Solution-robust estimation of strategic models

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Abstract

A number of normal-form games with parametric payoff functions have the feature that all solutions can be characterized by a collection of (semi)parametric functions that contain information about the payoff parameters and serve as *sufficient statistics*. Under additional exclusion restrictions involving selection mechanisms, these games can be expressed as semiparametric multiple index models and payoff parameters can be estimated using existing methods.

Keywords: Estimation of games, uncertain behavior, multiple solutions, multiple index models.
JEL classification: C1, C14, C57.

1 Introduction

A solution in a strategic-interaction model is associated with a distribution of outcomes conditional on state variables. In many examples of normal-form games with parametric payoff functions, the distribution of outcomes associated with each solution can be characterized by a collection of (semi)parametric functions of observables that serve effectively as sufficient statistics. If we impose additional exclusion restrictions on the otherwise unknown solution-selection mechanisms, point-identification and estimation of payoff parameters can proceed by using these sufficient statistics as control functions and applying existing methods for multiple index models. The approach described here allows for the existence of multiple solutions and multiple candidate behavioral models.

2 Basic general features of our model

We will describe the general features of the type of models we have in mind, and we will present some examples of well-known models that fit this description. Our focus here will

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be normal-form games with a discrete action space.

2.1 Normal-form description of the game

We have a collection of players, $p = 1,\ldots,\mathcal{P}$, each making a discrete choice within a finite action space $\mathcal{Y}_p$. The joint action space is $\mathcal{Y} \equiv \mathcal{Y}_1 \times \cdots \times \mathcal{Y}_{\mathcal{P}}$. Each player p is equipped with a parameterized payoff function $u_p(y; X_p, \varepsilon_p, \theta_{p_0})$, where $y \equiv (y_1,\ldots,y_{\mathcal{P}})$ is a particular action profile, X_p and ε_p denote observable and unobservable (to the econometrician) payoff shifters, and θ_{p_0} denotes a finite-dimensional parameter. We will let $Y \equiv (Y_1,\ldots,Y_{\mathcal{P}})$ denote the outcome of the game. The parameter of interest is $\theta_0 \equiv \cup_{p=1}^{\mathcal{P}} \theta_{p_0}$ in a setting where the joint distribution of $\varepsilon \equiv \cup_{p=1}^{\mathcal{P}} \varepsilon_p$ is unknown. Grouping $X \equiv \cup_{p=1}^{\mathcal{P}} X_p$, we will assume that the researcher observes a random sample $(Y_i, X_i)_{i=1}^{n}$ produced by a model with the features we will describe below.

2.2 A collection of candidate behavioral models

We will focus on a setting where the researcher is uncertain about the true behavioral model (e.g, complete-information Nash equilibrium, rationalizability, Nash bargaining, Bayesian Nash equilibrium, etc.), but has pre-specified a collection of *candidate* behavioral models, indexed as $b = 1,\ldots,\mathcal{B}$. We allow for the possibility that different observations in the sample are produced by different behavioral models, or that every observation is produced by the same behavioral model. Each behavioral model b is described by the following features.

2.2.1 Characterization of solutions

The solutions (e.g, Nash equilibria) produced by behavioral model b can be characterized by partitioning $Supp(\varepsilon)$ into $\overline{R}_b$ mutually exclusive *regions*, which we will denote as $\mathcal{R}_{b,1}(X,\theta_0),\ldots,\mathcal{R}_{b,\overline{R}_b}(X,\theta_0)$. Each region can be described (semi)parametrically from our normal-form parameterization and the behavioral features of model b. We will index these regions by $r_b = 1,\ldots,\overline{R}_b$. If $\varepsilon \in \mathcal{R}_{b,r_b}(X,\theta_0)$, the behavioral model has $\overline{S}_{b,r_b} \geq 1$ solutions (e.g, Nash equilibria), which we will index as $s_{r_b} = 1,\ldots,\overline{S}_{b,r_b}$. Each solution s_{r_b} produces a conditional distribution for $Y|(X,\varepsilon)$, denoted by $\sigma_{b,r_b}^{s_{r_b}}(Y|X,\varepsilon,\theta_0)$, so $\sigma_{b,r_b}^{s_{r_b}}(y|X,\varepsilon,\theta_0)$ describes the probability $Pr(Y = y|X,\varepsilon)$ produced by solution s_{r_b}. Each $\sigma_{b,r_b}^{s_{r_b}}(\cdot|X,\varepsilon,\theta_0)$ can be described (semi)parametrically from our normal-form parameterization and the behavioral features of model b.

2.2.2 A collection of sufficient statistics

For each behavioral model b, there exists a collection of functions $g_b(X,\theta_0) \in \mathbb{R}_{d_b}$, which can be described (semi)parametrically from our normal-form parameterization and the behavioral features of the model, such that,

$$\begin{aligned} \mathcal{R}_{b,r_b}(X,\theta_0) &= \mathcal{R}_{b,r_b}(g_b(X,\theta_0)) \ \forall \ r_b = 1,\ldots,\overline{R}_b. \\ \sigma^{s_{r_b}}_{b,r_b}(Y|X,\varepsilon,\theta_0) &= \sigma^{s_{r_b}}_{b,r_b}(Y|g_b(X,\theta_0),\varepsilon) \ \forall \ r_b = 1,\ldots,\overline{R}_b, \ s_{r_b} = 1,\ldots,\overline{S}_{b,r_b}. \end{aligned} \tag{1}$$

Thus, the functions $g_b(X,\theta_0)$ serve as *sufficient statistics* for X in the characterization of the solutions produced by behavioral model b.

3 Examples

Consider the following parameterized 2×2 normal-form game,

	$Y_2 = 1$	$Y_2 = 0$
$Y_1 = 1$	$W_1'\gamma_{10} + Z_1'\Delta_{10} - \varepsilon_1$, $W_2'\gamma_{20} + Z_2'\Delta_{20} - \varepsilon_2$	$W_1'\gamma_{10} - \varepsilon_1$, 0
$Y_1 = 0$	0, $W_2'\gamma_{20} - \varepsilon_2$	0, 0

Group $X \equiv (W_1, Z_1, W_2, Z_2)$ and $\varepsilon \equiv (\varepsilon_1, \varepsilon_2)$. Let $\theta_0 \equiv (\gamma_{10}, \Delta_{10}, \gamma_{20}, \Delta_{20})$. For illustration, suppose the strategic interaction effects satisfy $Z_1'\Delta_{10} \le 0$, $Z_2'\Delta_{20} \le 0$, so we have a game of strategic substitutes. The distribution of ε is nonparametrically specified, but we will maintain that $\varepsilon|X$ has an absolutely continuous distribution with respect to Lebesgue measure. Consider the following collection of possible behavioral models.

3.1 Complete-information Nash equilibrium

Suppose players have complete information (i.e, observe the realization of payoffs in the 2×2 payoff matrix) and that they play Nash equilibrium (NE) strategies, allowing for mixed-strategies. Denote this as behavioral model $b = 1$. The regions, outcome probabilities and sufficient statistics described in equation (1) are given as follows.

3.1.1 Regions

There are five relevant regions, given as follows.

$$
\begin{aligned}
\mathcal{R}_{1,1}(X,\theta_0) &\equiv \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 \le W_1'\gamma_{10} + Z_1'\Delta_{10}\ ,\ \varepsilon_2 > W_2'\gamma_{20} + Z_2'\Delta_{20}\Big\}\\
&\quad \cup \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ W_1'\gamma_{10} + Z_1'\Delta_{10} < \varepsilon_1 \le W_1'\gamma_{10}\ ,\ \varepsilon_2 > W_2'\gamma_{20}\Big\},\\
\mathcal{R}_{1,2}(X,\theta_0) &\equiv \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 > W_1'\gamma_{10}\ ,\ \varepsilon_2 > W_2'\gamma_{20}\Big\},\\
\mathcal{R}_{1,3}(X,\theta_0) &\equiv \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 > W_1'\gamma_{10}\ ,\ \varepsilon_2 \le W_2'\gamma_{20}\Big\}\\
&\quad \cup \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ W_1'\gamma_{10} + Z_1'\Delta_{10} < \varepsilon_1 \le W_1'\gamma_{10}\ ,\ \varepsilon_2 \le W_2'\gamma_{20} + Z_2'\Delta_{20}\Big\},\\
\mathcal{R}_{1,4}(X,\theta_0) &\equiv \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 \le W_1'\gamma_{10} + Z_1'\Delta_{10}\ ,\ \varepsilon_2 \le W_2'\gamma_{20} + Z_{20}'\Delta_{20}\Big\},\\
\mathcal{R}_{1,5}(X,\theta_0) &\equiv \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ W_1'\gamma_{10} + Z_1'\Delta_{10} < \varepsilon_1 \le W_1'\gamma_{10}\ ,\ W_2'\gamma_{20} + Z_2'\Delta_{20} < \varepsilon_2 \le W_2'\gamma_{20}\Big\}.
\end{aligned}
$$

Since the boundaries of these regions have Lebesgue measure zero, the distinction between weak and strict inequalities in their descriptions is not relevant.

3.1.2 Solutions, outcome distributions and sufficient statistics

Regions $\mathcal{R}_{1,1}(X,\theta_0)$, $\mathcal{R}_{1,2}(X,\theta_0)$, $\mathcal{R}_{1,3}(X,\theta_0)$ and $\mathcal{R}_{1,4}(X,\theta_0)$ have a unique pure-strategy Nash equilibrium (PSNE) each, given by $\{(1,0)\}$, $\{(0,0)\}$, $\{(0,1)\}$ and $\{(1,1)\}$, respectively. Region $\mathcal{R}_{1,5}(X,\theta_0)$ has three solutions: PSNE $\{(0,1),(1,0)\}$ and a mixed-strategy Nash equilibrium (MSNE) with mixing probabilities $Pr(Y_1=1) = (\varepsilon_2 - W_2'\gamma_{20})/Z_2'\Delta_{20}$ and $Pr(Y_2 = 1) = (\varepsilon_1 - W_1'\gamma_{10})/Z_1'\Delta_{10}$. The outcome distributions for each solution in this region are $\sigma^1_{1,5}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y=(0,1)\}$, $\sigma^2_{1,5}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y=(1,0)\}$ and,

$$
\sigma^3_{1,5}(y|X,\varepsilon,\theta_0) = \left(\frac{\varepsilon_2 - W_2'\gamma_{20}}{Z_2'\Delta_{20}}\right)^{y_1}\cdot\left(1-\frac{\varepsilon_2 - W_2'\gamma_{20}}{Z_2'\Delta_{20}}\right)^{1-y_1}\cdot\left(\frac{\varepsilon_1 - W_1'\gamma_{10}}{Z_1'\Delta_{10}}\right)^{y_2}\cdot\left(1-\frac{\varepsilon_1 - W_1'\gamma_{10}}{Z_1'\Delta_{10}}\right)^{1-y_2}
$$

By inspection of the regions and solution distributions, a collection of sufficient statistics that satisfy (1) are $g_1(X,\theta_0) = (W_1'\gamma_{10},\ Z_1'\Delta_{10},\ W_2'\gamma_{20},\ Z_2'\Delta_{20})$.

3.2 Rationalizability with pure strategies and complete information

Suppose players choose pure strategies with the only restriction that they are rationalizable, and let us refer to this as behavioral model $b=2$. Using iterated dominance, it is easy to find the relevant regions and the predicted solutions for this game.

3.2.1 Regions, solutions, outcome distributions and sufficient statistics

The relevant regions are the same as those of Nash equilibrium. Furthermore, rationalizability has the same solutions and outcome distributions as Nash equilibrium for regions $r = 1,2,3,4$. Rationalizability has *four* solutions in the multiple-equilibrium region $\mathcal{R}_{2,5}(X,\theta_0)$, given by $\{(0,1),(1,0),(0,0),(1,1)\}$ (every outcome of the game is rationalizable there). Thus, the outcome distributions for the solutions in this region are $\sigma^1_{2,5}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (0,1)\}$, $\sigma^2_{2,5}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (1,0)\}$, $\sigma^3_{2,5}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (0,0)\}$, $\sigma^4_{2,5}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (1,1)\}$. This model has the same sufficient statistics as Nash equilibrium, so $g_2(X,\theta_0) = (W_1'\gamma_{10}, Z_1'\Delta_{10}, W_2'\gamma_{20}, Z_2'\Delta_{20})$.

3.3 Nash bargaining with transferable utility

Suppose behavioral model $b = 3$ assumes cooperative behavior characterized by transferable utility and Nash bargaining. Efficiency predicts then that players choose the outcome Y that maximizes total value (i.e, the sum of players' payoffs).

3.3.1 Regions

This behavioral model has four relevant regions.

$$
\begin{aligned}
\mathcal{R}_{3,1}(X,\theta_0) \equiv &\Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 \le W_1'\gamma_{10} + Z_1'\Delta_{10} + Z_2'\Delta_{20}\ ,\ \varepsilon_2 > W_2'\gamma_{20} + Z_1'\Delta_{10} + Z_2'\Delta_{20}\Big\} \\
&\cup \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ W_1'\gamma_{10} + Z_1'\Delta_{10} + Z_2'\Delta_{20} < \varepsilon_1 \le W_1'\gamma_{10}\ ,\ \varepsilon_2 > W_2'\gamma_{20} - W_1'\gamma_{10} + \varepsilon_1\Big\}, \\
\mathcal{R}_{3,2}(X,\theta_0) \equiv &\Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 > W_1'\gamma_{10}\ ,\ \varepsilon_2 > W_2'\gamma_{20}\Big\}, \\
\mathcal{R}_{3,3}(X,\theta_0) \equiv &\Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 > W_1'\gamma_{10}\ ,\ \varepsilon_2 \le W_2'\gamma_{20}\Big\} \\
&\cup \Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ W_1'\gamma_{10} + Z_1'\Delta_{10} + Z_2'\Delta_{20} < \varepsilon_1 \le W_1'\gamma_{10}\ ,\ \varepsilon_2 \le W_2'\gamma_{20} - W_1'\gamma_{10} + \varepsilon_1\Big\}, \\
\mathcal{R}_{3,4}(X,\theta_0) \equiv &\Big\{(\varepsilon_1,\varepsilon_2)\in\mathbb{R}^2\colon\ \varepsilon_1 \le W_1'\gamma_{10} + Z_1'\Delta_{10} + Z_2'\Delta_{20}\ ,\ \ ,\ \varepsilon_2 \le W_2'\gamma_{20} + Z_1'\Delta_{10} + Z_2'\Delta_{20}\Big\}
\end{aligned}
$$

3.3.2 Solutions, outcome distributions and sufficient statistics

This behavioral model produces a unique solution for each region. Region $\mathcal{R}_{3,1}(X,\theta_0)$ predicts $Y = (1,0)$, so $\sigma^1_{3,1}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (1,0)\}$. Region $\mathcal{R}_{3,2}(X,\theta_0)$ predicts $Y = (0,0)$, and $\sigma^1_{3,2}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (0,0)\}$. The unique solution in region $\mathcal{R}_{3,3}(X,\theta_0)$ is $Y = (0,1)$, yielding $\sigma_{3,3}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (0,1)\}$. The unique solution in region $\mathcal{R}_{3,4}(X,\theta_0)$ is $(1,1)$, so $\sigma_{3,4}(y|X,\varepsilon,\theta_0) = \mathbb{1}\{y = (1,1)\}$. The strategic interaction effects are only relevant here through the combined effect $Z_1'\Delta_{10} + Z_2'\Delta_{20}$. The requirements in (1) are satisfied in this model by the collection of sufficient statistics $g_3(X,\theta_0) = (W_1'\gamma_{10}, W_2'\gamma_{20}, Z_1'\Delta_{10} + Z_2'\Delta_{20})$.

3.4 Incomplete-information Bayesian Nash equilibrium

Suppose behavioral model $b = 4$ assumes that X is observed by both players, but $(\varepsilon_1, \varepsilon_2)$ are private information. Suppose player p conditions her beliefs on X and on the event that $Y_p = 1$, and that beliefs solve Bayesian Nash equilibrium (BNE) self-consistency conditions. As shown in Aradillas-López (2010), assuming that p conditions her beliefs on the event $Y_p = 1$ rather than directly on the realization of ε_p allows for the joint distribution of $(\varepsilon_1, \varepsilon_2)$ to be left nonparametrically specified. For a given X, let $\{(\pi^*_{1s}(X, \theta_0), \pi^*_{2s}(X, \theta_0)\}^{\overline{S}}$ denote all $\overline{S} \geq 1$ solutions for BNE subjective beliefs for $Pr(Y_1 = 1|X, Y_2 = 1)$ and $Pr(Y_2 = 1|X, Y_1 = 1)$, respectively. For each BNE solution s, let $g^s_p(X, \theta_0) \equiv W'_p \gamma_{p0} + (Z'_p \Delta_{p0}) \cdot \pi^*_{-ps}(X, \theta_0)$. If BNE s is selected, player p's behavior would be described as $Y_p = \mathbb{1}\{g^s_p(X, \theta_0) - \varepsilon_p \geq 0\}$.

3.4.1 Regions, solutions, outcome distributions and sufficient statistics

For any X, the relevant region is simply $\mathbb{R}^2$, and the outcome distribution for solution s is $\sigma^s_{4,1}(y|X, \varepsilon, \theta_0) = \prod_{p=1}^2 \mathbb{1}\{g^s_p(X, \theta_0) - \varepsilon_p \geq 0\}^{y_p} \times \mathbb{1}\{g^s_p(X, \theta_0) - \varepsilon_p < 0\}^{(1-y_p)}$. Grouping $g^s(X, \theta_0) \equiv (g^s_1(X, \theta_0), g^s_2(X, \theta_0))$, this model's sufficient statistics are $g(X, \theta_0) \equiv \{g^s(X, \theta_0)\}_{s=1}^{\overline{S}}$, which can be estimated semiparametrically as shown in Aradillas-López (2010).

4 Testable implications with multiple candidate solutions

Instead of assuming a single behavioral model, we pre-specify a collection of candidate behavioral models, $b = 1, \ldots \mathcal{B}$, and group their sufficient statistics $g(X, \theta_0) \equiv \cup_{b=1}^{\mathcal{B}} g_b(X, \theta_0)$.

4.1 Introducing behavioral and solution-selection mechanisms

There is an unobserved behavior selection mechanism $\xi \in \{1, \ldots, \mathcal{B}\}$, where $\xi = b$ indicates that behavioral model b has been selected before players make their choices. For each behavioral model b, if $\xi = b$ and $\varepsilon \in \mathcal{R}_{b,r_b}(g_b(X, \theta_0))$, a solution selection mechanism λ_{b,r_b} selects one among the existing solutions inside region $\mathcal{R}_{b,r_b}(g_b(X, \theta_0))$. Thus, $\lambda_{b,r_b} \in \{1, \ldots, \overline{S}_{b,r_b}\}$, where $\lambda_{b,r_b} = s_{r_b}$ indicates that solution s_{r_b} has been selected. For any $y \in \mathcal{Y}$, our model then yields,

$$\mathbb{1}\{Y = y\} = \sum_{b=1}^{\mathcal{B}} \mathbb{1}\{\xi = b\} \sum_{r_b=1}^{\overline{R}_b} \mathbb{1}\{\varepsilon \in \mathcal{R}_{b,r_b}(g(X, \theta_0))\} \sum_{s_{r_b}=1}^{\overline{S}_{b,r_b}} \mathbb{1}\{\lambda_{b,r_b} = s_{r_b}\} \cdot \mathbb{1}\{Y_{(s_{r_b})} = y\}, \tag{2}$$

where $Y_{(s_{r_b})}$ is the (potential) outcome of solution s_{r_b}

4.2 An exclusion restriction that yields a multiple index model

Group $\lambda \equiv \{\lambda_{b,r_b}: b = 1,\ldots,\mathcal{B}, r_b = 1,\ldots,\overline{R}_b\}$. While the the structural properties of the game depend on X only through $g(X,\theta_0)$, to obtain testable implications we will assume that $g(X,\theta_0)$ is also a sufficient statistic for the distribution of $(\varepsilon,\xi,\lambda)|X$.

Restriction R1 (an exclusion restriction) The distribution of $(\varepsilon,\xi,\lambda)|X$ is unknown, but it satisfies the following exclusion restrictions. $\varepsilon|(\xi,X) \sim \varepsilon|g(X,\theta_0)$, $\xi|X \sim \xi|g(X,\theta_0)$, and $\lambda|(\xi,\varepsilon,X) \sim \lambda|(\xi,g(X,\theta_0))$.

The next result follows from Restriction R1 and the structure of our general model.

Proposition 1 *Let* $\omega_{b,r_b}^{s_{r_b}}(g(X,\theta_0)) \equiv Pr\Big(\xi = b,\ \lambda_{b,r_b} = s_{r_b} \,|\, g(X,\theta_0)\Big)$*, and for any* $y \in \mathcal{Y}$*, let*

$$\eta_{b,r_b}^{s_{r_b}}(y|g(X,\theta_0)) \equiv E[\mathbb{1}\{\varepsilon \in \mathcal{R}_{b,r_b}(g(X,\theta_0))\} \cdot \sigma_{b,r_b}^{s_{r_b}}(y|g_b(X,\theta_0),\varepsilon) \mid g(X,\theta_0)]$$

In a model satisfying the structural properties in Section 2 and Restriction R1,

$$Pr(Y=y|X) = Pr(Y=y|g(X,\theta_0)) = \sum_{b=1}^{\mathcal{B}} \sum_{r_b=1}^{\overline{R}_b} \sum_{s_{r_b}=1}^{\overline{S}_{b,r_b}} \omega_{b,r_b}^{s_{r_b}}(g(X,\theta_0)) \cdot \eta_{b,r_b}^{s_{r_b}}(y|g(X,\theta_0)) \quad \forall\, y \in \mathcal{Y} \tag{3}$$

The proof follows by using iterated expectations and equations (1) and (2).

4.2.1 A weaker version of Restriction R1

We can assume that the exclusion restrictions described hold conditional on an observable "instrument" Z in addition to our sufficient statistics. In the end, this would modify (3) to $Pr(Y=y|X,Z) = Pr(Y=y|g(X,\theta_0),Z)$.

5 Identification and estimation of θ_0

Once we arrive at the result in (3), ours becomes a special case of a semiparametric multiple index model. As such, identification and estimation of θ_0 can be approached using a number of existing methods, including semiparametric least squares (Ichimura and Lee (1991), Ichimura (1993), Donkers and Schafgans (2008)) or pairwise-difference methods (Honoré and Powell (1994), Honoré and Powell (2005), Aradillas-López, Honoré, and Powell (2007), Aradillas-López (2012)).

5.1 Identifiability of θ_0

Like any other multiple index model, the exclusion restriction in (3) may not be enough to identify θ_0 if any of our candidate behavioral models b is such that $E[g_b(X,\theta_0)|g_b(X,\theta)] = g_b(X,\theta_0)$ w.p.1 for some $\theta \neq \theta_0$. In this case, our first step would be to restrict the parameter space Θ. This is equivalent to the well known location and scale normalizations in semiparametric linear index models. A restriction of Θ is a transformation (reparameterization) $\beta(\cdot)$ that maps each $\theta \in \Theta$ to a lower-dimensional parameter $\beta(\theta) \equiv \beta$. Following the reparameterization, our sufficient statistics are $g(X,\beta(\theta)) \equiv \cup_{b=1}^{\mathcal{B}} g_b(X,\beta(\theta))$. Local identifiability of $\beta_0 \equiv \beta(\theta_0)$ would require that, $\forall\ \theta \in \Theta$, $\exists\ \epsilon > 0$ such that, for each candidate behavioral model b,

$$Pr\Big(E[g_b(X,\beta(\theta))|g_b(X,\beta(\theta'))] \neq g_b(X,\beta(\theta))\Big) > 0 \quad \forall\ \theta' \neq \theta\colon \|\theta - \theta'\| < \epsilon. \tag{4}$$

If we maintain that a subset of candidate behavioral models occur with strictly positive probability in the data, (4) would only need to be satisfied for this subset, which could potentially allow us to identify more features of θ_0. Inspecting the sufficient statistics of the behavioral examples in Section 3, we can see that assuming that BNE occurs in the data with strictly positive probability can potentially identify more features about θ_0, such as a constant strategic-interaction effects.

5.2 Estimation

Following any necessary reparameterization and denoting $\beta(\theta) \equiv \beta$, estimation of $\beta_0 \equiv \beta(\theta_0)$ can proceed applying a number of existing estimation methods suited for semiparametric multiple index models (cited above). The indices in this case are the the sufficient statistics of each candidate behavioral model, which are semiparametrically estimable for any β, and given by $\widehat{g}(X,\beta) \equiv \cup_{b=1}^{\mathcal{B}} \widehat{g}_b(X,\beta)$. Consistent specification tests can be pursued adapting methods suited for multiple index models, which include Fan and Li (1996), Zheng (1996) and Aradillas-López (2012, Section 4).

6 Concluding remarks

The structure of a fairly general class of parameterized normal-form games, combined with testable exclusion restrictions involving unobserved selection mechanisms, allow us to treat their estimation as that of a multiple index model while remaining robust to the existence of multiple solutions and multiple candidate behavioral models.

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