Empirical Supplement for the paper "Inference in Ordered Response Games with Complete Information".*

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Abstract

This supplement includes all the results from the Monte Carlo experiments as well as additional results from the empirical application of a multiple entry game between Lowe's and Home Depot.

1 Monte Carlo experiments

This section is motivated by two goals. First, analyzing the properties of our econometric approach and second, studying the consequences of incorrectly estimating a true ordered-response game as a binary-choice game. We find that our methodology performs well in our designs, while a binary-choice misspecification carries severe bias, both for the strategic-interaction parameters and for the non-strategic payoff payoff parameters.

1.1 Experiment designs

We focus on designs that resemble the features of our empirical application, where we model multiple entry decisions between competitors Lowe's and Home Depot. To this end, we use the same

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number of observable payoff shifters and the same parametrization as our empirical application. In our empirical application, our shifters are: population (X^{pop}), total payroll per capita (X^{pay}), land area (X^{area}), and distance (X_j^{dist}) to the nearest distribution center of player j for $j = \{1, 2\}$. X includes 5 covariates, 3 common to each player (X^{pop}, X^{pay}, X^{area}), as well as the player-specific distances to their own distribution centers (X_j^{dist}). All these covariates are, by nature, nonnegative. We generated them here as jointly-distributed log-Normal random variables with mean and variance-covariance matrix matching the sample mean and the sample variance-covariance matrix observed in the data of our empirical application. We employ the parametric specification described in the paper. We have,

$$\pi_j\left(Y, X_j, U_j\right) = Y_j \times \left(\delta + X^{pop} \cdot \beta^{pop} + X^{pay} \cdot \beta^{pay} + X^{area} \cdot \beta^{area} + X_j^{dist} \cdot \beta^{dist} - \Delta_j Y_{-j} - \eta Y_j + U_j\right),$$

with intercept and slope coefficients: $\delta = 2$, $\beta^{pop} = 2$, $\beta^{pay} = 0.25$, $\beta^{area} = 0.25$ and $\beta^{dist} = -0.5$. The strategic interaction parameters were fixed at $\Delta_1 = 1$ and $\Delta_2 = 2$. All these parameter values were chosen because they were interior points in the estimated CS of our empirical application.

The strategy space was capped at $M_1 = M_2 = 100$, which was sufficiently large to be non-binding in all of our simulations. The unobserved payoff shifters U_1 and U_2 were logistically distributed with a joint CDF given by the FGM copula described in the paper. In our designs, we considered two values for the copula coefficient, $\lambda = \{0.5, -0.5\}$, with resulting correlation coefficients between U_1 and U_2 of 0.153 and −0.153, respectively. Denote

$$W_j = X^{pop} \cdot \beta^{pop} + X^{pay} \cdot \beta^{pay} + X^{area} \cdot \beta^{area} + X_j^{dist} \cdot \beta^{dist}$$

as the non-strategic, observable payoff "index" for player j. The indices W_1 and W_2 are highly positively correlated, with $\rho(W_1, W_2) = 0.840$. Let

$$\overline{W} = E\left[X^{pop}\right] \cdot \beta^{pop} + E\left[X^{pay}\right] \cdot \beta^{pay} + E\left[X^{area}\right] \cdot \beta^{area} + \frac{1}{2}\left(E\left[X_1^{dist}\right] + E\left[X_2^{dist}\right]\right) \cdot \beta^{dist} \qquad (1)$$

$\overline{W}$ is the average of the non-strategic component of payoff shifters (without the intercept) between both players. Its true value is $\overline{W} = 5.463$ in our designs. We will construct CIs for $\overline{W}$ as a way to evaluate our method.

The value of the concavity coefficient η has a significant effect on the properties of the equilibrium outcomes: small values of η imply "flatter" payoff functions, favoring the existence of multiple equilibria, and a richer range of equilibrium outcomes. The opposite is true for large values of η, which strengthen the concave nature of payoffs with respect to players' own actions. We used multiple values of η in our designs. We considered $\eta = \{0.25, 0.75, 1.5, 4.5\}$. Thus, our experiments included *eight* MC designs in total.

In each simulation we solved the game, looking for all PSNE, with an equilibrium selection rule that chose an equilibrium at random, assigning uniform probability to each of the existing PSNE. The equilibrium selection device was independent from all other covariates in the game. Table 5 summarizes the population properties of our MC designs. As we see there, smaller values of η (the payoff concavity coefficient) lead to the prevalence of multiple equilibria, and to a richer range of possible outcomes, while the opposite is true for larger values of η. Note that, in spite of having a game of strategic substitutes, the correlation in outcomes Y_1 and Y_2 can range from negative to positive depending on the degree of concavity of payoffs.

1.1.1 Comparison with a binary-choice game

One of our goals is to investigate the consequences of misspecifying a true ordinal game with three or more actions as a binary-choice one. A binary game ignores the intensive-margin of strategic interaction; that is, the fact that players care not only about whether other firms "enter" (the extensive-margin choice), but the *intensity* with which they compete. With strategic substitutes, a true binary-choice game with the same parameter values we use for our payoff functions would produce probabilities of entry that are much larger than the ones produced by an ordinal game[1]. Our experiments find that, in order to have a true binary choice game that replicates (approximately) the same probabilities of entry as our ordinal game, the binary choice game would have to be produced by payoffs that are systematically lower than those of the ordinal game.

Table 6 illustrates this fact. As we show there, in order to have a true binary choice game that produces entry probabilities within the range of those of our ordinal-game designs, the binary choice game would have to be produced by payoff functions where $\overline{W}$ is significantly smaller than $\overline{W} = 5.463$ (the value in our designs). In fact, the binary game would have to be produced by payoffs where $\overline{W} \approx -1.8$. Intuitively, we can anticipate that misspecifying a true ordered-response game as a binary-choice one will produce estimates for $\overline{W}$ that are biased downwards. This intuition will be confirmed by our results.

1.1.2 Observable features of a true ordered-response game

Currently, there do not seem to exist formal econometric tests that can help select between binary-choice and ordinal games with complete-information. Even though a formal test is outside the scope of our paper, it is still interesting to ask whether, in our designs, there exist features of the data that can hint at the true game being ordinal rather than binary.

Let $d_i \equiv 1[Y_i \neq 0]$. Table 7 presents $Pr(d_i = 1|Y_j, W_i = median(W_i))$ for different values of Y_j. As we can see there, these probabilities are monotonically decreasing in Y_j in all cases studied, and this is true for both players. This pattern was preserved for other values of W_i, such as the 25th

[1] In a binary game, the incentive to enter depends only on whether the opponent entered. In an ordered game like ours, the incentive to enter is different if the opponent opened one store versus more stores.

and 75th quantiles. While this monotonic feature arises in our designs and we do not formalize it as a general result, it can be nevertheless a potentially useful, auxiliary diagnostic tool. In the empirical application we find evidence in our data that is consistent with this property.

1.2 Results

We apply our methodology to each of our designs and we compare them to the results derived from misspecifying the game as being binary and estimating it by MLE (with the correct specification for the joint distribution of U_1 and U_2) as in Bresnahan and Reiss (1991).

1.2.1 Results from a binary-game misspecification

As the discussion above anticipated, a binary-choice misspecification leads to a systematic *downward* bias in players' estimated payoffs. As Table 8 shows, this misspecification also produced poor coverage probabilities for the strategic interaction parameters (Δ_1, Δ_2). Table 9 shows the extent to which estimated payoffs are downward-biased. It includes 95% CIs for the MLE estimates of the index $\overline{W}$. As we anticipated, these intervals are systematically shifted to the left of the true value of $\overline{W}$. The degree of bias is more pronounced for the designs where the underlying game has more widespread multiple equilibria (i.e, when the concavity coefficient η is smaller).

1.2.2 Results from our method when the model is correctly specified as an ordered-response game

The kernels and bandwidths employed are exactly as described in the empirical section of the paper. The class of test-sets we use in our procedure are also exactly as described there. For brevity, we refer the reader to that section for the details. Table 10 presents the coverage probability of our CS for the subset of parameters $(\beta, \Delta_1, \Delta_2)$. The results presented there correspond to projections, for that subset of parameters, of the overall CS for the entire parameter vector. Because these are projections, the coverage probability is (for large enough sample sizes) larger than the target nominal coverage probability of 95%. We find that, for very small sample sizes ($n = 250$), our approach has under-coverage when the underlying game has many equilibria and a very rich support of outcomes ($\eta = 0.25$). This problem was largely absent for all the other values of η used, and it disappeared quickly for $\eta = 0.25$ for moderately large sample sizes ($n \geq 500$). When the sample size is $n = 1000$ (very close to the sample size of $n = 954$ in our empirical application), our approach has very good coverage for these parameters across the board for all the designs we employed.

Table 11 presents a 95% CI for $\overline{W}$ from our results, constructed once again as the projection of our overall CS. We find that our approach performs well, generating CI's that contain the true value of $\overline{W}$ in all the designs and for all sample sizes analyzed; furthermore, the resulting CI's are not too wide, and they consistently shrink as n increases. The sign of the correlation in the unobserved shocks (the sign of λ) had no significant or systematic impact in the performance of our method given the values analyzed for λ. The inferential results of our approach when the game is correctly specified as ordinal stand in sharp contrast with the results from the misspecified binary game.

2 Application to a Multiple Entry Game between Home Depot and Lowe's. Additional results excluded from the main paper

Here we present and discuss results from our empirical application that were excluded from the main paper for the sake of brevity.

2.1 Profiled likelihood function and evidence of point-identification of θ_1

Figure 1 in Section B depicts the profiled log-likelihood function for individual parameters in θ_1. The figure supports the point identification result for the subvector θ_1 from the conditional probability of the event $Y = (0,0)$ provided in the paper.

2.2 Two-dimensional joint confidence sets for payoff parameters

Section B includes graphical depictions of pairwise 95% joint confidence sets (CS) for our payoff parameters, which were obtained as projections from the 95% CS estimated for the entire parameter vector. As figures 2, 3 and 4 show, there is no evidence of "holes". As is commonly the case in these problems, the border of each CS is a bit more "fuzzy" than the interior. However, a quick visual inspection shows that these CS are quite informative in many cases.

2.3 Comparison of the propensity of equilibrium selection across specific outcome profiles

As we described in the main paper, our results can also be used for inference about economic quantities of interest such as the propensity of equilibrium selection (i.e, the likelihood that the underlying equilibrium selection mechanism will choose a particular profile y conditional on y being a PSNE). As in the paper, we denote this propensity as $P_{\mathcal{M}}(y)$. We conducted a comparison for this propensity across specific outcomes; we excluded these results in the main paper for the sake of brevity but we include them here. Figure 5 makes such comparisons by plotting $\widehat{P_{\mathcal{M}}}(y;\theta)$ for each $\theta \in CS_{1-\alpha}$. As Figure 3 shows, $CS_{1-\alpha}$ includes parameter values θ for which $\Delta_1 = 0$. For such values, the optimal decision of Lowes does not depend on the actions of Home Depot; by strict concavity of payoffs, this eliminates the possibility of multiple equilibria for any such θ.

Consequently for such parameter values the propensity to select equilibria is always equal to one for any outcome. This explains why the upper bound for $P_{\mathcal{M}}(y)$ in our CS is always 1 for any y.

However, as Figure 5 shows, our results yield nontrivial lower bounds for these propensities and they also allow us to make comparisons across different outcomes to try to understand whether firms have a particular preference towards certain equilibrium outcomes. The comparisons in parts (A)-(C) of Figure 5 can be summarized as follows:

(A) *Equilibria with at most one store by each firm:* We compare the propensity of equilibrium selection for the outcomes $(0,1)$, $(1,0)$ and $(1,1)$. Our results yield two findings: (i) Comparing equilibria where only one store is opened, there is a higher selection propensity for Lowe's to have the only store than for Home Depot. (ii) There is a greater selection propensity for the equilibrium in which both firms operate one store than those where only one firm does.

(B) *Equilibria with a monopolist opening multiple stores:* We focus on the outcomes $(0,2)$, $(2,0)$, $(0,3)$ and $(3,0)$. Our results indicate that the selection propensity is higher for the outcome in which Lowe's operates two stores than those where Home Depot operates two stores. Our findings regarding selection propensities for $(0,3)$ and $(3,0)$ were less conclusive.

(C) *Equilibria where both firms enter with the same number of stores:* We focus on the outcomes $(1,1)$, $(2,2)$ and $(3,3)$. Although not illustrated in the figure, the propensity to select symmetric equilibria where both firms are present appeared to be comparably higher than the propensity to select equilibria where there is only one firm in the market. For most $\theta \in CS_{1-\alpha}$, the outcome $(1,1)$ was the most favored.

2.4 Counterfactual equilibrium selection rules

As we discussed in the main paper, our framework allows us to study the likelihood that other outcomes could have co-existed as equilibria along with the outcomes actually observed in each market in the data. With this information at hand we can do counterfactual analysis based on pre-specified (by us) equilibrium selection mechanisms. Here we generate counterfactual outcomes in each market based on four hypothetical equilibrium selection rules. We focus our analysis on those markets where at least one firm entered and each firm opened at most 15 stores.[2] This accounts for approximately 70% of the entire sample.

(A) Selection rule favoring Lowe's. For each market i, a counterfactual outcome $y_i^c \equiv \left(y_{i1}^c, y_{i2}^c\right)$ was generated through the following steps:

[2]Recall again that observing $(0,0)$ in a given market implies that no other counterfactual equilibrium was possible.

1.– Find all the outcomes y for which

$$\overline{P}_{\mathcal{E}}(y|Y_i, X_i) = \max\{P_{\mathcal{E}}(y|Y_i, X_i, \theta) : \theta \in CS_{1-\alpha}\}$$

was at least 95%. Here $P_{\mathcal{E}}(y|Y_i, X_i, \theta)$ is as defined in the main test in Section 7.5.2. In words, $\overline{P}_{\mathcal{E}}(y|Y_i, X_i)$ is the largest possible value consistent with θ in our CS for the probability that y is an equilibrium simultaneously with Y_i conditional on the observed outcome Y_i and market covariates X_i. If there are no such outcomes, then set $y_i^c = Y_i$. Otherwise proceed to step 2.

2.– Choose the outcome y with the largest number of Lowe's stores. If there are ties, choose the one with the largest number of Home Depot stores.

(B) Selection rule favoring Home Depot. Same as (A), but switching the roles of Home Depot and Lowe's.

(C) Selection rule favoring entry by both firms and largest total number of stores. Here we took the following steps to determine y_i^c:

1.– As in (A) and (B), look for all the outcomes y for which $\overline{P}_{\mathcal{E}}(y|Y_i, X_i) \geq 0.95$. If no such $y \neq Y_i$ exists, set $y_i^c = Y_i$. Do the same if no y was found where *both* firms enter. Otherwise proceed to step 2.

2.– Among the outcomes y found in step 1, look for the one that maximizes the total number of stores $y_1 + y_2$. If there are ties, then choose the one that minimizes $|y_1 - y_2|$. If more than one such outcome exists, choose randomly among them using uniform probabilities.

(D) Selection rule favoring symmetry. Each y_i^c was generated as follows:

1.– As in (A)-(C), look for all the outcomes y for which $\overline{P}_{\mathcal{E}}(y|Y_i, X_i) \geq 0.95$. If no such $y \neq Y_i$ exists, set $y_i^c = Y_i$. Otherwise proceed to step 2.

2.– Among the outcomes y found in step 1, look for the one that minimizes $|y_1 - y_2|$. If more than one such outcomes exist, choose randomly among them using uniform probabilities.

Examining Table 1, the pattern of market outcomes that results from counterfactual selection rules (A), (B) and (C) is decisively different from the features of the observed outcomes in the data. This is less so for selection rule (D). Table 1 also suggests that a selection mechanism which maximizes the total number of stores in each market (rule (C)) would produce a pattern of outcomes heavily biased in favor of Lowe's. Overall, among these counterfactual experiments, the one employing selection rule (D) favoring symmetry most closely matches the observed pattern of store profiles in the data.

Table 1: Results of counterfactual equilibrium selection experiments

	Observed data†		Selection rules (A)		(B)		(C)		(D)	
	Y_1	Y_2	y_1^c	y_2^c	y_1^c	y_2^c	y_1^c	y_2^c	y_1^c	y_2^c
Average	1.76	1.62	4.72	0.41	0.66	2.85	3.27	1.02	1.79	1.67
Median	1	1	1	0	0	1	1	1	1	1
75^{th} percentile	2	1	5	0	1	3	2	1	2	1
90^{th} percentile	4	4	13	1	1	7	10	1	4	4
95^{th} percentile	6	7	20	1	1	15	18	2	8	7
Total	1,180	1,090	3,014	319	283	2,062	2,121	730	1,172	1,120
$\%(y_1 > y_2)$	47%		64%		17%		44%		33%	
$\%(y_1 = y_2)$	23%		21%		28%		42%		48%	

(†) The markets considered in this experiment where those where at least one firm entered and each firm opened at most 15 stores. This included approx. 70% of the entire sample.

2.5 Counterfactual experiments: cooperative behavior

Our results allow us to analyze counterfactual alternatives to noncooperative behavior. Here, we consider a simple cooperative counterfactual scenario in which the firms maximize the sum of their payoff functions, assigning equal weight to each. This produces an outcome on the frontier of the set of feasible firm payoffs. We refer to this counterfactual as "cooperative behavior."

Fix a parameter value θ and focus on market i. Let (y_i, x_i, u_i) denote the realizations of (Y, X, U) in that market. Let $\left(y_1^e(x_i, u_i; \theta), y_2^e(x_i, u_i; \theta)\right)$ be an element of

$$\underset{(y_1, y_2)}{\arg\max} \Big[\pi_1 (y_1, y_2, x_{1,i}, u_{1,i}; \theta) + \pi_2 (y_1, y_2, x_{2,i}, u_{2,i}; \theta)\Big],$$

so that $\left(y_1^e(x_i, u_i; \theta), y_2^e(x_i, u_i; \theta)\right)$ denotes an action profile maximizing the sum of firm payoffs. Recall from above that $P_U(\mathcal{R}_\theta(y, x); \theta)$ denotes the probability that y is an equilibrium outcome given $X = x$. We are interested in the following two functionals,

$$\overline{y}_j^e(y_i, x_i, \theta) = \int\limits_{u \in \mathcal{R}_\theta(y_i, x_i)} \frac{y_j^e(x_i, u; \theta) f(u; \lambda) du}{P_U(\mathcal{R}_\theta(y_i, x_i); \theta)} \quad \text{for } j = 1, 2.$$

$$P^e(y_i, x_i, \theta) = \int\limits_{u \in \mathcal{R}_\theta(y_i, x_i)} \frac{1\Big[(y_{1,i}, y_{2,i}) = \left(y_1^e(x_i, u; \theta), y_2^e(x_i, u; \theta)\right)\Big] f(u; \lambda) du}{P_U(\mathcal{R}_\theta(y_i, x_i); \theta)}.$$

Conditional on $X = x_i$ and conditional on y_i being an equilibrium outcome, $\overline{y}_j^e(y_i, x_i, \theta)$ is the expected cooperative choice for j and $P^e(y_i, x_i, \theta)$ is the probability that the outcome observed in the i^{th} market is the cooperative outcome.

We apply this analysis to the 308 (out of 954) markets that had a single store in our sample. We use S_1 to denote this collection of markets. Our goal is to compare observed market outcomes to those that would be obtained under cooperative behavior, and in particular to determine whether cooperation would lead to more stores in the markets in S_1. Let $\left[\underline{T}_1, \overline{T}_1\right]$ denote a 95% CI for the total number of stores we would observe in the markets in S_1 under a cooperative regime. Of particular interest to us is how 308 (the actual number of stores observed in S_1) compares to this CI. Our results yielded $\left[\underline{T}_1, \overline{T}_1\right] = \left[308, 445.10\right]$. Note first that the number of stores observed in these markets corresponds to the lower bound we would observe under cooperation. This is by construction, since the number of stores in markets in S_1 could only be lower if the outcome were $(0,0)$, which would necessarily produce lower total payoff (specifically zero) than the observed single-entrant PSNE outcome, as otherwise it would not have been a PSNE. On the other hand, a market in which equilibrium resulted in a single entrant could have resulted in more stores under cooperation if the firm that did not enter would find it more profitable to operate multiple stores absent the presence of the firm that actually entered. Our analysis reveals that we could have as many as 45% more expected stores under this counterfactual scenario. Table 2 summarizes some of the main findings.

Table 2: Summary of counterfactual results under cooperation.

• There exist at least 96 markets (out of 308) where Home Depot had more stores (and Lowe's had fewer stores) than the expected outcome under cooperation. The number of such markets could be as large as 110.
• There existed parameter values in our confidence set for which *every* market in S_1 had fewer total stores than under cooperation.
• The expected number of total stores under cooperation would increase from 1 to at least 2 in as many as 85 markets.
• There were 286 markets for which we could not reject that $P^e(y_i, x_i) < 50\%$, 93 markets for which we could not reject that $P^e(y_i, x_i) < 10\%$ and 47 markets for which we could not reject that $P^e(y_i, x_i) < 5\%$. There were 15 markets for which we could not reject that $P^e(y_i, x_i) = 0$.

Our results suggest that in this market segment noncooperative behavior has led to less entry by Lowe's and greater entry by Home Depot than would be optimal under the counterfactual cooperative regime. These results are in line with some of our findings depicted in Figure 5 showing a higher propensity to select equilibria favoring Lowe's in markets with at most one store.

2.6 Counterfactual experiment: Monopolistic behavior

Our results also allow us to analyze the implications of how each of these firms would behave if their opponent left the industry. For firm j in the i^{th} market let

$$y_j^m(x_{j,i}, u_{j,i}; \theta) = \arg\max_{y_j} \pi_j\Big(\big(y_j, 0\big), x_{j,i}, u_{j,i}; \theta\Big)$$

denote the optimal choice if j is the monopolist in market i. Let

$$\overline{y}_j^m(y_i, x_i, \theta) = \int_{u_j \in \mathcal{R}_\theta(y_i, x_i)} \frac{y_j^m\left(x_{j,i}, u_j; \theta\right) f_j\left(u_j\right) du_j}{P_U\left(\mathcal{R}_\theta(y_i, x_i); \theta\right)}$$

denote the expected monopolistic choice firm j would make in market i given that the observed outcome there is a PSNE. We constructed a 95% confidence set for this expected choice for every market in our sample. Our main finding is that there is a stark contrast in the monopolistic behavior of both firms. While Lowe's would enter many markets where it has no current presence if Home Depot dropped out of the industry, the opposite is not true: Home Depot would concentrate its presence in relatively fewer markets, remaining out of multiple markets where it currently has no presence. Lowe's on the other hand would spread its presence over a larger geographic area including smaller markets. Table 3 summarizes some of our main findings.

Table 3: Summary of counterfactual results under monopolistic behavior.

• There exist at least 119 markets where Lowe's is currently absent where it would enter if it were a monopolist.
• We could not reject that Home Depot would not enter any market where it is currently absent if it were a monopolist.
• In our data there were 251 markets with no stores. If Lowe's were a monopolist, this number would increase to no more than 257. In contrast, if Home Depot were a monopolist this number could grow to as many as 465 markets (almost half of the total markets in our data).
• There exist 3,483 stores in our data. If Lowe's were a monopolist the expected number of stores would be at least 2,130. If Home Depot were a monopolist, this number could fall as low as 1,860, constituting approximately a 50% drop).

In summary, a sizeable number of markets that are currently served by Lowe's (as many as 214) could go unserved by Home Depot if the latter were a monopolist. In contrast, Lowe's would enter almost every market where Home Depot has a presence, staying out of at most 6 such markets.

While our model does not reveal the source of asymmetry in its predictions of monopolistic behavior, these predictions align with differences in store branding. Fernando (2015) notes that Home Depot's stores are more geared toward professional customers, with an "industrial asthetic" and some shelves that can only be reached by using forklifts. Lowe's on the other hand is more friendly to the typical nonprofessional home improvement customer, featuring, "more elaborate floor displays or themed products such as patio sets or holiday decor items." A similar view of the stores' distinguishing features is given by Mitchell (2015). It may then be plausible that Lowe's stores are more suited to certain rural markets with a relatively low density of construction professionals, which Home Deport would not find profitable. Although not directly captured by our model, this conjecture offers a possible explanation for the observed difference in monopolistic behavior.

2.7 Estimation as a binary entry game

One of the goals of our Monte Carlo experiments was to study the consequences of misspecifying a true ordinal game as a binary-choice one. Our designs, modeled to mimic the properties of the data used in this empirical application showed two main consequences derived from a binary choice misspecification: (i) a systematic downward bias in the estimates of non-strategic payoff components, and (ii) poor coverage of confidence sets for strategic interaction coefficients. We revisit this issue here by estimating our model as a binary-choice (entry) game under the assumption of strategic substitutes. As we did in our Monte Carlo experiments, following Bresnahan and Reiss (1991) we estimate the model by using MLE where the outcome variable is the number of entrants (zero, one or two) in each market. As we did in our Monte Carlo experiments, we aggregate the estimation results of non-strategic payoff components through their average. Parallel to our definition of $\overline{W}$ in (1), let

$$\widehat{\overline{W}} = \overline{X}^{pop} \cdot \beta^{pop} + \overline{X}^{pay} \cdot \beta^{pay} + \overline{X}^{area} \cdot \beta^{area} + \frac{1}{2}\left(\overline{X}_1^{dist} + \overline{X}_2^{dist}\right) \cdot \beta^{dist} \tag{2}$$

The point estimate for $\widehat{\overline{W}}$ when we model the game as a binary-action entry game was 5.905, with a 95% CI (taking the sample means of the payoff shifters as fixed) of [5.121, 6.688]. In contrast, a 95% CI for $\overline{W}$ using our approach[3] was [5.975, 8.914], which excluded the binary choice point-estimate, although it had some overlap with the CI in that case. Overall, the feature of estimated non-strategic payoff components "shifted to the left" under a binary choice specification that we observed in our Monte Carlo experiments appears to be present in our empirical application. Lastly, we compared the inferential results for the strategic coefficients. Misspecification in our Monte Carlo experiments produced CS for (Δ_1, Δ_2) that undercovered (or excluded) the

[3]This was computed as the projection of our CS over $\overline{W}$, taking the sample means of payoff shifters as fixed.

true parameter values systematically. One way to evaluate this is to compare our CS with that of the binary-choice MLE specification. In this case, a 95% CI (truncated at 0) was [0, 1.189] for Δ_1, and [0, 0.274] for Δ_2. In particular, the CI for Δ_2 is entirely disjoint with the CI that follows from our results. Figure 6 compares the joint (Δ_1, Δ_2) CS in both cases (truncated at zero since we maintain strategic substitutes). The figure shows very clearly how the confidence sets are disjoint, which is entirely consistent with the pattern we observed in our Monte Carlo experiments when an ordered-response game is misspecified as a binary one. Lastly, let $d_i = 1[Y_i \geq 1]$ denote the decision of "entry". In the Monte Carlo experiment designs we used, we found evidence that $Pr\left(d_i = 1|Y_j, X\right)$ – the conditional probability of entry (as a binary choice) given the rival's intensity of entry Y_j – is nonincreasing in Y_j when the true underlying game is ordinal instead of binary. While this does not constitute a proper specification test but rather a feature of our designs (which were motivated by our empirical data), it is useful to revisit this here. Table 4 displays the observed probabilities conditional on $POPULATION$ (market size) being between the 45th and the 55th percentiles (i.e, around the median market size). The probabilities shown there display the same decreasing monotonicity of firms' conditional entry probabilities in their rival's competitive intensity produced by ordered-response games with the types of designs analyzed in Section 1.[4] We reiterate that, while these features do not constitute in any way a formal specification test, they are nevertheless consistent with the properties[5] of true ordered-response games with the types of designs analyzed in Section 1.

Table 4: Probability of entry ($d_i = 1$) conditional on the number of stores of the opponent (Y_j) for markets whose size (population) was between the 45th and the 55th percentiles.

Player 1 (Lowe's)	$Y_2 = 1$	$Y_2 = 2$	$Y_2 = 3$	$Y_2 \geq 4$
$Pr(d_1 = 1\|Y_2)$	46.5%	0.3%	0%	0%

Player 2 (Home Depot)	$Y_1 = 1$	$Y_1 = 2$	$Y_1 = 3$	$Y_1 \geq 4$
$Pr(d_2 = 1\|Y_1)$	32.2%	20.0%	1.2%	0%

[4]For other quantiles of POPULATION these probabilities immediately died off to zero.

[5]While there were quantiles of POPULATION for which the probabilities shown in 4 increased, in all cases they were nonincreasing for $Y_j \geq 3$.

A Monte Carlo Experiment Tables

Table 5: Summary statistics for our Monte Carlo designs[†].

	Positive correlation in unobserved payoff shocks: $\lambda = 0.5$ and $\rho(U_1, U_2) = 0.153$				Negative correlation in unobserved payoff shocks: $\lambda = -0.5$ and $\rho(U_1, U_2) = -0.153$			
	Value of η (concavity coefficient)				Value of η (concavity coefficient)			
	0.25	0.75	1.50	4.50	0.25	0.75	1.50	4.50
$\rho(Y_1, Y_2)$	−0.37	−0.23	0.63	0.74	−0.39	−0.26	0.58	0.72
$Pr(Y_1 = 0, Y_2 = 0)$	4%	6%	10%	42%	4%	5%	9%	40%
$Pr(Y_1 \geq 2)$	51%	61%	40%	7%	52%	61%	40%	7%
$Pr(Y_2 \geq 2)$	45%	24%	24%	6%	44%	26%	25%	6%
$Pr(Y_1 \geq 4)$	45%	35%	10%	1%	46%	36%	10%	1%
$Pr(Y_2 \geq 4)$	41%	9%	4%	1%	41%	11%	4%	1%
$F_{Y_1}^{-1}(0.75)$	9	5	2	1	9	5	2	1
$F_{Y_2}^{-1}(0.75)$	9	1	1	1	9	2	2	1
$F_{Y_1}^{-1}(0.95)$	24	11	5	2	24	11	5	2
$F_{Y_2}^{-1}(0.95)$	24	4	3	2	24	5	3	2
Pr(multiple eqbia)	0.69	0.21	0.17	0.03	0.67	0.20	0.16	0.03

(†) Probabilities computed from 5 million simulations.

Table 6: Approximating the outcome probabilities of an ordered-response game with a binary-choice game.

	Positive correlation in unobserved payoff shocks: $\lambda = 0.5$ and $\rho(U_1, U_2) = 0.153$			Negative correlation in unobserved payoff shocks: $\lambda = -0.5$ and $\rho(U_1, U_2) = -0.153$		
	Binary-choice game with same parameters as the ordered-response game ($\overline{W} = 5.463$)	Binary-choice game with alternative slope coefficients ($\overline{W} = -1.835$)	Range of probabilities[¶] for the ordered response games in our Monte Carlo designs.	Binary-choice game with same parameters as the ordered-response game ($\overline{W} = 5.463$)	Binary-choice game with alternative slope coefficients ($\overline{W} = -1.835$)	Range of probabilities[¶] for the ordered response games in our Monte Carlo designs.
Pr(duopoly)	67.2%	23.2%	[2.9%, 37.8%]	66.1%	20.2%	[2.8%, 35.5%]
Pr(monopoly)	29.1%	60.7%	[33.5%, 92.7%]	30.5%	65.5%	[36.3%, 93.3%]
Pr(no entrant)	3.7%	16.1%	[4.4%, 42.1%]	3.4%	14.3%	[3.9%, 40.3%]

(†) Probabilities computed from 5 million simulations.

(¶) Probability range taken over the corresponding range of values $\eta \in \{0.25, 0.75, 1.5, 4.5\}$ for the concavity coefficient in our Monte Carlo designs.

Table 7: Relation between players' extensive margin decision (binary choice "entry" decision) and the opponent's intensive margin choice in our designs

Player 1	$\eta = 0.25$	$\eta = 0.75$	$\eta = 1.50$	$\eta = 4.50$
$Pr(d_1 = 1 \mid Y_2 = 1, W_1 = median(W_1))$	72.6%	82.3%	75.4%	7.1%
$Pr(d_1 = 1 \mid Y_2 = 2, W_1 = median(W_1))$	61.3%	46.3%	32.6%	4.0%
$Pr(d_1 = 1 \mid Y_2 = 3, W_1 = median(W_1))$	43.4%	16.6%	12.2%	0.3%
$Pr(d_1 = 1 \mid Y_2 \geq 4, W_1 = median(W_1))$	2.7%	3.0%	3.5%	0%

Player 2	$\eta = 0.25$	$\eta = 0.75$	$\eta = 1.50$	$\eta = 4.50$
$Pr(d_2 = 1 \mid Y_1 = 1, W_2 = median(W_2))$	86.0%	73.6%	58.4%	5.8%
$Pr(d_2 = 1 \mid Y_1 = 2, W_2 = median(W_2))$	46.4%	18.7%	13.0%	3.4%
$Pr(d_2 = 1 \mid Y_1 = 3, W_2 = median(W_2))$	8.3%	1.4%	1.5%	0%
$Pr(d_2 = 1 \mid Y_1 \geq 4, W_2 = median(W_2))$	0.1%	0.1%	0.1%	0%

- Probabilities computed from 5 million simulations.
- Values shown correspond to $\lambda = 0.5$ (positive correlation in unobserved payoff shocks). The same type of monotonic pattern was observed for $\lambda = -0.5$.

Table 8: Results from Estimating the Misspecified Binary Choice Game.

Empirical coverage probability[§] for the strategic-interaction coefficients (Δ_1, Δ_2) of the MLE-based analytical CS with nominal Coverage Probability: 95%								
	Positive correlation in unobserved payoff shocks: $\lambda = 0.5$ and $\rho(U_1, U_2) = 0.153$				Negative correlation in unobserved payoff shocks: $\lambda = -0.5$ and $\rho(U_1, U_2) = -0.153$			
Sample size	Value of η (concavity coefficient)				Value of η (concavity coefficient)			
	0.25	0.75	1.50	4.50	0.25	0.75	1.50	4.50
250	1.3%	42.1%	90.9%	93.6%	1.7%	46.5%	91.3%	94.7%
500	0%	4.0%	65.6%	91.3%	0.3%	5.6%	80.3%	93.4%
1000	0%	0.3%	26.4%	88.9%	0%	0.7%	33.1%	91.5%

(§) Let $\widehat{\Delta} \equiv (\widehat{\Delta}_1, \widehat{\Delta}_2)$ denote the binary-game MLE estimated strategic-interaction coefficients, and let $\Delta^0 = (\Delta_1^0, \Delta_2^0)$ denote their true values. Let $\widehat{\Sigma}_\Delta$ denote the estimated MLE variance-covariance matrix of $\widehat{\Delta}$. The entries in the table correspond to the observed frequency (over 500 Monte Carlo simulations) with which the test-statistic $J_\Delta = n \cdot (\widehat{\Delta} - \Delta^0)' \widehat{\Sigma}_\Delta^{-1} (\widehat{\Delta} - \Delta^0)$ was below the χ_2^2 95% critical value. This is the frequency with which the true value Δ^0 of the strategic-coefficients was included in the analytical, MLE-based 95% CS in the (misspecified) binary-game.

Table 9: Results from Estimating the Misspecified Binary Choice Game.

95% confidence interval‡ for $\widehat{\overline{W}}$ (500 Monte Carlo simulations) True value: $\overline{W} = 5.463$								
	Positive correlation in unobserved payoff shocks: $\lambda = 0.5$ and $\rho(U_1, U_2) = 0.153$				Negative correlation in unobserved payoff shocks: $\lambda = -0.5$ and $\rho(U_1, U_2) = -0.153$			
	Value of η (concavity coefficient)				Value of η (concavity coefficient)			
Sample size	0.25	0.75	1.50	4.50	0.25	0.75	1.50	4.50
250	$[-2.82, 0.62]$	$[-2.40, 0.97]$	$[0.86, 3.19]$	$[3.04, 5.99]$	$[-3.25, 0.68]$	$[-2.57, 0.99]$	$[0.77, 3.00]$	$[3.23, 6.22]$
500	$[-1.48, 0.44]$	$[-1.69, 0.77]$	$[1.10, 2.76]$	$[3.29, 5.66]$	$[-1.77, 0.42]$	$[-1.74, 0.78]$	$[1.09, 2.60]$	$[3.76, 5.69]$
1000	$[-0.91, 0.17]$	$[-1.26, 0.25]$	$[1.32, 2.33]$	$[3.68, 5.28]$	$[-1.09, 0.17]$	$[-1.18, 0.30]$	$[-1.30, 2.29]$	$[3.73, 5.38]$

(‡) The bounds reported in the table correspond to the observed 2.5^{th} and 97.5^{th} quantiles of $\widehat{\overline{W}} = \overline{X}\widehat{\beta}$, where $\widehat{\beta}$ is to the binary-game MLE estimate of β and $\overline{X}$ corresponds to the true means of our generated payoff shifters.

Table 10: Results from our methodology.

Empirical Coverage Probability for the slopes and strategic-interaction coefficients† $(\beta, \Delta_1, \Delta_2)$. Nominal Coverage Probability: 95%								
	Positive correlation in unobserved payoff shocks: $\lambda = 0.5$ and $\rho(U_1, U_2) = 0.153$				Negative correlation in unobserved payoff shocks: $\lambda = -0.5$ and $\rho(U_1, U_2) = -0.153$			
	Value of η (concavity coefficient)				Value of η (concavity coefficient)			
Sample size	0.25	0.75	1.50	4.50	0.25	0.75	1.50	4.50
250	25.0%	82.0%	95.9%	96.7%	31.3%	87.3%	97.0%	96.5%
500	84.4%	98.5%	97.3%	97.8%	89.4%	99.1%	97.7%	97.1%
1000	98.6%	99.1%	98.5%	98.9%	99.0%	99.7%	98.3%	97.8%

(†) Results correspond to the projection of our CS for the subvector of parameters $(\beta, \Delta_1, \Delta_1)$. Accordingly, the table reports the observed frequency (over 500 Monte Carlo simulations) with which the true values of $(\beta, \Delta_1, \Delta_1)$ were included in our estimated 95% CS for the entire parameter vector.

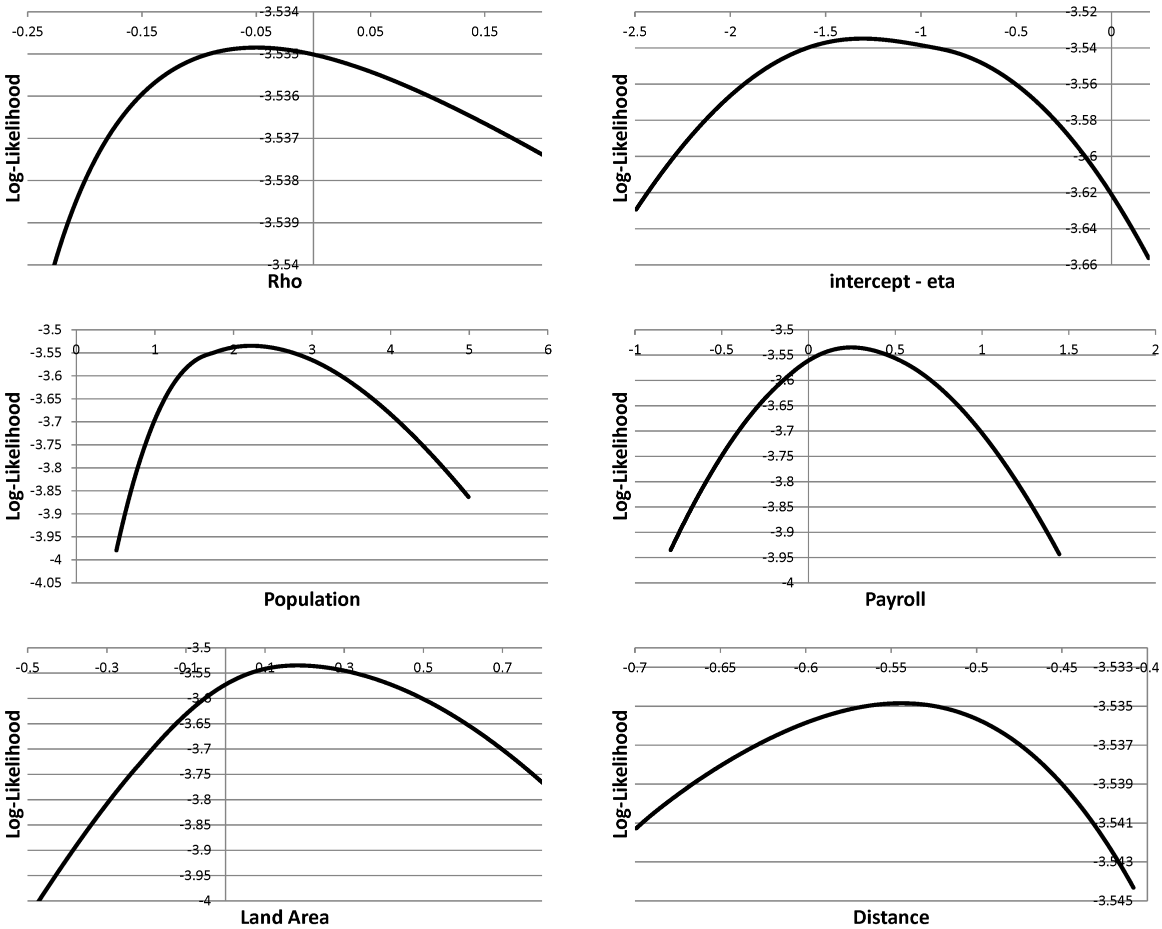
Table 11: Results from our methodology.

95% confidence interval[◇] for $\overline{W}$ (500 Monte Carlo simulations) True value: $\overline{W} = 5.463$								
	Positive correlation in unobserved payoff shocks: $\lambda = 0.5$ and $\rho(U_1, U_2) = 0.153$				Negative correlation in unobserved payoff shocks: $\lambda = -0.5$ and $\rho(U_1, U_2) = -0.153$			
	Value of η (concavity coefficient)				Value of η (concavity coefficient)			
Sample size	0.25	0.75	1.50	4.50	0.25	0.75	1.50	4.50
250	[3.63, 8.27]	[3.61, 8.45]	[3.48, 8.56]	[3.74, 8.66]	[3.55, 8.48]	[3.49, 9.17]	[3.67, 8.19]	[3.81, 7.97]
500	[3.88, 8.02]	[3.76, 8.02]	[3.90, 7.91]	[3.94, 7.96]	[3.64, 8.11]	[3.63, 8.24]	[3.89, 8.15]	[3.88, 7.96]
1000	[4.05, 7.86]	[4.00, 7.85]	[3.95, 7.77]	[4.15, 6.95]	[3.95, 7.81]	[3.99, 7.80]	[4.30, 7.74]	[4.50, 7.33]

(◇) Results shown are based on the projection of our 95% CS for $\overline{W} = \overline{X}\beta$. For the s^{th} Monte Carlo simulation, we compute the $\overline{W}_L^s$ and $\overline{W}_U^s$ as the smallest and largest values, respectively, of $\overline{X}\beta$, taken over all the β's that were included in our estimated 95% CS. The lower and upper bounds in the intervals reported in the table correspond, respectively, to the smallest value of $\overline{W}_L^s$, and the largest value of $\overline{W}_U^s$ observed in our 500 Monte Carlo simulations. As in the binary-game results reported in Table 9, the values used for $\overline{X}$ correspond to the true means of our generated payoff shifters.

B Empirical application auxiliary figures

Figure 1: Profiled log-likelihood for each parameter in θ_1



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Mitchell, M. (2015). What's the difference between lowe's and home depot? http://www.seethewhizard.com/blog/whats-the-difference-between-lowes-and-home-depot/. Accessed: 2016-07-29.

Figure 2: Joint 95% confidence regions for slopes, intercept, and η

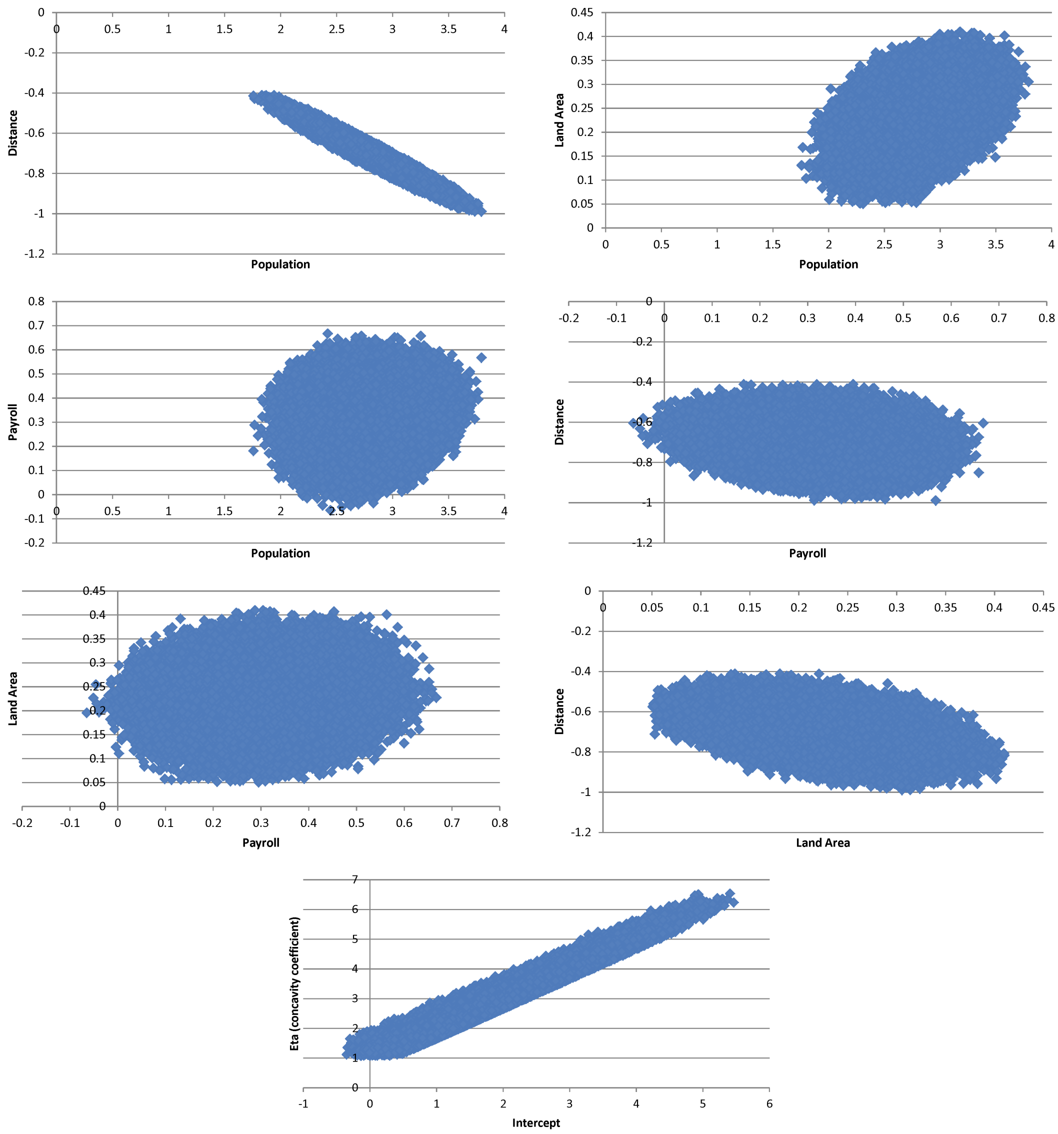


Figure 3: Joint 95% confidence region for strategic interaction coefficients

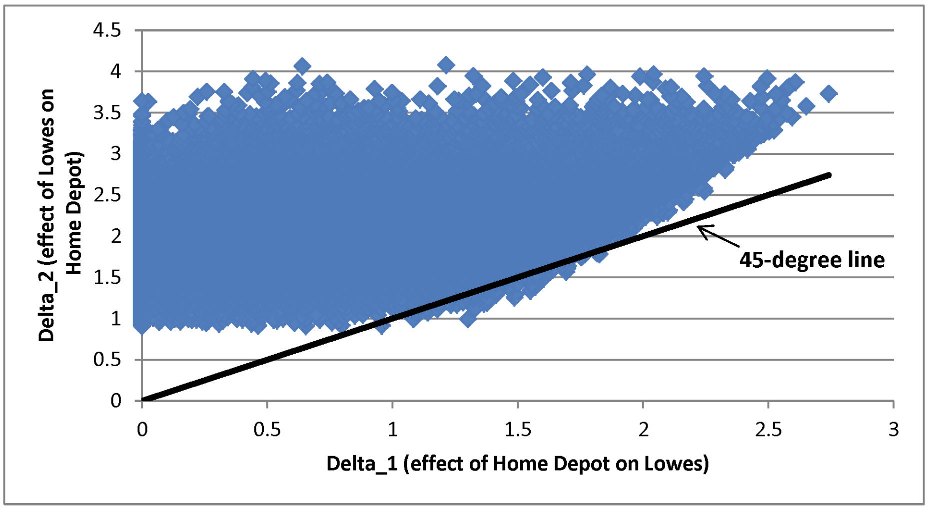


Figure 4: Joint 95% confidence region for strategic interaction coefficients and slope parameters

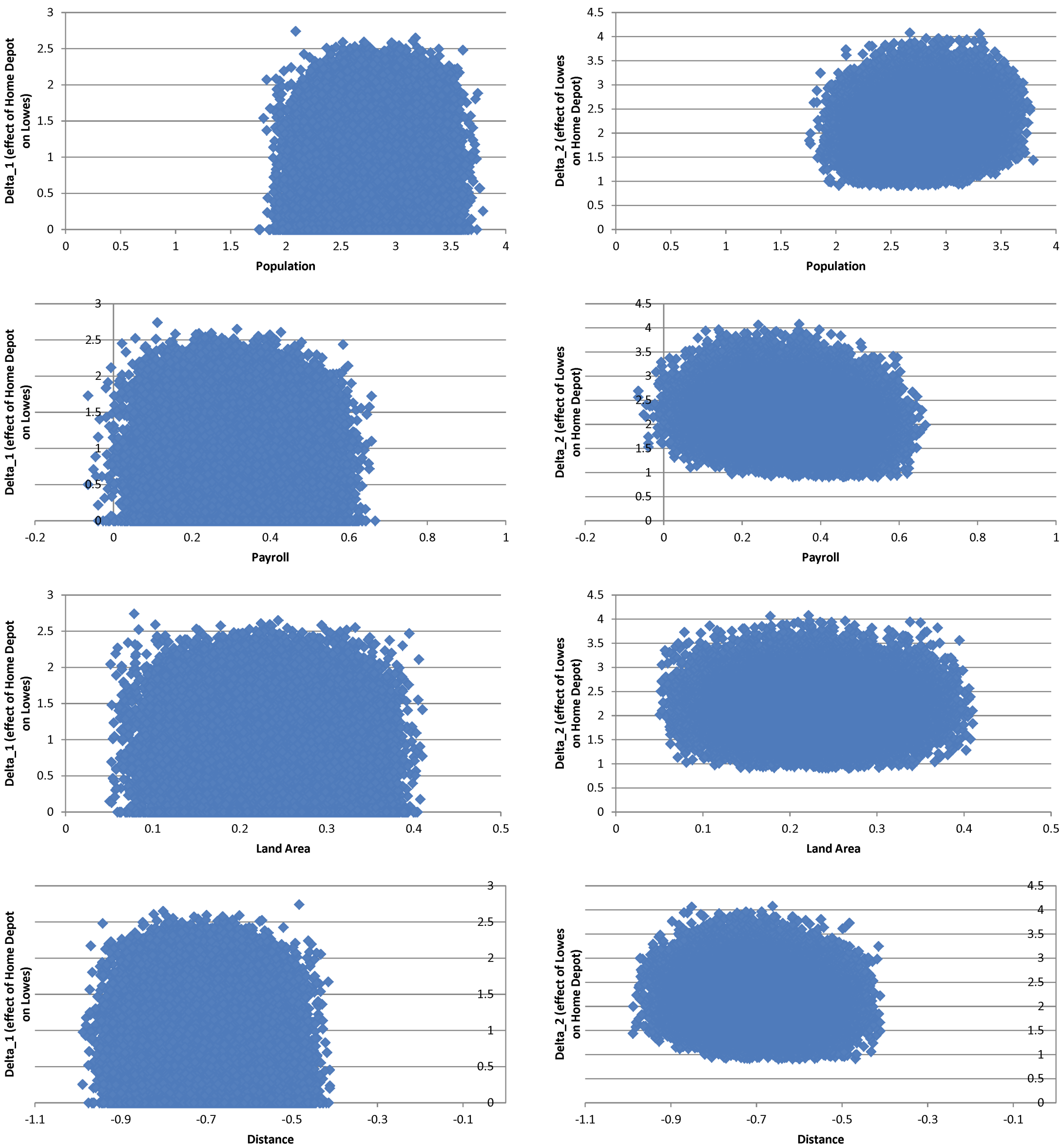
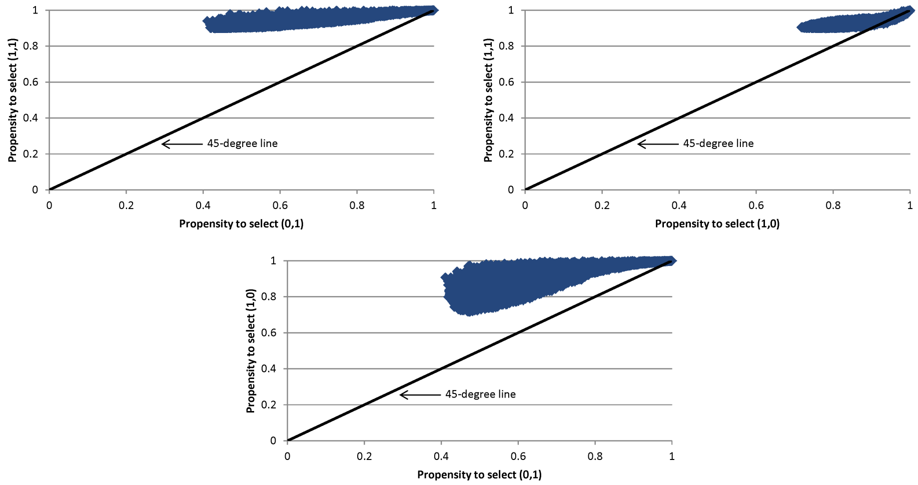
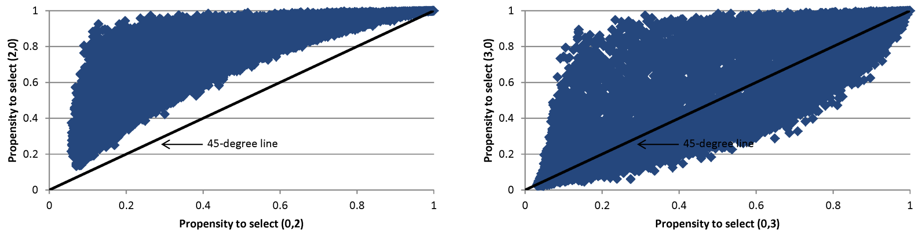


Figure 5: Confidence sets for estimated propensities of equilibrium selection

(A) Equilibria with at most one store by each firm



(B) Equilibria with a monopolist opening multiple stores



(C) Equilibria where both firms enter with the same number of stores

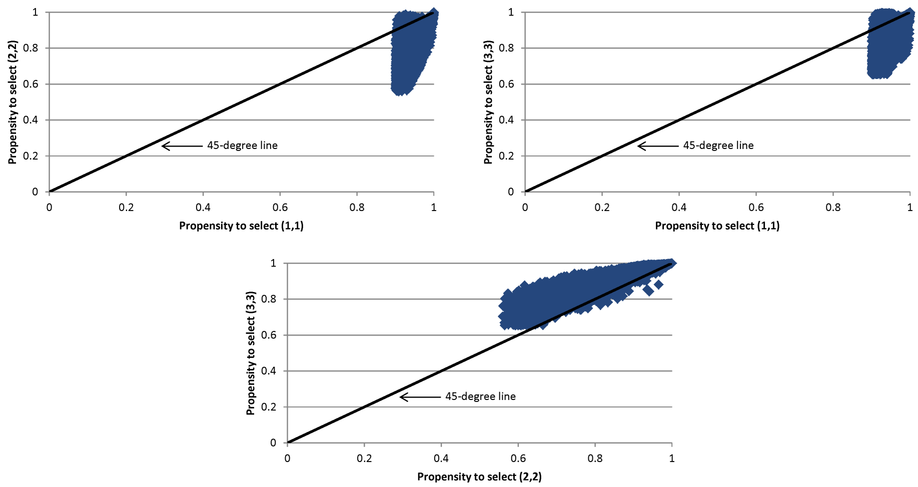


Figure 6: Confidence sets for strategic interaction coefficients (target coverage 95%). Comparison of our results with those derived from a binary-choice specification