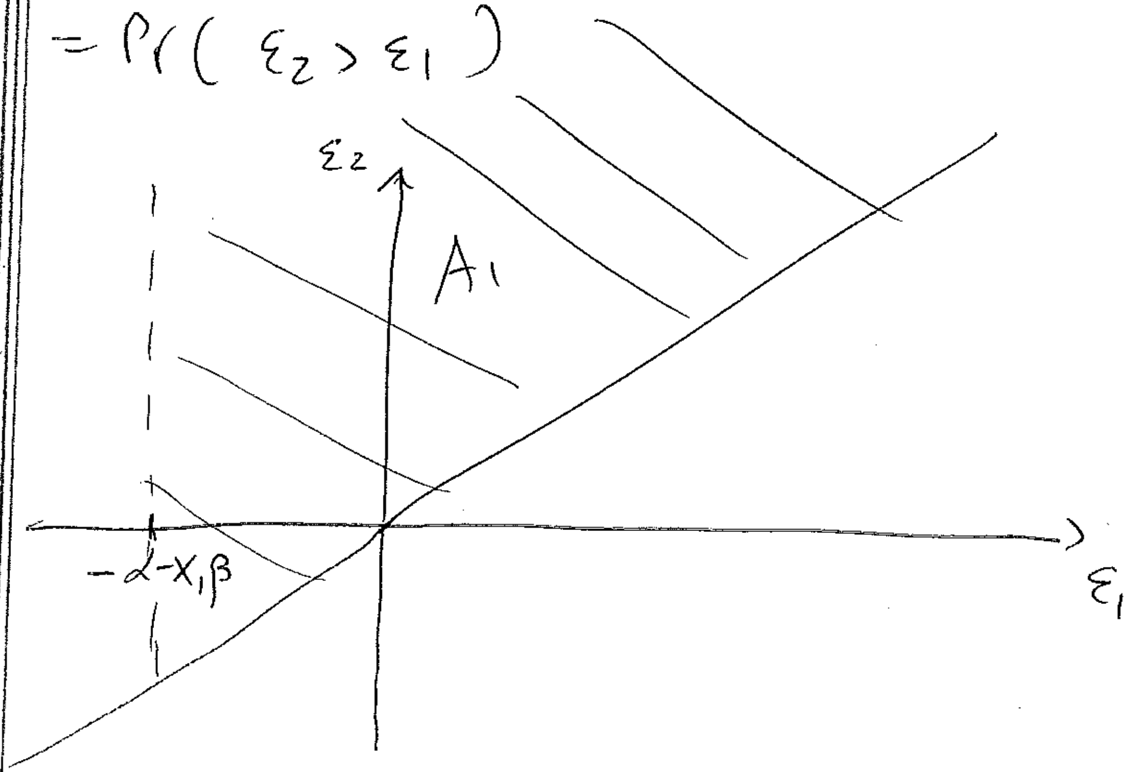
$B_1 = \{(Y_1^*, Y_2^*) : Y_1^* > 0,\ -\Delta x'\beta < Y_2^* < Y_1^* - \Delta x'\beta\}$

$Pr(Y_1^* > 0) = Pr(\alpha + X_1\beta + \varepsilon_1 > 0)$
$= Pr(\varepsilon_1 > -\alpha - X_1\beta)$

$Pr(Y_2^* > Y_1^* - \Delta X\beta)$

$= Pr(\cancel{\alpha} + \cancel{X_2\beta} + \varepsilon_2 > \cancel{\alpha} + \cancel{X_1\beta} + \varepsilon_1 - X_1\beta + \cancel{X_2\beta})$

$= Pr(\varepsilon_2 > \varepsilon_1)$



IV

$$Pr(-\Delta X'\beta < Y_2^* < Y_1^* - \Delta X'\beta)$$

$$= Pr(-X_1''\beta + \cancel{X_2\beta} < \alpha + \cancel{X_2}\beta + \varepsilon_2$$

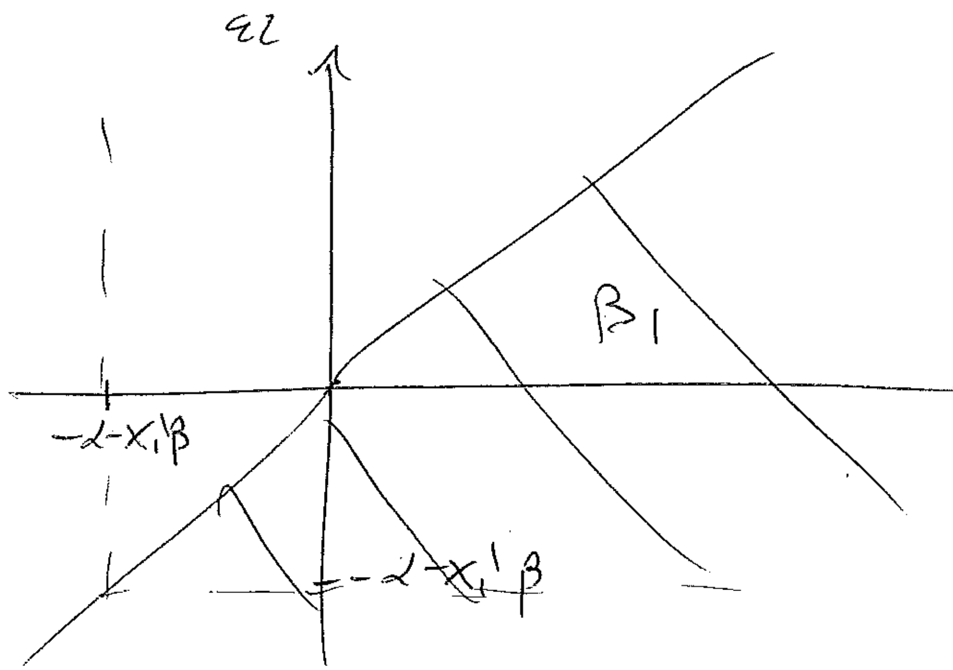
AND

$$\cancel{\alpha} + \cancel{X_2\beta} + \varepsilon_2 < \cancel{\alpha} + \cancel{X_1\beta} + \varepsilon_1 - \cancel{X_1\beta} + \cancel{X_2\beta})$$

$$= Pr(-X_1\beta - \alpha < \varepsilon_2 \text{ AND } \varepsilon_2 < \varepsilon_1)$$

$$\Rightarrow P((Y_1^*, Y_2^*) \in B_1 | X, \alpha)$$

$$= P(\varepsilon_1 > -\alpha - X_1'\beta \text{ AND } -\alpha - X_1\beta < \varepsilon_2 \text{ AND } \varepsilon_2 < \varepsilon_1$$



$$\Rightarrow \Pr[(y_1^*, y_2^*) \in A_1 \mid X, \alpha]$$

$$= \Pr[(y_1^*, y_2^*) \in B_1 \mid X, \alpha]$$

where

$$A_1 \equiv \{(y_1^*, y_2^*) : y_1^* > \max\{\Delta x'\beta, 0\},\ y_2^* > y_1^* - \Delta x'\beta\}$$

$$A_2 \equiv \{(y_1^*, y_2^*) : y_1^* > \max\{\Delta x'\beta, 0\},\ \max\{0, -\Delta x'\beta\} < y_2^* < y_1^* - \Delta x'\beta\}$$

$\Rightarrow$

(1) $$E[\{\Pr((y_1, y_2) \in A_1 \mid X) - \Pr((y_1, y_2) \in B_1 \mid X)\} \cdot \Delta X] = 0$$

→ Next: Expected vertical distance from (y_1^*, y_2^*) in A_1 to LL' equals the expected horizontal distance from (y_1^*, y_2^*) in B_1 to LL', with both expectations conditional on (X_1, X_2).

VI

- For (Y_1^*, Y_2^*) in A_1, the vertical distance to LL' is

$$-(Y_1^* - Y_2^* - \Delta X \beta)$$

- In B_1, the horizontal distance is

$$(Y_1^* - Y_2^* - \Delta X \beta)$$

$\Rightarrow$

$$E[\mathbb{1}\{(Y_1,Y_2)\in A_1\}\cdot(Y_1-Y_2-\Delta X\beta)\mid X]$$

$$= - E[\mathbb{1}\{(Y_1,Y_2)\in B_1\}\cdot(Y_1-Y_2-\Delta X\beta)\mid X]$$

$\Rightarrow$ moment condition:

(2) $$E\Big[\{\mathbb{E}[\mathbb{1}\{(Y_1,Y_2)\in A_1\}\cdot(Y_1-Y_2-\Delta X\beta)\mid X] + E[\mathbb{1}\{(Y_1,Y_2)\in B_1\}\cdot(Y_1-Y_2-\Delta X\beta)\mid X]\}\cdot\Delta X\Big]=0$$

VII

• Censored: $\begin{cases} A_1 \longrightarrow A_1 \cup A_2 \\ B_1 \longrightarrow B_1 \cup B_2 \end{cases}$

(1) now becomes:

(1') $$E\left[\left(1\!\!1\{(Y_1, Y_2) \in A_1 \cup A_2\} - 1\!\!1\{(Y_1, Y_2) \in B_1 \cup B_2\} \right) \Delta X \right] = 0$$

(2) now becomes:

$$\begin{aligned} E\Big[\Big\{ & 1\!\!1\{(Y_1, Y_2) \in A_1\} \cdot (Y_1 - Y_2 - \Delta X \beta) \\ & - 1\!\!1\{(Y_1, Y_2) \in A_2\} \cdot (Y_2 - \max\{0, -\Delta X \beta\}) \\ & + 1\!\!1\{(Y_1, Y_2) \in B_1\} \cdot (Y_1 - Y_2 - \Delta X \beta) \\ & + 1\!\!1\{(Y_1, Y_2) \in B_2\} \cdot (Y_1 - \max\{0, \Delta X \beta\}) \Big\} \Delta X \Big] = 0 \end{aligned}$$

(a)

$$A_1 = \{(y_1^*, y_2^*) : y_1^* > \max\{\Delta X\beta, 0\},\ y_2^* > y_1^* - \Delta X\beta\}$$
$$A_2 = \{(y_1^*, y_2^*) : y_1^* \le \max\{\Delta X\beta, 0\},\ y_2^* > \max\{0, -\Delta X\beta\}\}$$

$$B_1 = \{(y_1^*, y_2^*) : y_1^* > \max\{\Delta X\beta, 0\},\ \max\{0, -\Delta X\beta\} < y_2^* < y_1^* - \Delta X\beta\}$$

$$B_2 = \{(y_1^*, y_2^*) : y_1^* > \max\{\Delta X\beta, 0\},\ y_2^* \le \max\{0, -\Delta X\beta\}\}$$

- Vertical distance from any point in A1 to LL' line:

$$-(Y_1^* - Y_2^* - \Delta X\beta) \Big\} \text{ this is } > 0$$

- Horizontal distance from any point in B1 to LL' line:

$$(Y_1^* - Y_2^* - \Delta X\beta) \Big\} \text{ this is } > 0$$

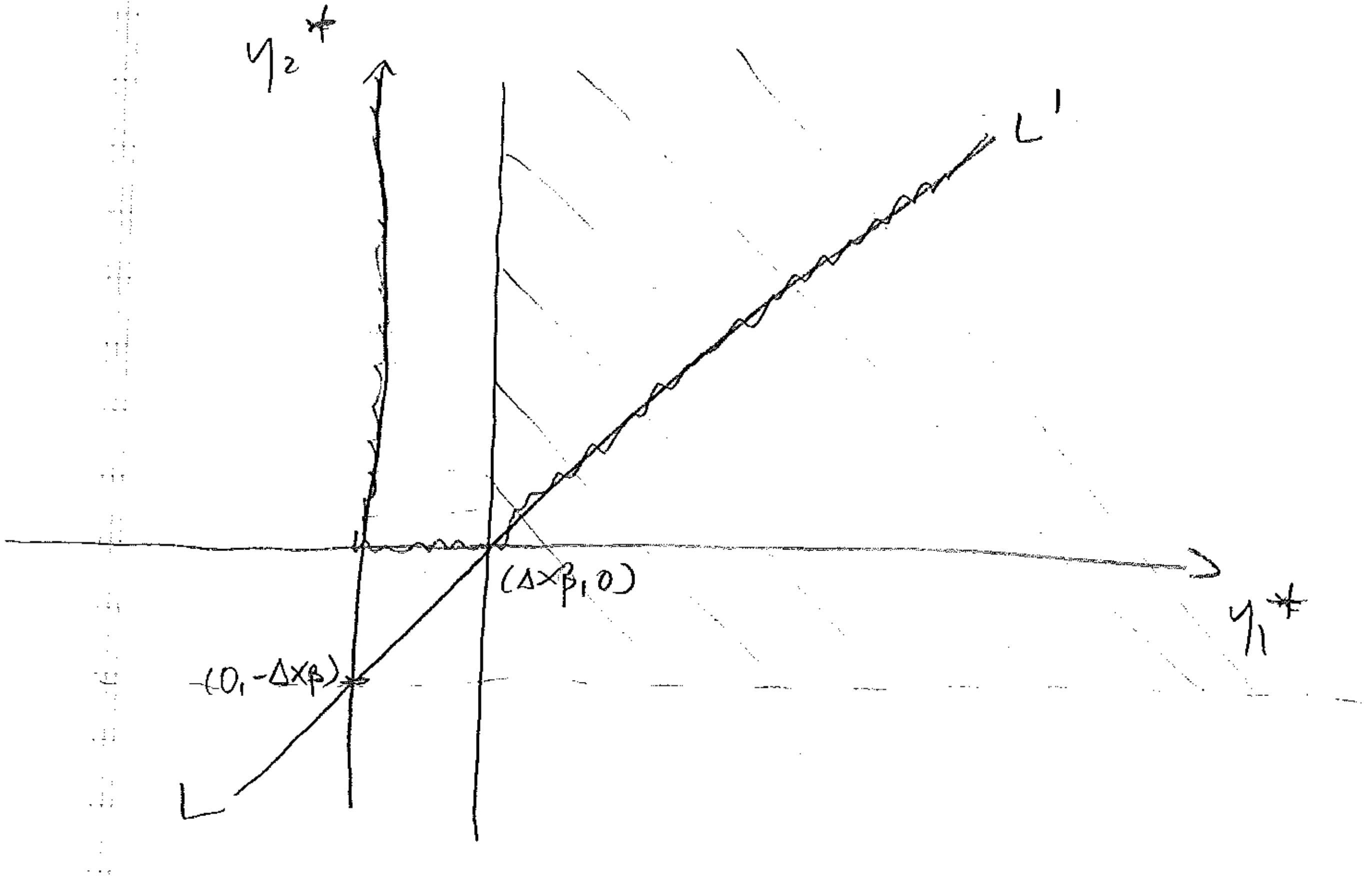
$$E[(1\{(Y_1^*, Y_2^*) \in A_1\} - 1\{(Y_1^*, Y_2^*) \in B_1\}) \cdot \Delta X] = 0$$

$(Y_1 - Y_2 - \Delta X \beta)$

5

$$- E [\mathbb{1}\{(Y_1, Y_2) \in A_1\} \cdot (Y_1 - Y_2 - \Delta X \beta) \Delta X]$$

$$= - E [\mathbb{1}\{ Y_1 > \max\{\Delta X \beta, 0\}\} \cdot \mathbb{1}\{ Y_2 > Y_1 - \Delta X \beta \} \cdot (Y_1 - Y_2 - \Delta X \beta) \Delta X]$$



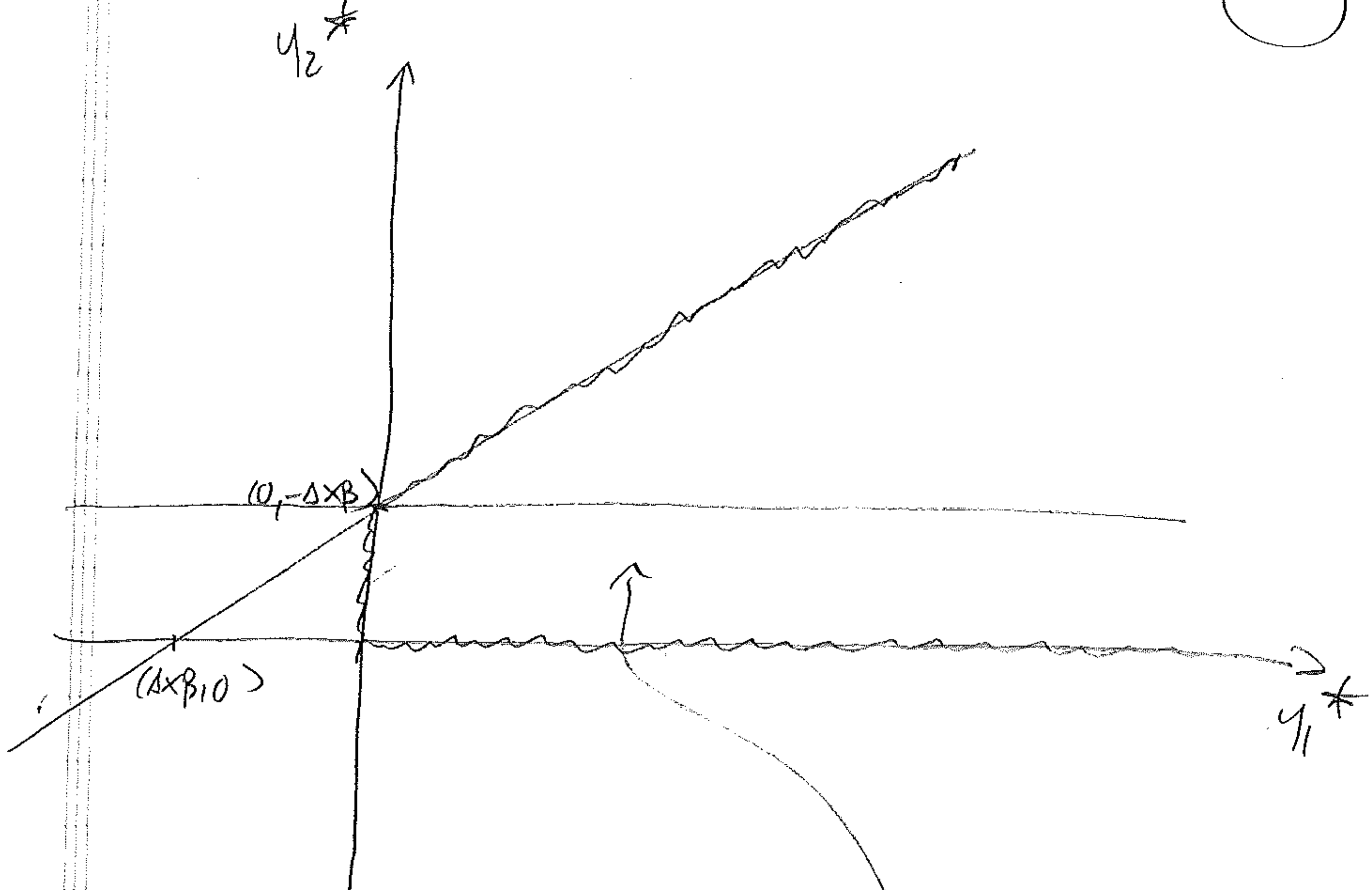
$$|\Delta Y - \Delta X b| \cdot \mathbb{1}\{ Y_1 > \Delta X b, Y_2 > -\Delta X b \}$$

$$= |\Delta Y - \Delta X b| \cdot \mathbb{1}\{ Y_1 > \Delta X b, Y_2 \geq 0 \}$$

$$+ |Y_1| \cdot \mathbb{1}\{ Y_1 \geq \Delta X b, Y_2 < -\Delta X b \}$$

$$+ |Y_2| \cdot \mathbb{1}\{ Y_1 < \Delta X b, Y_2 \geq -\Delta X b \}$$

(C)



Consistency: need optimal β_0 to not be such that we lie here w.p. 1

Note: Individual-specific dummy variables lead to inconsistent estimation. Use ~~the~~ trimmed-LAD approach in censored/truncated panel data models.