Nonparametric tests for strategic interaction effects with rationalizability

Andrés Aradillas-López*

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Abstract

We introduce the first nonparametric tests for the presence and the sign of strategic interaction effects in discrete 2×2 games of complete information under the assumption of rationalizable behavior, which includes Nash Equilibrium as a special case but allows for incorrect beliefs. Our tests assume the existence of an observable covariate with a positive stochastic relationship with the payoffs of a particular player.

Keywords: *Nonparametric testing; discrete games; rationalizability.*
JEL Codes: C14, C57.

1 A 2×2 game

Consider the following normal-form game.

	$Y_2 = 1$	$Y_2 = 0$
$Y_1 = 1$	$T_1 + D_1$, $T_2 + D_2$	T_1, 0
$Y_1 = 0$	0, T_2	0, 0

We will treat (T_1, D_1, T_2, D_2) as unobserved random variables. D_j and T_j measure the strategic and the non-strategic portions of player j's payoffs, respectively. We refer to (D_1, D_2) as the strategic-interaction effects.

2×2 static simultaneous games were first analyzed econometrically in Bjorn and Vuong (1984). Other well known papers in the literature which focused on them include Bresnahan and Reiss (1990), Tamer (2003) and many others. This body of work focuses on either parametric models and/or on the assumption of Nash Equilibrium (NE) behavior. We present here the first nonparametric tests that do not rely on (but allow for) NE behavior. Our discussion will provide a roadmap of what would be required to extend our results beyond 2×2 games.

*email: aaradill@psu.edu. Pennsylvania State University. Department of Economics, 518 Kern Graduate Building, University Park, PA 16802

2 Rationalizable actions

Let us maintain expected-utility maximizing players and let us predict rational behavior based on the following assumptions:

1. The players in the game are rational in the sense that they do not play dominated strategies.

2. Each player believes that the other player is rational.

3. Each player believes that the other player believes this, and so on ad infinitum.

The strategies that survive this iterative thinking process are rationalizable strategies, consistent with common knowledge of rationality.

Assumption 1 *The realization of* (T_1, T_2, D_1, D_2) *and the normal-form payoffs are known to both players (i.e, the game is played with complete information). Players choose their actions simultaneously. Players are allowed to randomize their actions but player* j *is assumed to play* $Y_j \in \{0, 1\}$ *with nonzero probability only if* Y_j *is rationalizable.*

Assumption 1 includes Nash Equilibrium (NE) as a special case, but it allows for incorrect beliefs as long as they are consistent with rationalizability. This solution concept was analyzed in Aradillas-López and Tamer (2008) in the context of parametric models. To my knowledge, this paper contains the first nonparametric testable implications in discrete games based solely on rationalizability as opposed to Nash Equilibrium behavior.

Denote $(V)_+ \equiv \max\{V, 0\}$ and $(V)_- \equiv \min\{V, 0\}$. $Y_j = 1$ is dominated for player p iff $T_j + (D_j)_+ < 0$. $Y_j = 0$ is dominated iff $T_j + (D_j)_- > 0$. If $T_j + (D_j)_+ \geq 0$ and $T_j + (D_j)_- \leq 0$ for $p = \{1, 2\}$, both actions are rationalizable for each player and all four outcomes of the game are rationalizable.

Suppose $Y_j + (D_j)_+ < 0$. Then $Y_j = 1$ is dominated for player p. In this case, the rationalizable actions are:

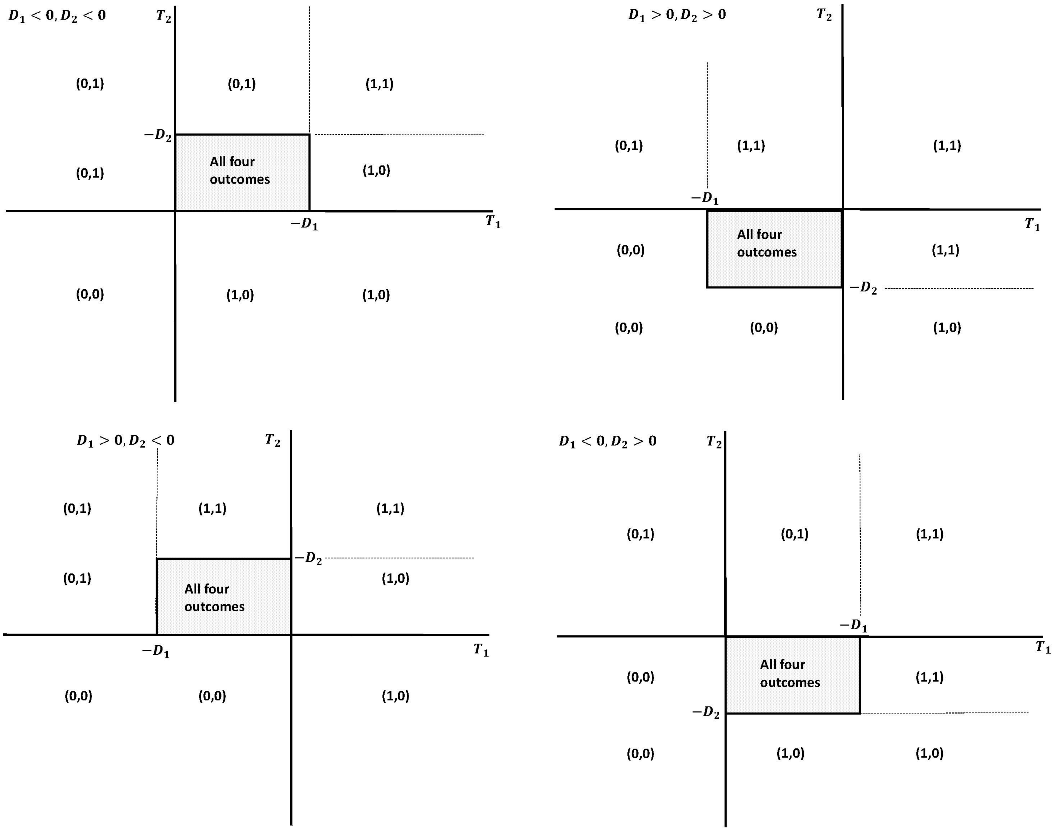
- $(Y_j = 0, Y_\ell = 1)$ if $T_\ell > 0$,
- $(Y_j = 0, Y_\ell = 0)$ if $T_\ell < 0$,
- $(Y_j = 0, Y_\ell = 0)$ and $(Y_j = 0, Y_\ell = 1)$ if $T_\ell = 0$.

Suppose $Y_j + (D_j)_- > 0$. Then $Y_j = 0$ is dominated for player p. In this case, the rationalizable actions are:

- $(Y_j = 1, Y_\ell = 1)$ if $T_\ell + D_\ell > 0$,
- $(Y_j = 1, Y_\ell = 0)$ if $T_\ell + D_\ell < 0$,
- $(Y_j = 1, Y_\ell = 0)$ and $(Y_j = 1, Y_\ell = 1)$ if $T_\ell + D_\ell = 0$.

Figure 1 summarizes the regions of rationalizable actions in the (T_1, T_2) space.

Figure 1: Rationalizable actions



3 Observables

We focus on a setting where the researcher observes the outcome of the game (Y_1, Y_2) and a collection of covariates (X, Z_1) related to players' unobserved payoffs and strategic-interaction effects. $Z_1 \in \mathbb{R}$ is assumed to have a positive stochastic relationship with T_1 in a way described below. Henceforth $\text{Supp}(U)$ denotes the support of the random variable U. Lower case u denotes a particular value of the random variable U.

Assumption 2

(i) T_1, T_2 *are jointly continuously distributed conditional on* (D_2, D_1, X, Z_1).

(ii) Let $H_1(\cdot|T_2, D_2, D_1, X, Z_1)$ *denote the cdf of* T_1 *conditional on* (T_2, D_2, D_1, X, Z_1). *With probability one (w.p.1) in* (T_2, D_2, D_1, X, Z_1),

$$0 < H_1(0|T_2, D_2, D_1, X, Z_1) < 1, \quad \textit{and} \quad 0 < H_1(-D_1|T_2, D_2, D_1, X, Z_1) < 1.$$

(iii) For almost every (a.e) $(t_1, t_2, d_2, d_1, x) \in Supp\,(T_1, T_2, D_2, D_1, X)$ *and a.e* $z_1, z_1' \in Supp(Z_1)$,

$$z_1 > z_1' \implies H_1(t_1|t_2, d_2, d_1, x, z_1) < H_1(t_1|t_2, d_2, d_1, x, z_1').$$

That is, $H_1(t_1|t_2, d_2, d_1, x, z_1)$ *is strictly decreasing in* z_1 *for all* $z_1 \in Supp(Z_1)$.

(iv) $T_2, D_2|X, Z_1, D_1 \sim T_2, D_2|X, D_1$ *and* $D_1|X, Z_1 \sim D_1|X$

(v) W.p.1 in X: $Pr\,(T_2 < (-D_2)_-|X) > 0$, $Pr\,(T_2 > (-D_2)_+|X) > 0$ *and, unless* $Pr\,(D_2 = 0|X) = 1$, *we also have* $Pr\,((-D_2)_- \leq T_2 \leq (-D_2)_+|X) > 0$.

Example 1 Suppose $T_1 = m_1(X_1, Z_1) + \varepsilon_1$ and $T_2 = m_2(X_2) + \varepsilon_2$. Let $X \equiv (X_1, X_2)$ and suppose,

- *For a.e* $x_1 \in Supp(X_1)$, $m_1(x_1, z_1)$ is strictly increasing in z_1 for a.e $z_1 \in \text{Supp}(Z_1)$.
- $\text{Supp}(T_1) = \text{Supp}(\varepsilon_1)$ and $\text{Supp}(T_2) = \text{Supp}(\varepsilon_2)$ (e.g, $\text{Supp}(\varepsilon_j) = \mathbb{R}$).
- $\varepsilon_1, \varepsilon_2|X, Z_1, D_1, D_2 \sim \varepsilon_1, \varepsilon_2|X, D_1, D_2$. Let $F_{\varepsilon_1|\varepsilon_2,X,D_1,D_2}(\cdot|\varepsilon_2, X, D_1, D_2)$ denote the cdf of $\varepsilon_1|\varepsilon_2, X, D_1, D_2$.
- $F_{\varepsilon_1|\varepsilon_2,X,D_1,D_2}(\,\cdot\,|\varepsilon_2, X, D_1, D_2)$ is strictly increasing everywhere on $\text{Supp}(\varepsilon_1)$.

In this case, we have

$$H_1(t_1|t_2, d_2, d_1, x, z_1) = F_{\varepsilon_1|\varepsilon_2,X,D_1,D_2}\left(t_1 - m_1(x, z_1)|t_2 - m_2(x_2), x, d_1, d_2\right),$$

which is decreasing in z_1 for all $z_1 \in \text{Supp}(Z_1)$. This example is compatible with the commonly assumed parametrization $T_1 = X_1'\beta_1 + Z_1 \cdot \beta_1^z + D_1 + \varepsilon_1$ and $T_2 = X_2'\beta_2 + D_2 + \varepsilon_2$, where (D_1, D_2) are fixed strategic-interaction parameters. Assumption 2(iii) presupposes that $\beta_1^z > 0$.

4 Testable implications of strategic-interaction effects

Let $\mathcal{R} = [(-D_1)_-\,,\,(-D_1)_+] \times [(-D_2)_-\,,\,(-D_2)_+]$. From Figure 1, all four outcomes are rationalizable when $(T_1, T_2) \in \mathcal{R}$. For $j = 1, 2$, let

$$\pi_j(X, Z_1) = Pr\,(Y_j = 1|(T_1, T_2) \in \mathcal{R},\, X, ,\, Z_1)\,.$$

(π_1, π_2) summarize players' rationalizable selection mechanism in $\mathcal{R}$. We make no assumptions about it. Denote

$$P_j(X, Z_1) = Pr(Y_j = 1|X, Z_1), \quad \text{and} \quad P_j(X) = Pr(Y_j = 1|X).$$

Fix $(x, z_1) \in \text{Supp}(X, Z_1)$. By Assumptions 1-2, we have

$$
\begin{aligned}
&P_1(x, z_1) = \\
&1 - E\left[H_1\left(-D_1 | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{T_2 > (-D_2)_+\right\} | X = x\right] \\
&- E\left[H_1\left(0 | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{T_2 < (-D_2)_-\right\} | X = x\right] \\
&- E\left[H_1\left((-D_1)_+ | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{(-D_2)_- \leq T_2 \leq (-D_2)_+\right\} | X = x\right] \times \Big(1 - \pi_1(x, z_1)\Big) \\
&- E\left[H_1\left((-D_1)_- | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{(-D_2)_- \leq T_2 \leq (-D_2)_+\right\} | X = x\right] \times \pi_1(x, z_1), \\
\\
&P_2(x, z_1) = \\
&Pr(T_2 > -D_2 | X = x) \\
&+ \Big\{E\left[H_1\left((-D_1)_- | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{0 \leq T_2 \leq -D_2\right\} | X = x\right] \\
&\quad - E\left[H_1\left((-D_1)_+ | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{-D_2 \leq T_2 \leq 0\right\} | X = x\right]\Big\} \times \Big(1 - \pi_2(x, z_1)\Big) \\
&+ \Big\{E\left[H_1\left((-D_1)_+ | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{0 \leq T_2 \leq -D_2\right\} | X = x\right] \\
&\quad - E\left[H_1\left((-D_1)_- | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{-D_2 \leq T_2 \leq 0\right\} | X = x\right]\Big\} \times \pi_2(x, z_1).
\end{aligned} \tag{4.1}
$$

Result 1 ***Implications of $D_2 = 0$ on $P_2(x, z_1)$:*** *If $Pr(D_2 = 0 | X = x) = 1$, then $P_2(x, z_1) = P_2(x)\ \forall\ z_1$. In particular, if $Pr(D_2 = 0) = 1$, then $P_2(X, Z_1) = P_2(X)$. Conversely, $P_2(X, Z_1) \neq P_2(X)$ implies $Pr(D_2 \neq 0) > 0$. However, without further assumptions, having $P_2(x, Z_1) = P_2(x)$ does not necessarily imply $Pr(D_2 = 0 | X = x) = 1$. We will describe below a sufficient condition (Assumption 3) that will ensure that, if $P_2(x, Z_1) = P_2(x)$, then we must have $Pr(D_2 = 0 | X = x) = 1$.*

Result 2 ***Implications of $D_2 \leq 0$ on $P_2(x, z_1)$:*** *Suppose $Pr(D_2 \leq 0 | X = x) = 1$. In this case (4.1) becomes*

$$
\begin{aligned}
P_2(x, z_1) = &Pr(T_2 > -D_2 | X = x) \\
&+ E\left[H_1\left((-D_1)_- | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{0 \leq T_2 \leq -D_2\right\} | X = x\right] \times \big(1 - \pi_2(x, z_1)\big) \\
&+ E\left[H_1\left((-D_1)_+ | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{0 \leq T_2 \leq -D_2\right\} | X = x\right] \times \pi_2(x, z_1).
\end{aligned}
$$

Since $\pi_2(x, z_1) \in [0, 1]$, lower and upper bounds for $P_2(x, z_1)$ are given by,

$$
\begin{aligned}
\underline{P}_2(x, z_1) &\equiv Pr(T_2 > -D_2 | X = x) + E\left[H_1\left((-D_1)_- | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{0 \leq T_2 \leq -D_2\right\} | X = x\right], \\
\overline{P}_2(x, z_1) &\equiv Pr(T_2 > -D_2 | X = x) + E\left[H_1\left((-D_1)_+ | T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{0 \leq T_2 \leq -D_2\right\} | X = x\right].
\end{aligned} \tag{4.2}
$$

- *If* $Pr(D_1 = 0|X = x) = 1$, *then* $P_2(x, z_1)$ *is strictly decreasing in* z_1. *In particular, if* $Pr(D_1 = 0) = 1$, *then* $P_2(X, z_1)$ *is strictly decreasing in* z_1 *for a.e* X.

- *If* $Pr(D_1 \neq 0|X = x) > 0$, *then without further restrictions,* $P_2(x, z_1)$ *can be decreasing or increasing in* z_1 *everywhere on* $Supp(Z_1)$. *More precisely, we can always characterize a rationalizable selection mechanism* π_2 *such that* $P_2(x, z_1)$ *is increasing in* z_1 $\forall\ z_1 \in Supp(Z_1)$ *and a selection mechanism* π_2 *such that* $P_2(x, z_1)$ *is decreasing in* z_1 $\forall\ z_1 \in Supp(Z_1)$.

However, we can obtain a more definitive monotonicity result for $P_2(x, z_1)$ *as a function of* z_1 *under the following assumption.*

Assumption 3 (x, z_1) *is such that,* $\exists\ b_1 > 0$*:* $\forall\ b \geq b_1$ *where* $z_1 + b \in Supp(Z_1)$,

$$H_1((-D_1)_+|T_2, D_2, D_1, x, z_1 + b) < H_1((-D_1)_-|T_2, D_2, D_1, x, z_1) \text{ for a.e } (T_2, D_2, D_1).$$

Example 1 (continued).- Assumption 3 would be satisfied if $\exists b_1 > 0$ such that $m_1(x_1, z_1 + b_1) - m_1(x_1, z_1) > (-D_1)_+ - (-D_1)_-\ \forall\ D_1 \in \text{Supp}(D_1)$ and $z_1 + b_1 \in \text{int}(\text{Supp}(Z_1))$. If $|D_1| \leq \overline{D}_1$ w.p.1, then Assumption 3 would be satisfied if $\exists b_1 > 0$ such that $m_1(x_1, z_1 + b_1) - m_1(x_1, z_1) > \overline{D}_1$. In the usual parametrization $T_1 = X_1'\beta_1 + Z_1 \cdot \beta_1^z + D_1 + \varepsilon_1$, with D_1 a fixed parameter, Assumption 3 would be satisfied for a given z_1 if $z_1 + \frac{|D_1|}{\beta_1^z} \in \text{Supp}(Z_1)$. In particular, if $\text{Supp}(Z_1)$ has no upper bound, then Assumption 3 would be satisfied for a.e z_1.

From the bounds in (4.2)*, Assumption 3 implies the following,*

- *If* $Pr(D_2 \leq 0|X = x) = 1$ *and* $Pr(D_2 < 0|X = x) > 0$, *then* $P_2(x, z_1 + b) < P_2(x, z_1)$ $\forall\ b \geq b_1$*:* $z_1 + b \in Supp(Z_1)$. *In particular, if* $Pr(D_2 \leq 0) = 1$ *and there exists a range of values* (x, z_1) *with positive probability measure where Assumption 3 holds and* $Pr(D_2 < 0|X = x) > 0$, *then there must exist* x *and* $z_1' > z_1$ *such that* $P_2(x, z_1') < P_2(x, z_1)$.

Result 3 ***Implications of*** $\boldsymbol{D_2 \geq 0}$ ***on*** $\boldsymbol{P_2(x, z_1)}$***:*** *Suppose* $Pr(D_2 \geq 0|X = x) = 1$. *Now* (4.1) *becomes*

$$\begin{aligned} P_2(x, z_1) = {} & Pr(T_2 > -D_2|X = x) \\ & - E\left[H_1\left((-D_1)_+|T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{-D_2 \leq T_2 \leq 0\right\}|X = x\right] \times \big(1 - \pi_2(x, z_1)\big) \\ & \left.- E\left[H_1\left((-D_1)_-|T_2, D_2, D_1, x, z_1\right) \cdot \mathbb{1}\left\{-D_2 \leq T_2 \leq 0\right\}|X = x\right]\right\} \times \pi_2(x, z_1). \end{aligned}$$

- *If* $Pr(D_1 = 0|X = x) = 1$, *then* $P_2(x, z_1)$ *is strictly increasing in* z_1. *In particular, if* $Pr(D_1 = 0) = 1$, *then* $P_2(X, z_1)$ *is strictly increasing in* z_1 *for a.e* X.

- *If $Pr(D_1 \neq 0|X = x) > 0$, $P_2(x, z_1)$ can be increasing or decreasing in z_1. However, if $Pr(D_2 \geq 0) = 1$ and Assumption 3 holds over a range of values (x, z_1) such that $Pr(D_2 > 0|X = x) > 0$, then there must exist x and $z_1' > z_1$ such that $P_2(x, z_1') > P_2(x, z_1)$.*

Result 4 ***Implications of D_1 and D_2 on $P_1(x, z_1)$***

- *If $Pr(D_1 = 0|X = x) = 1$ or $Pr(D_2 = 0|X = x) = 1$, then $P_1(x, z_1)$ is strictly increasing in z_1. In particular, if $Pr(D_1 = 0) = 1$ or $Pr(D_2 = 0) = 1$, then $P_1(X, z_1)$ is strictly increasing in z_1 for a.e X.*

- *$P_1(x, z_1)$ is not strictly decreasing in z_1 only if $Pr(D_1 \neq 0|X = x) > 0$ and $Pr(D_2 \neq 0|X = x) > 0$. In particular, $P_1(X, z_1)$ is not strictly increasing in z_1 only if $Pr(D_1 \neq 0$ and $D_2 \neq 0) > 0$.*

- *Unlike $P_2(x, z_1)$, the specific signs of D_1 and D_2 do not have qualitatively different implications for $P_1(x, z_1)$. The monotonicity of $P_1(x, z_1)$ with respect to z_1 can only help us infer whether strategic interaction is present, but not its sign.*

5 A menu of econometric tests

Our previous results provide a roadmap for econometric tests about different conjectures of strategic-interaction effects. These tests involve nonparametric functional equalities and/or inequalities, as well as exclusion restrictions, all of which can be implemented using existing econometric methods (e.g:Lee, Song, and Whang (2013), Aradillas-López, Gandhi, and Quint (2016), Lee, Song, and Whang (2018) for inequality tests, Fan and Li (1996) for equalities and exclusion restrictions).

5.1 Tests under Assumptions 1 and 2

A sufficient condition to determine the presence of strategic-interaction effects

Consider the following null hypothesis

$$H_0 : P_1(X, z_1') > P_1(X, z_1) \; \forall \; z_1' > z_1 \in \text{Supp}(Z_1), \; \text{a.e in } X.$$

Under Assumptions 1 and 2, H_0 can be rejected *only if* $Pr(D_1 \times D_2 \neq 0) > 0$. That is, rejection of H_0 reveals the presence of strategic-interaction effects for both players.

A test for $D_2 = 0$

Under Assumptions 1-2, a test of the conjecture $Pr(D_2 = 0|X = x) = 1$ can be done by testing the joint null hypothesis

$$H_0 : \begin{cases} P_2(x, z_1) = P_2(x) \ \forall \ z_1 \in \text{Supp}(Z_1), \\ P_1(x, z_1') > P_1(x, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1). \end{cases}$$

A test of the conjecture $Pr(D_2 = 0) = 1$ can be done by testing the joint null hypothesis

$$H_0 : \begin{cases} P_2(X, Z_1) = P_2(X), \\ P_1(X, z_1') > P_1(X, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1), \ \text{a.e in } X \end{cases}$$

A test for $D_1 = 0$

Under Assumptions 1-2, a test of the conjecture $Pr(D_1 = 0|X = x) = 1$ can be done by testing the null hypothesis

$$H_0 : P_1(x, z_1') > P_1(x, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1).$$

A test of the conjecture $Pr(D_1 = 0) = 1$ can be done by testing

$$H_0 : P_1(X, z_1') > P_1(X, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1), \ \text{a.e in } X.$$

A test for $D_1 = 0$ and $D_2 \leq 0$

Under Assumptions 1 and 2, a test for the conjecture $Pr(D_1 = 0, D_2 \leq 0|X = x) = 1$ can be done by testing the joint null hypothesis

$$H_0 : \begin{cases} P_2(x, z_1') \leq P_2(x, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1), \\ P_1(x, z_1') > P_1(x, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1). \end{cases}$$

A test for the conjecture $Pr(D_1 = 0, D_2 \leq 0) = 1$ can be done by testing the joint null hypothesis

$$H_0 : \begin{cases} P_2(X, z_1') \leq P_2(X, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1), \ \text{a.e in } X, \\ P_1(X, z_1') > P_1(X, z_1) \ \forall \ z_1' > z_1 \in \text{Supp}(Z_1), \ \text{a.e in } X, \end{cases}$$

A test for $D_1 = 0$, $D_2 \geq 0$

Under Assumptions 1 and 2, a test for the conjecture $Pr(D_1 = 0, D_2 \geq 0|X = x) = 1$ can be done by testing the joint null hypothesis

$$H_0 : \begin{cases} P_2(x, z_1') \geq P_2(x, z_1) \;\forall\; z_1' > z_1 \in \text{Supp}(Z_1), \\ P_1(x, z_1') > P_1(x, z_1) \;\forall\; z_1' > z_1 \in \text{Supp}(Z_1). \end{cases}$$

A test for the conjecture $Pr(D_1 = 0, D_2 \geq 0) = 1$ can be done by testing the joint null hypothesis

$$H_0 : \begin{cases} P_2(X, z_1') \geq P_2(X, z_1) \;\forall\; z_1' > z_1 \in \text{Supp}(Z_1), \text{ a.e in } X, \\ P_1(X, z_1') > P_1(X, z_1) \;\forall\; z_1' > z_1 \in \text{Supp}(Z_1), \text{ a.e in } X, \end{cases}$$

5.2 Tests under Assumptions 1, 2 and 3

A test to reject Assumptions 1- 3

Consider the null hypothesis

$$H_0 : P_1(X, z_1') \leq P_1(X, z_1) \;\forall\; z_1' > z_1 \in \text{Supp}(Z_1), \text{ a.e in } X.$$

Failure to reject H_0 would invalidate the joint validity of Assumptions 1- 3. In other words, there would not exist any range of values (x, z_1) such that all three assumptions are satisfied. These conditions would be invalidated for a particular (x, z_1) if $P_1(x, z_1') \leq P_1(x, z_1) \;\forall\; z_1' > z_1$.

A test for $D_2 < 0$

Take a given (x, z_1) and consider the conjecture of a strategic-substitute effect on Player 2: H^s : $Pr(D_2 \leq 0|X = x) = 1$, $Pr(D_2 < 0|X = x) > 0$. Now consider the null hypothesis

$$H_0 : P_2(x, z_1') \geq P_2(x, z_1) \;\forall\; z_1' > z_1.$$

If Assumptions 1- 3 hold for (x, z_1), then failure to reject H_0 would immediately invalidate H^s. More generally, failure to reject the null hypothesis

$$H_0 : P_2(X, z_1') \geq P_2(X, z_1) \;\forall\; z_1' > z_1 \in \text{Supp}(Z_1), \text{ a.e in } X,$$

would immediately reject the possibility that H^s holds for some (x, z_1). There cannot be a strategic-substitute effect on Player 2.

A test for $D_2 > 0$

Take a given (x, z_1) and consider the conjecture of a strategic-complement effect on Player 2: $H^c : Pr(D_2 \geq 0|X = x) = 1$, $Pr(D_2 > 0|X = x) > 0$. Now consider the null hypothesis

$$H_0 : P_2(x, z_1') \leq P_2(x, z_1) \; \forall \; z_1' > z_1.$$

If Assumptions 1- 3 hold for (x, z_1), then failure to reject H_0 would immediately invalidate H^c. More generally, failure to reject the null hypothesis

$$H_0 : P_2(X, z_1') \leq P_2(X, z_1) \; \forall \; z_1' > z_1 \in \text{Supp}(Z_1), \text{ a.e in } X,$$

would immediately reject the possibility that H^c holds for some (x, z_1). There cannot be a strategic-complement effect on Player 2.

5.3 Extensions: tests for mutual strategic substitutes, complements

If there exists an observable covariate z_2 for Player 2 with the same features described in Assumptions 2 and 3 (for z_1 and Player 1), then it is easy to deduce from the above results how we could test conjectures such as mutual-strategic substitute effects ($D_1 < 0, D_2 < 0$) or complement effects ($D_1 > 0, D_2 > 0$).

6 Concluding remarks

We described nonparametric tests for the presence and the sign of strategic interaction effects in 2×2 games of complete information under the basic assumption of rationalizable behavior and the presence of a regressor with a special type of positive stochastic relationship one of the players' payoffs. Our assumptions are testable. Extensions beyond the 2×2 can be undertaken once the regions of rationalizable choices are characterized. In such cases, our tests can be extended provided that there exist regressors positively associated with the payoffs of a subset of players in a way analogous to the one described here.

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